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Phase Plane Rotation as Variance Reduction Method in Monte Carlo Simulations of Axial-Symmetric Radiation Sources

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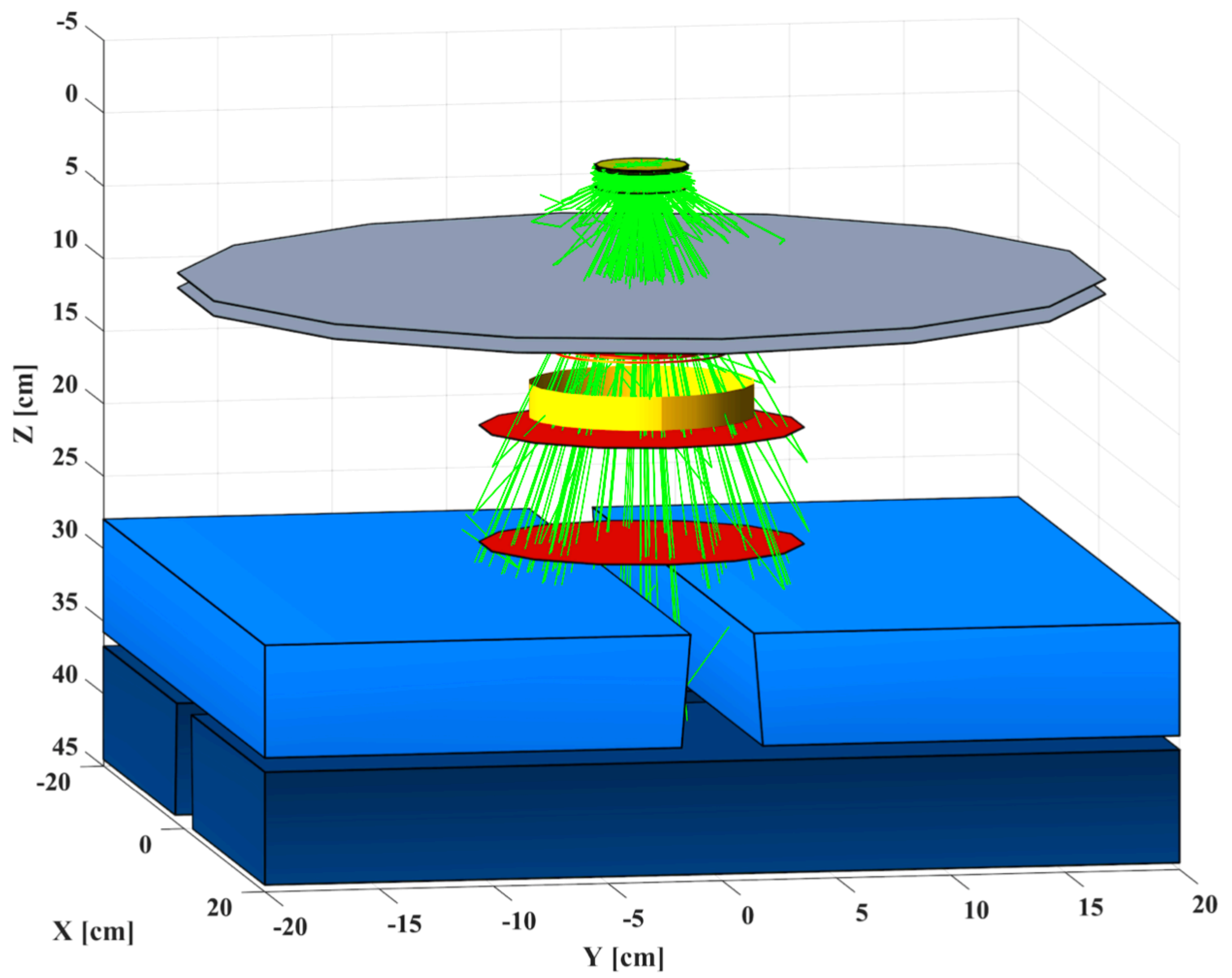
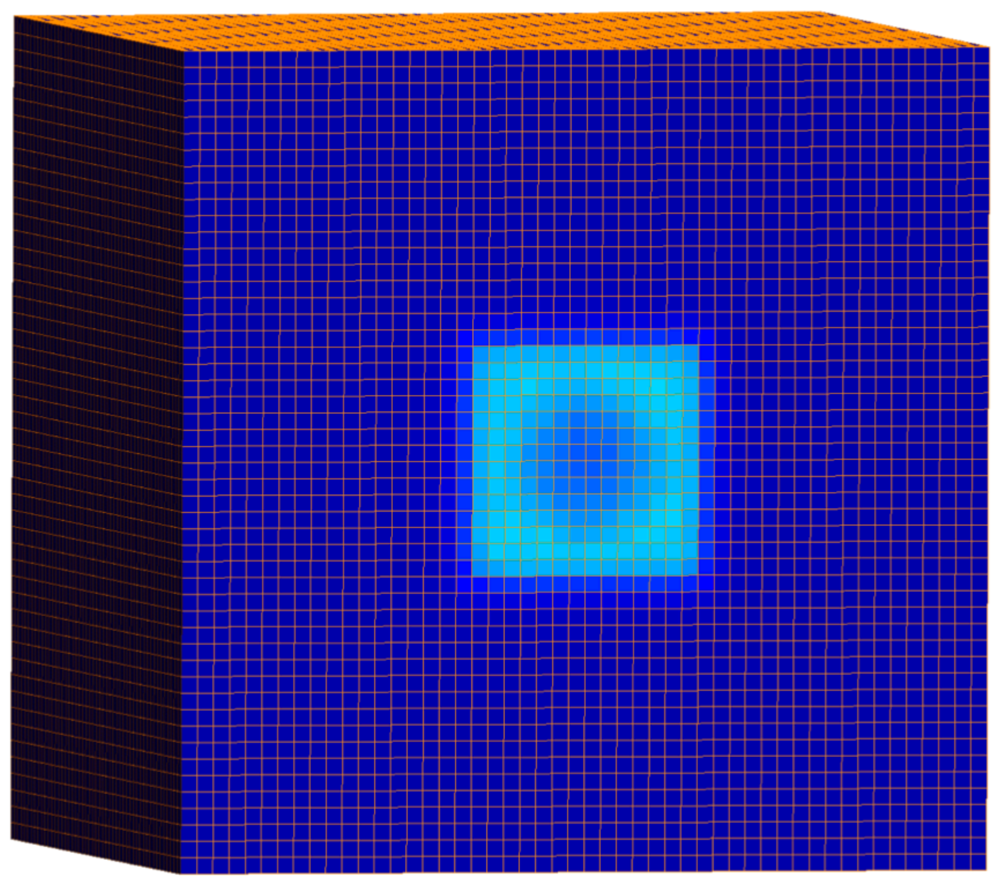
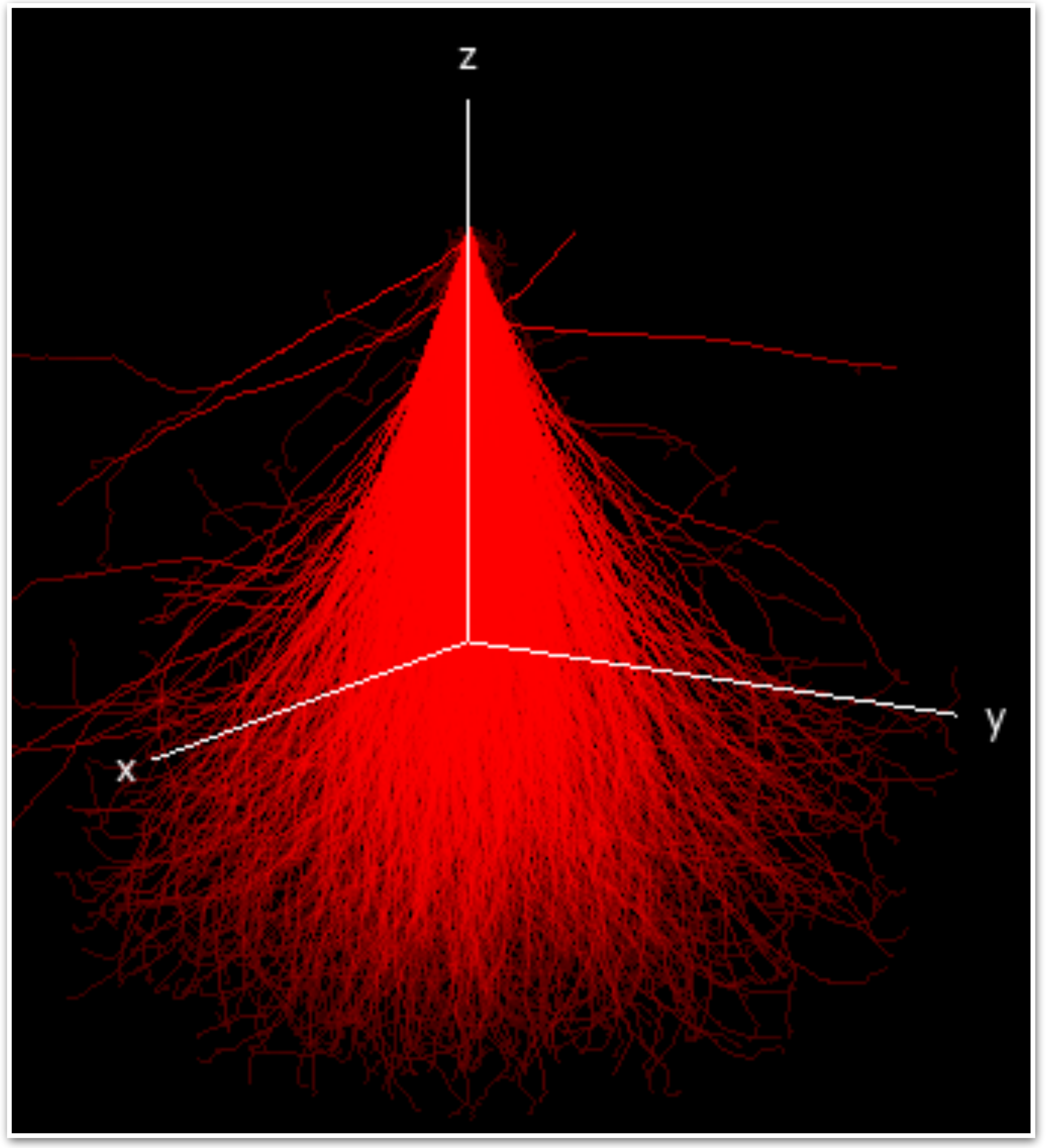
Monte Carlo Dosimetry

- Using the Monte Carlo method to simulate radiation transport and calculate dose
- *EGSnrc, GEANT4, PENELOPE, MNCP…*

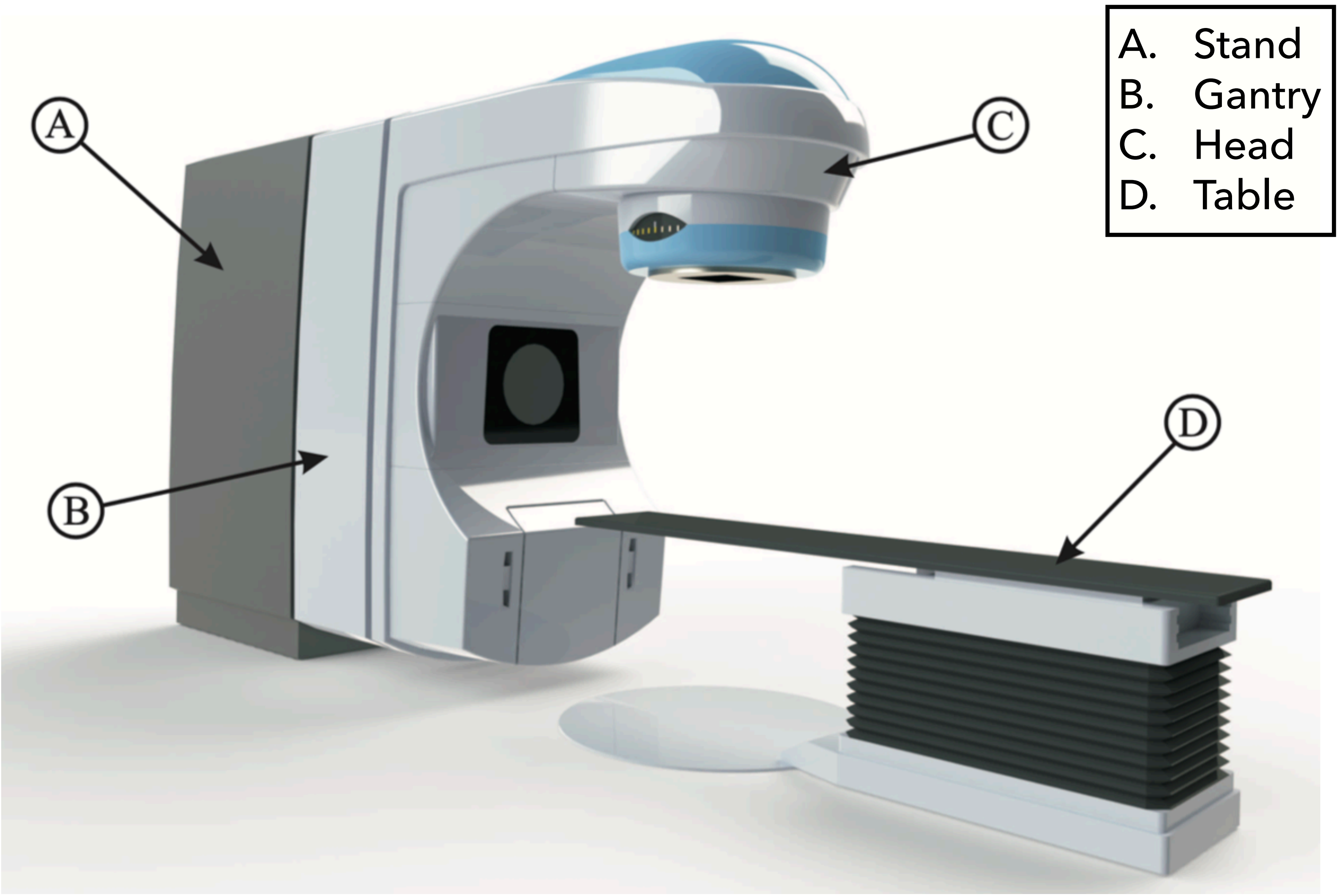






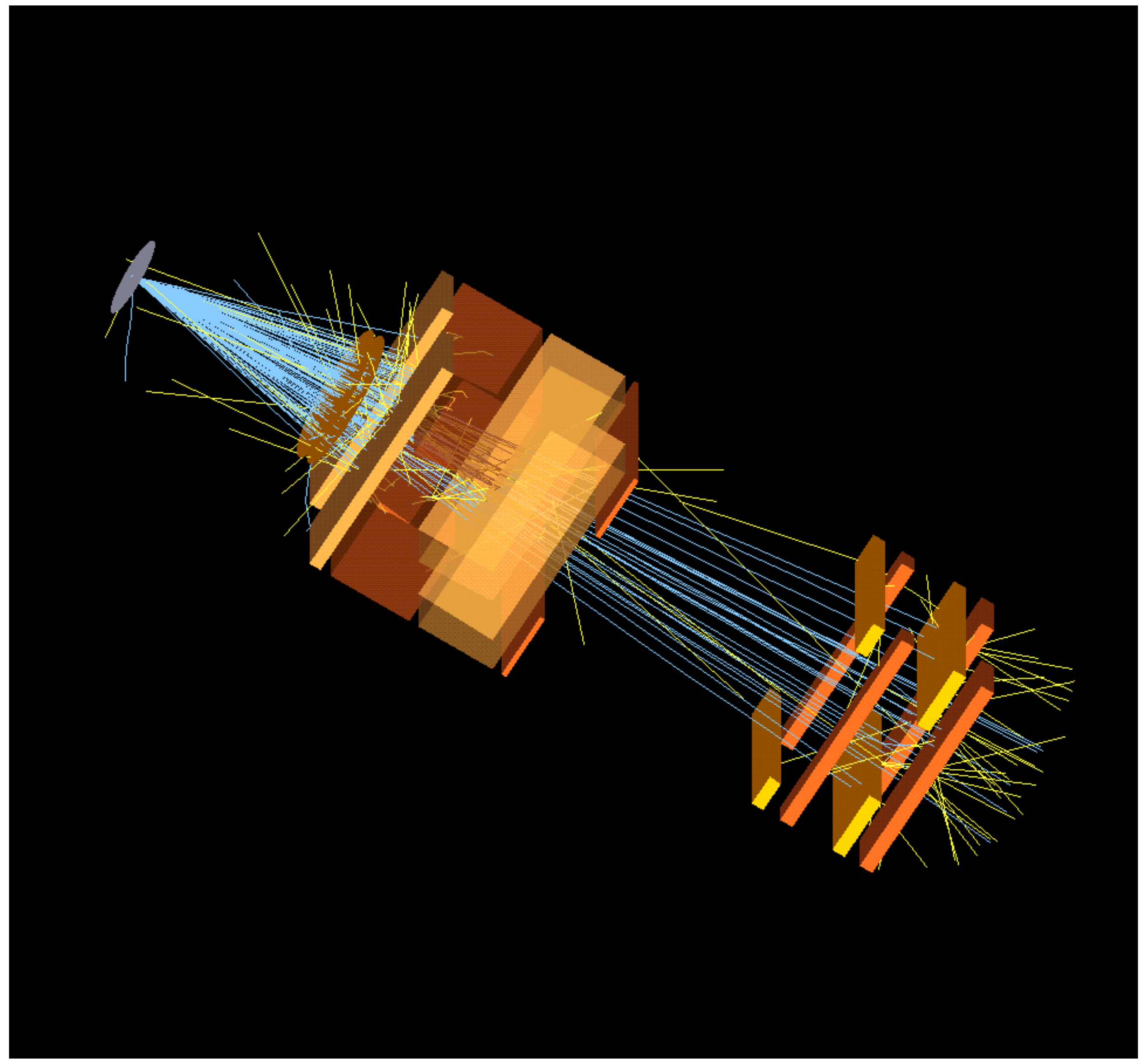
Linear accelerator MC simulation

Linear accelerator model



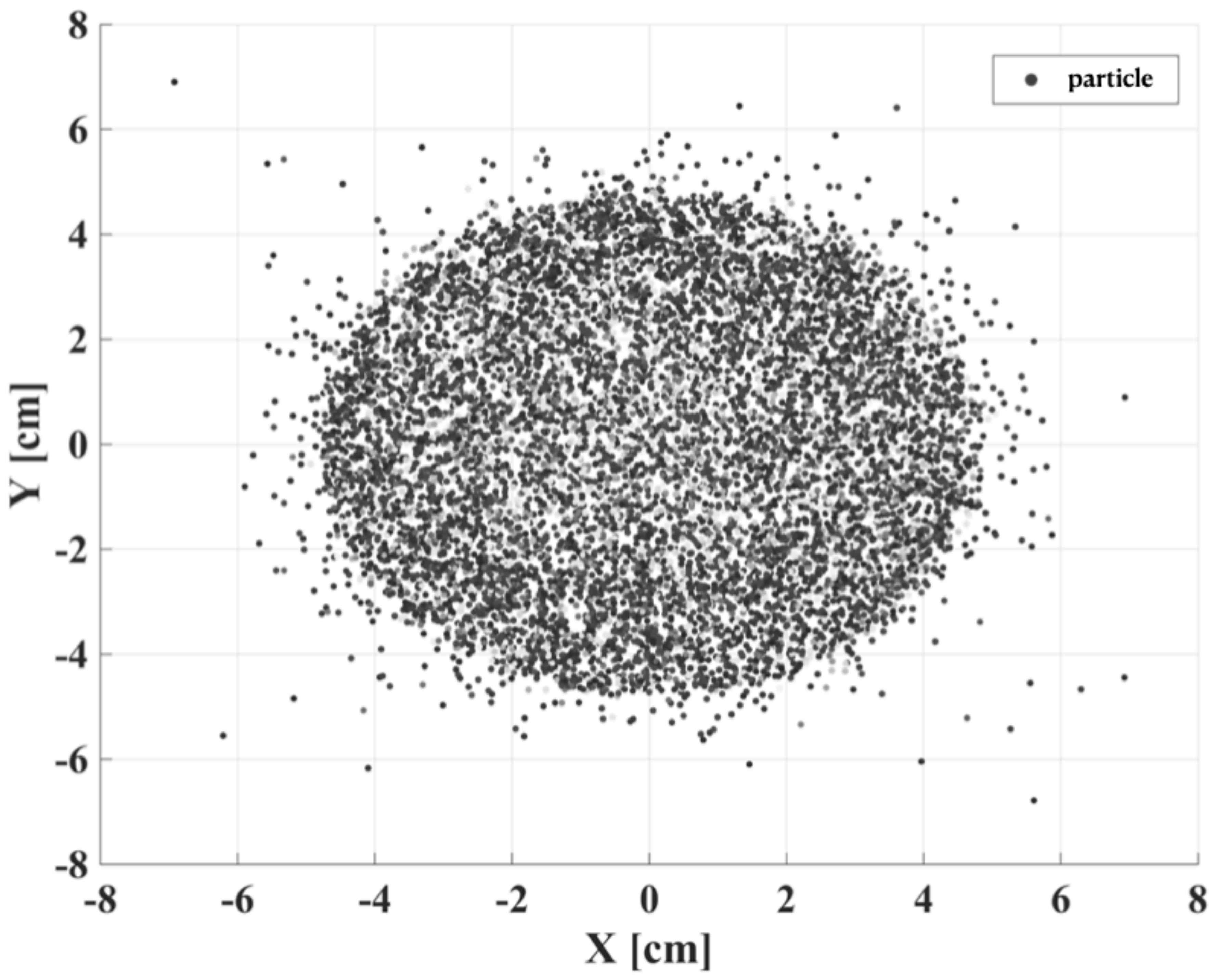
Accelerator head

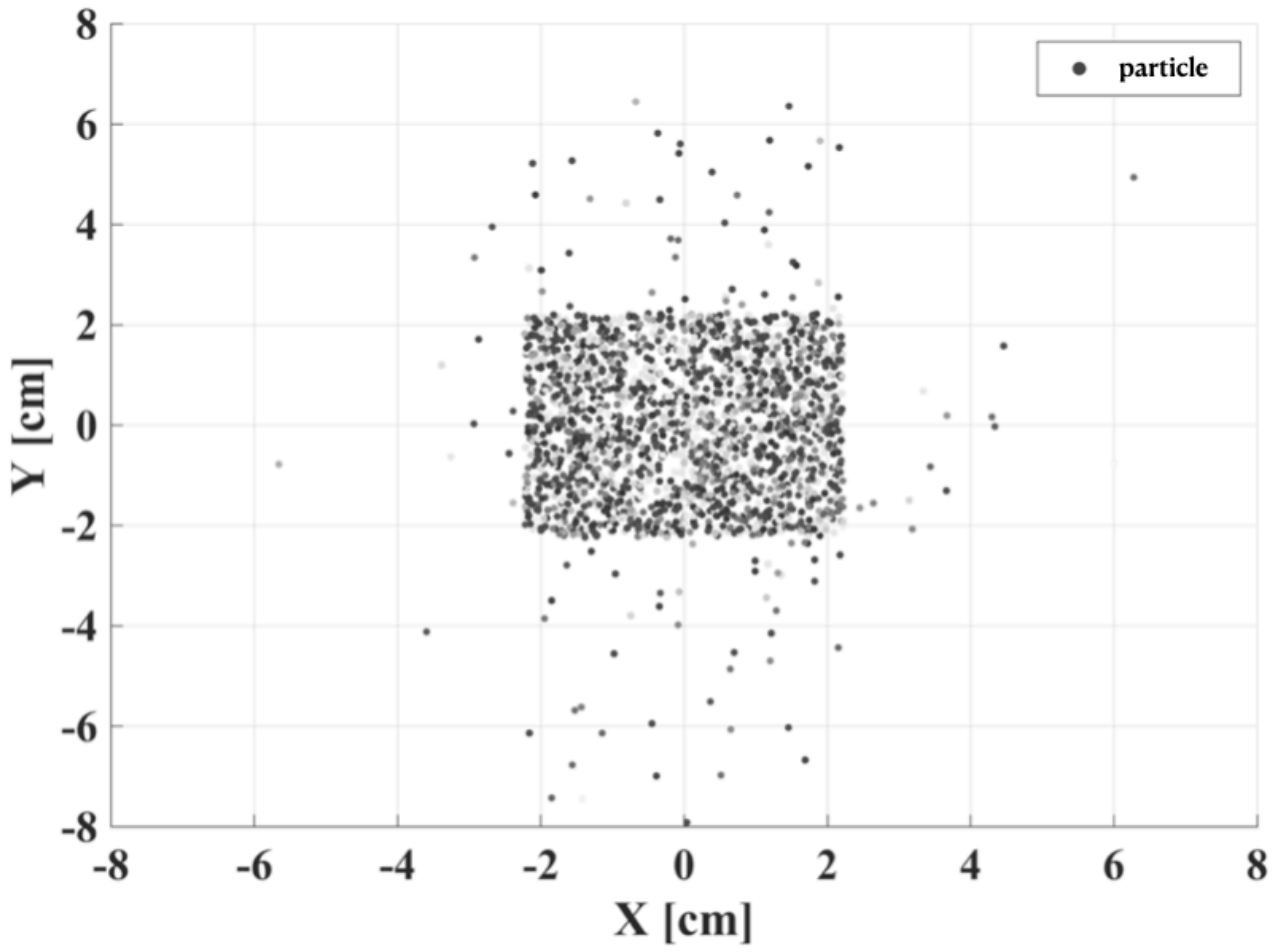
Simulation



Simulation output - Phase plane

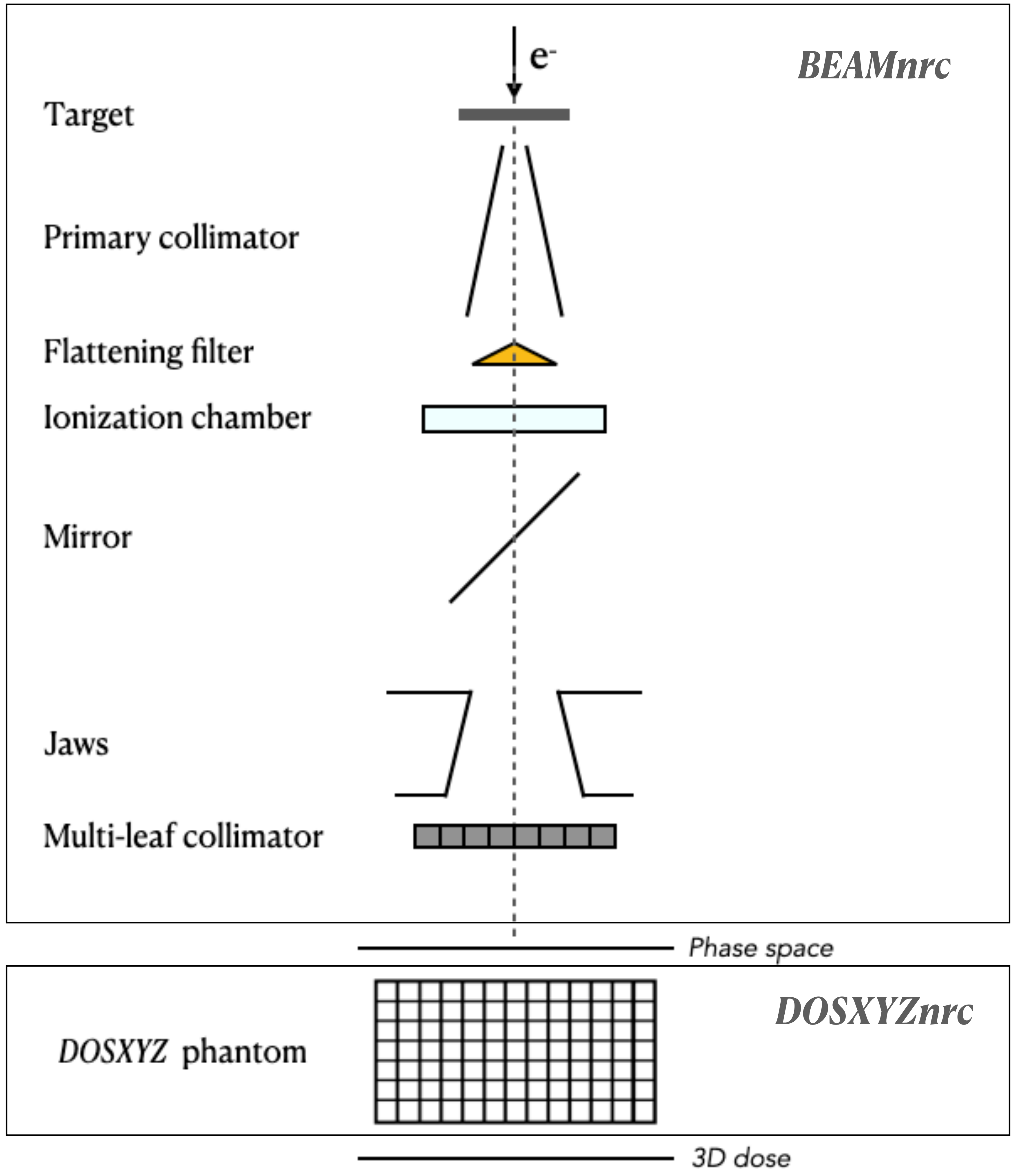
- Phase space for a given ***xy*** plane on a depth ***z***





EGSnrc

- Package for simulating photon and electron transport
- Developed in the 1970s at SLAC (Stanford Linear Accelerator Center)
- ***BEAMnrc*** - radiation source modelling
- ***DOSXYZnrc*** - dose deposition modelling
- Linear accelerator simulation ouput - phase space file



Sometimes simulations take too long

How efficient is my simulation ?

- Definition of Monte Carlo simulation efficiency:

$$\epsilon = \frac{1}{\sigma^2 T}$$

T: computing time required for obtaining variance σ^2

σ^2: variance of the desired quantity (in our case - dose)

How to increase the efficiency ?

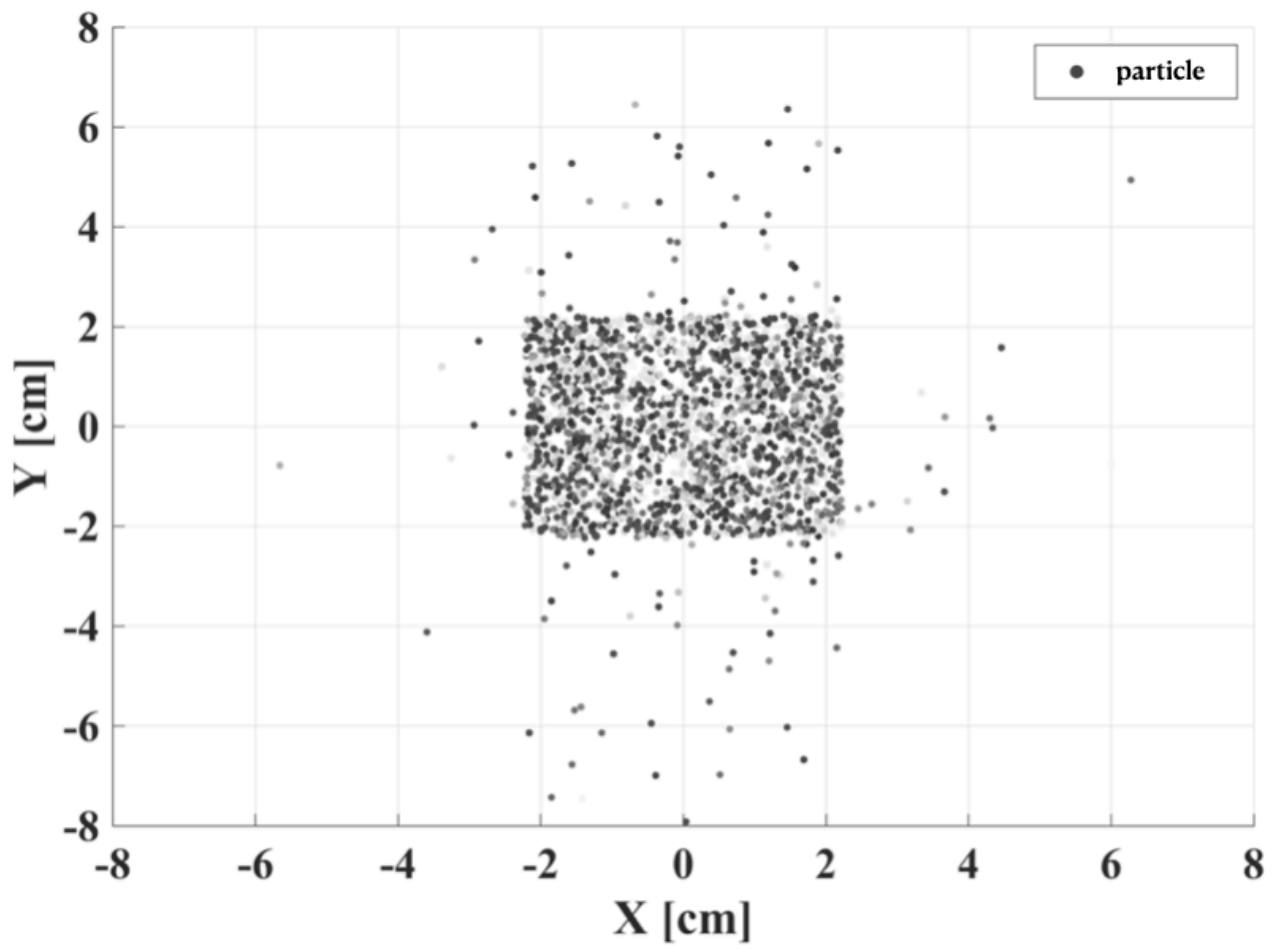
A: By reducing the computing time it takes to obtain a sufficiently small variance on the quantity of interest

Variance reduction methods

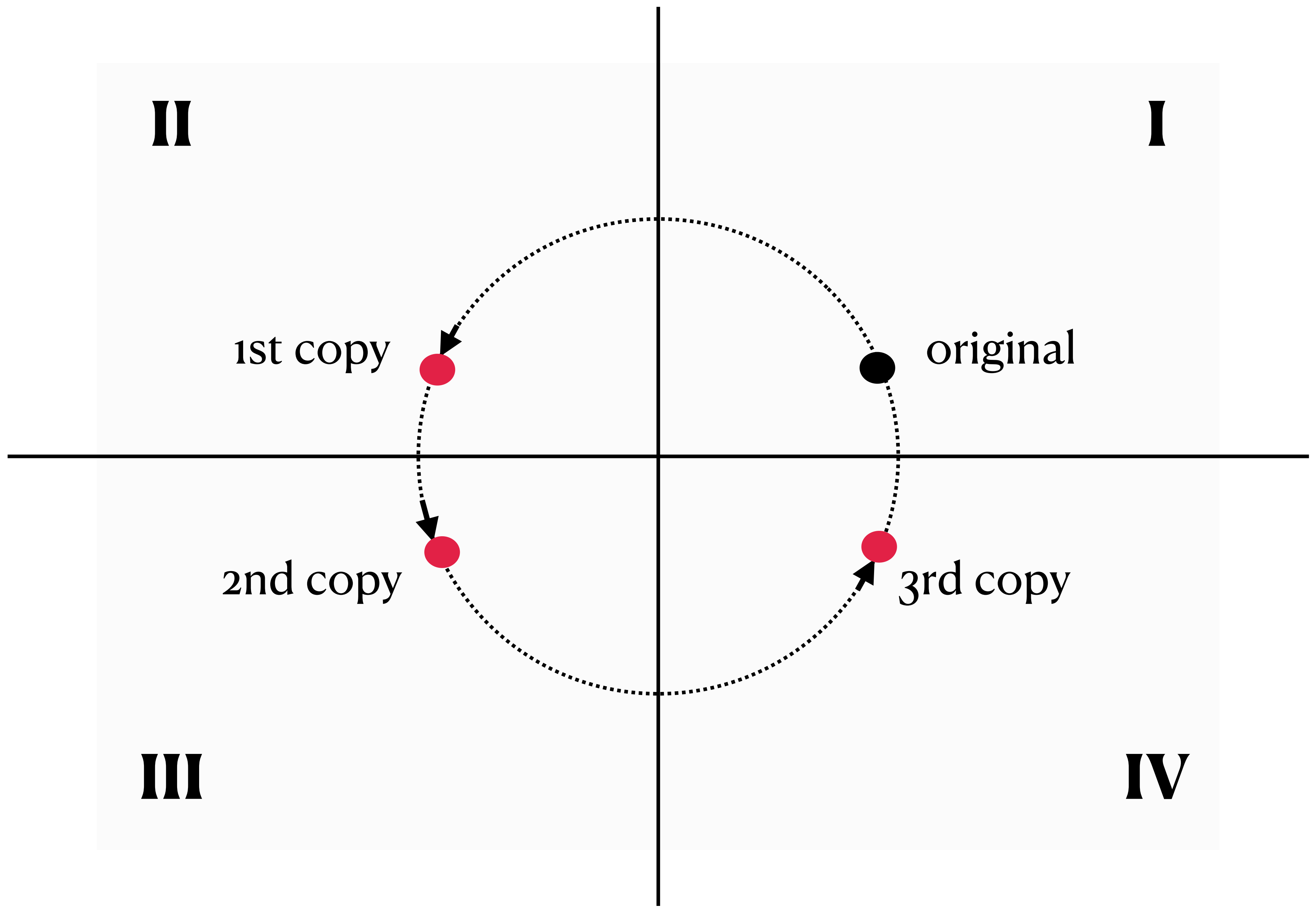
- ***The art of discarding superfluous calculations***
- Increased frequency of events and/or population -> better defined mean value -> less variance
- Elementary variance reduction - Particle recycling
- Geometry/Symmetry inspired variance reduction ?

Variance reduction inspired by symmetry

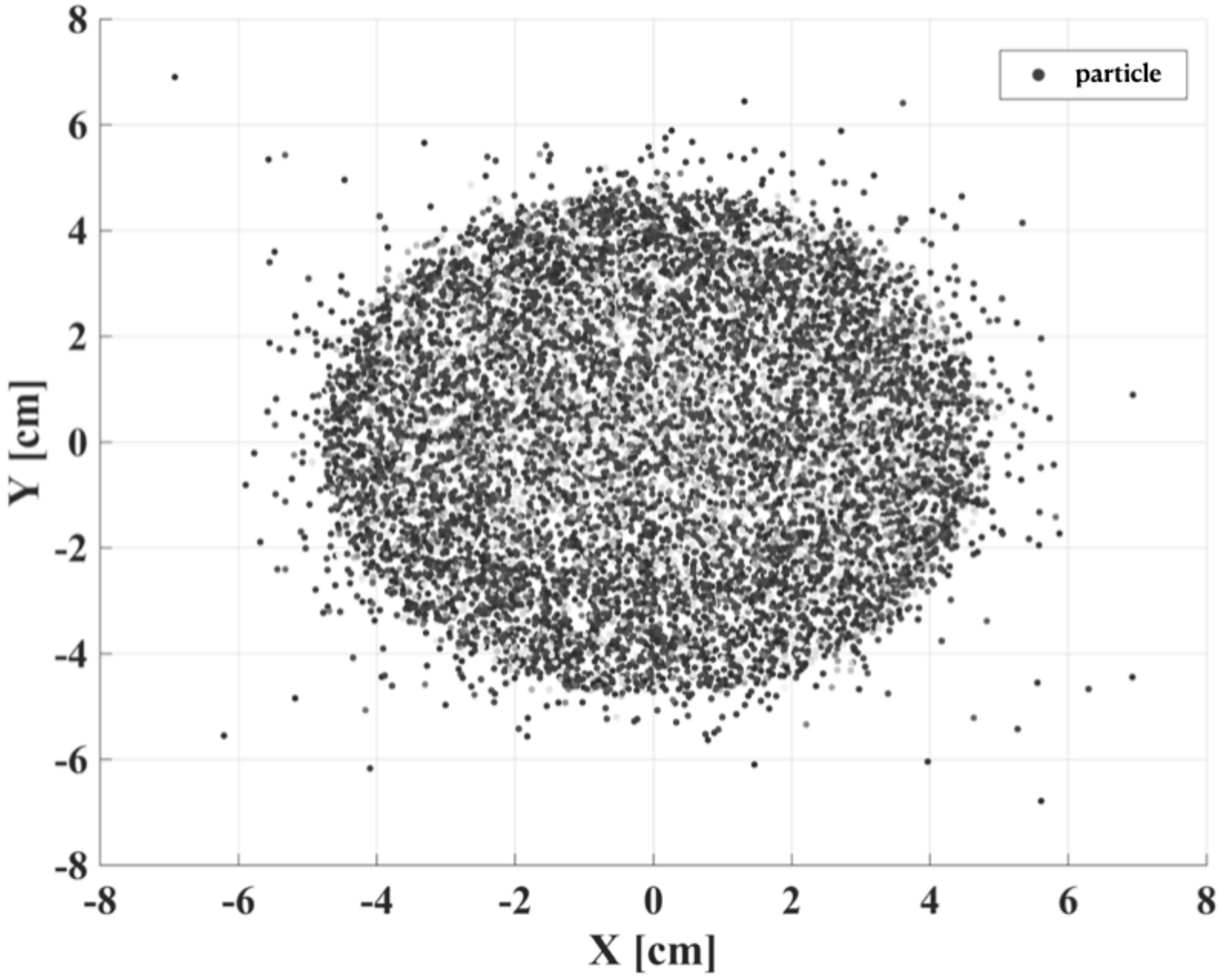
Rectangular phase plane



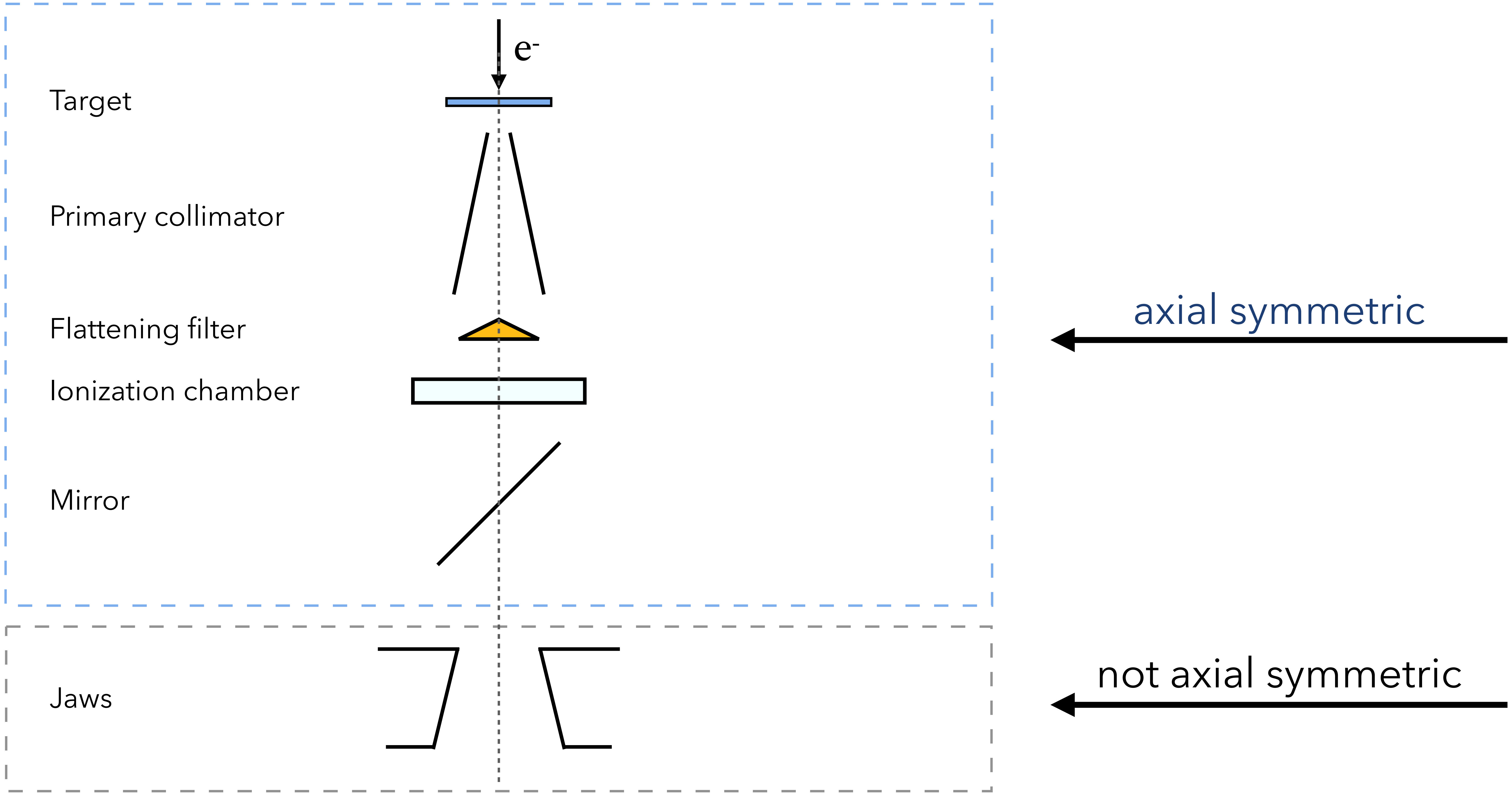
Particle redistribution mechanism



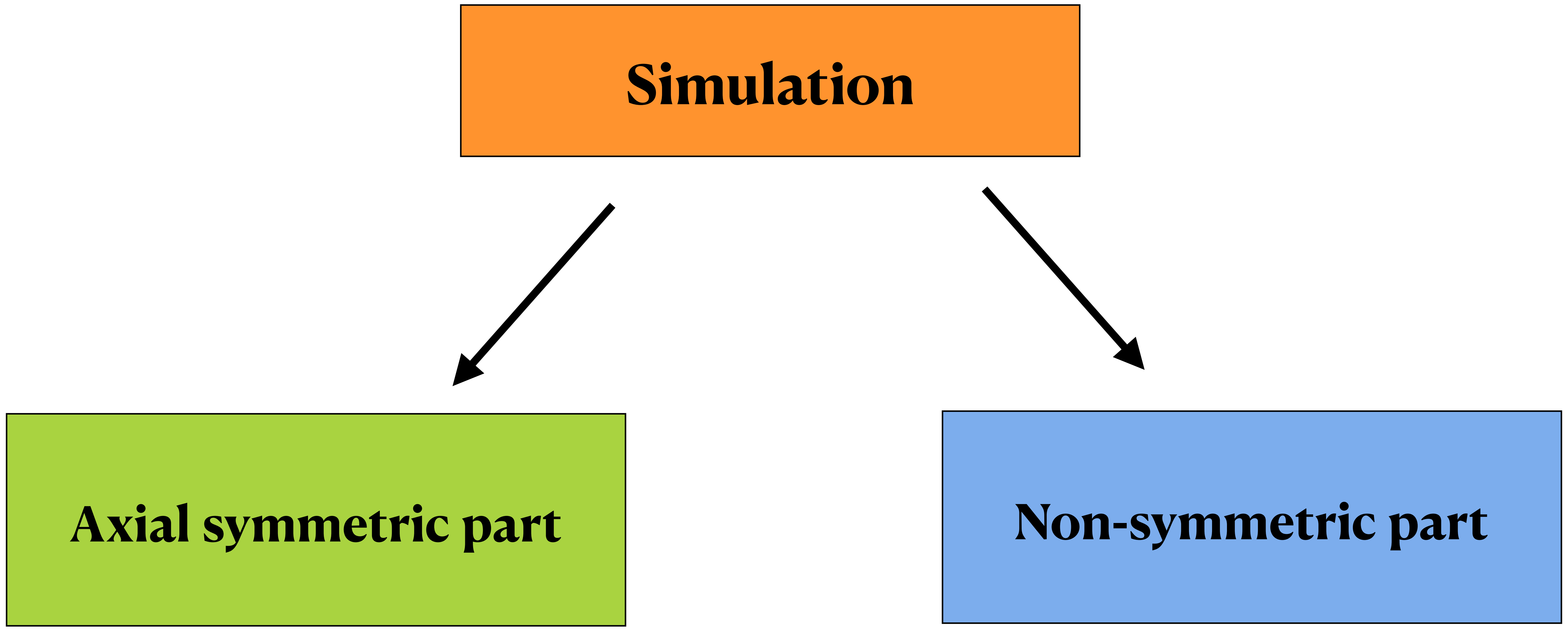
What about radial fields ?



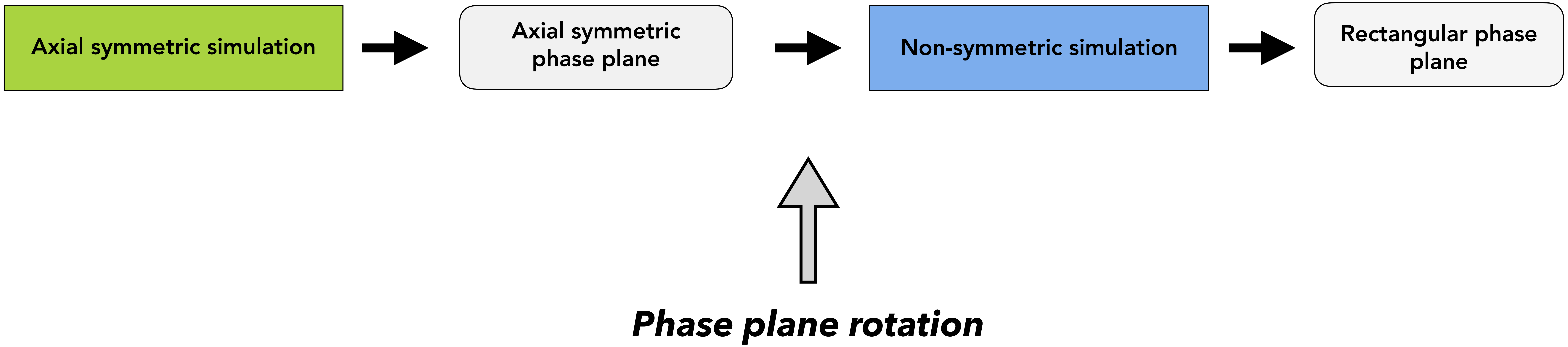
Linear accelerator partial axial symmetry



Splitting the simulation by symmetry



Utilizing the axial symmetry - phase plane rotation



Algorithm

***Twist*:**

1. Sample random angle:

$$\theta$$

2. Take particle with coordinates:

$$[x, y]$$

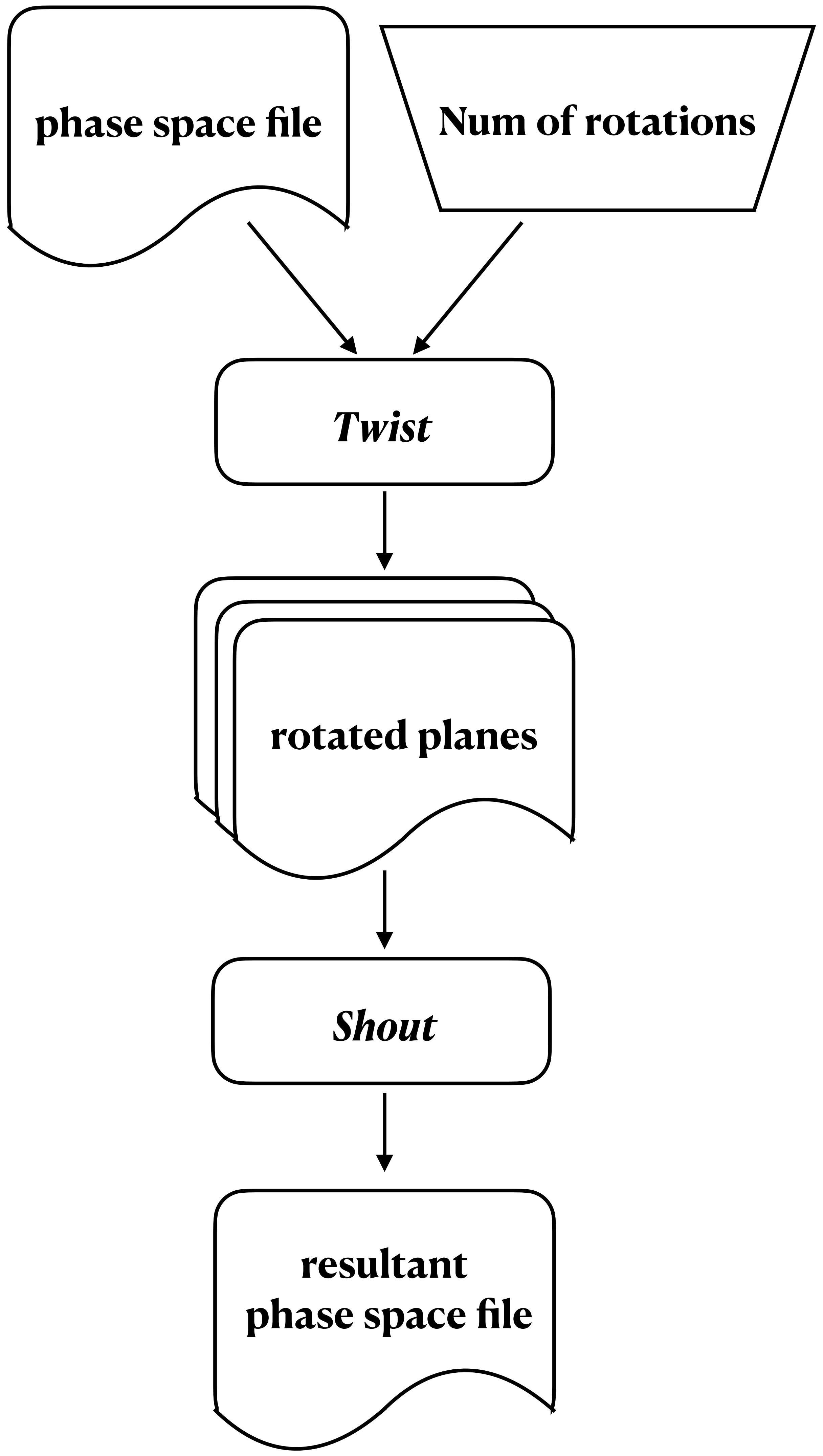
3. Calculate rotation produced particle coordinates

$$[x_R, y_R] = R_z(\theta) \cdot [x, y]$$

***Shout*:**

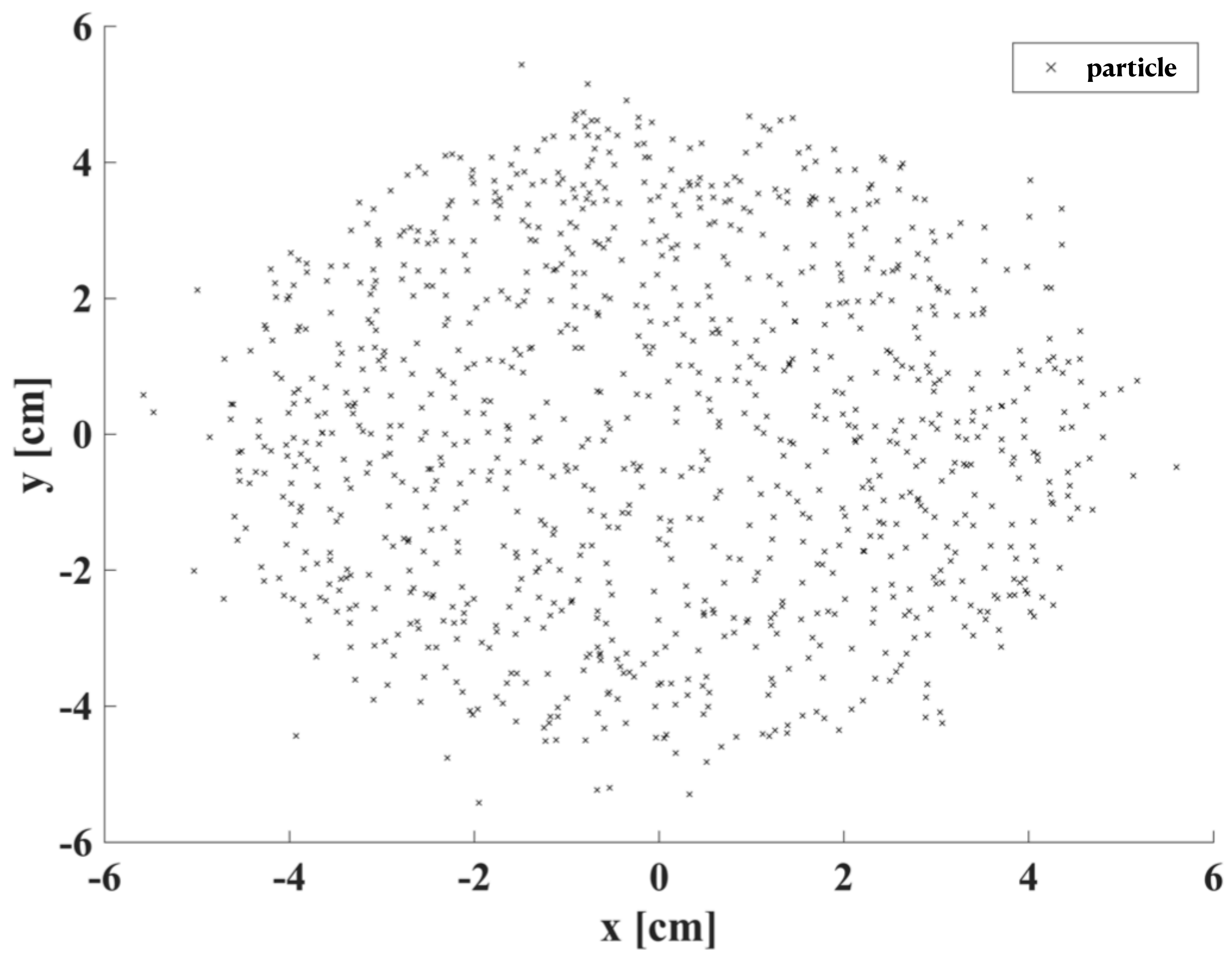
1. Take rotated phase space files

2. Merge particles into single phase space file

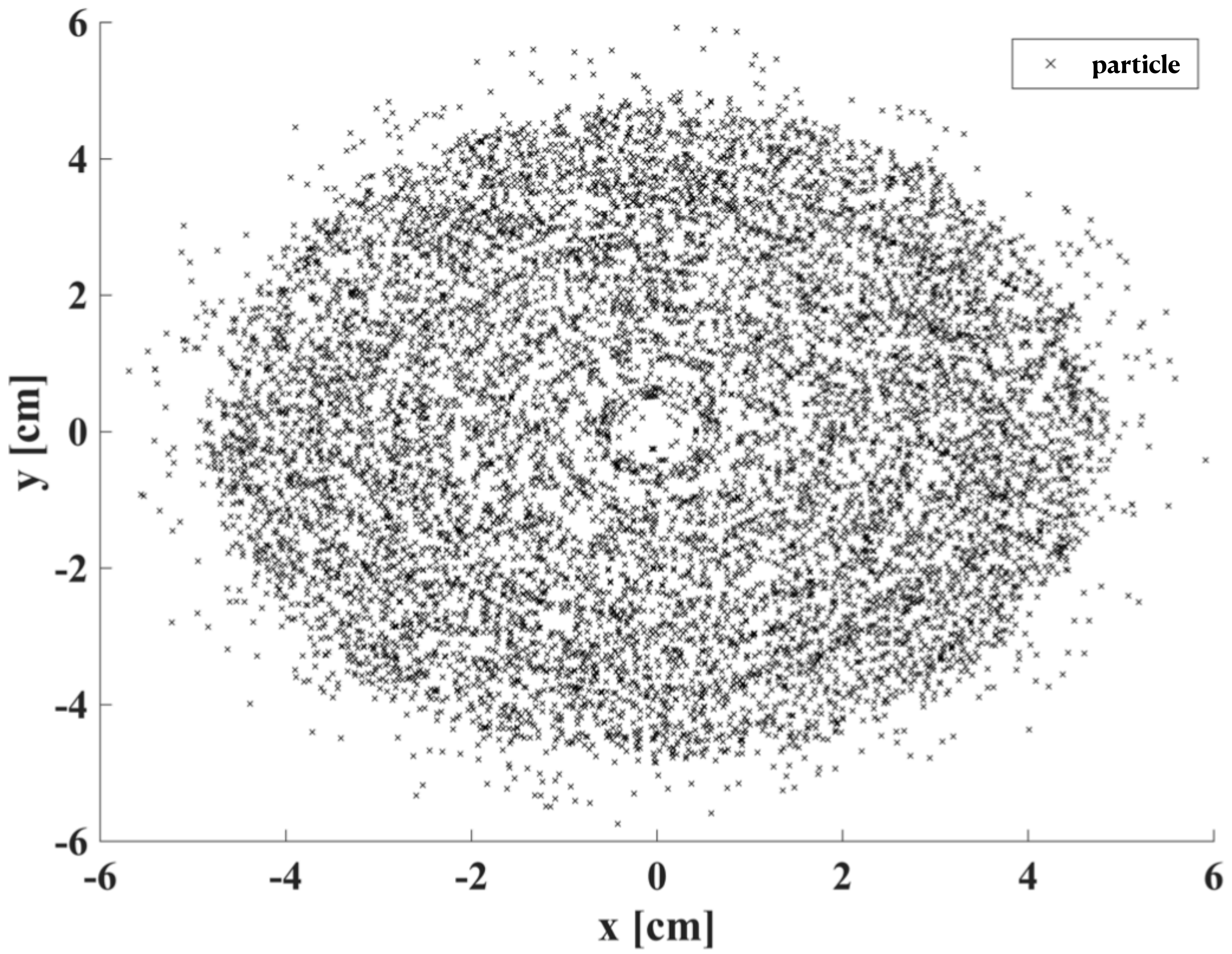


Rotation Artifacts

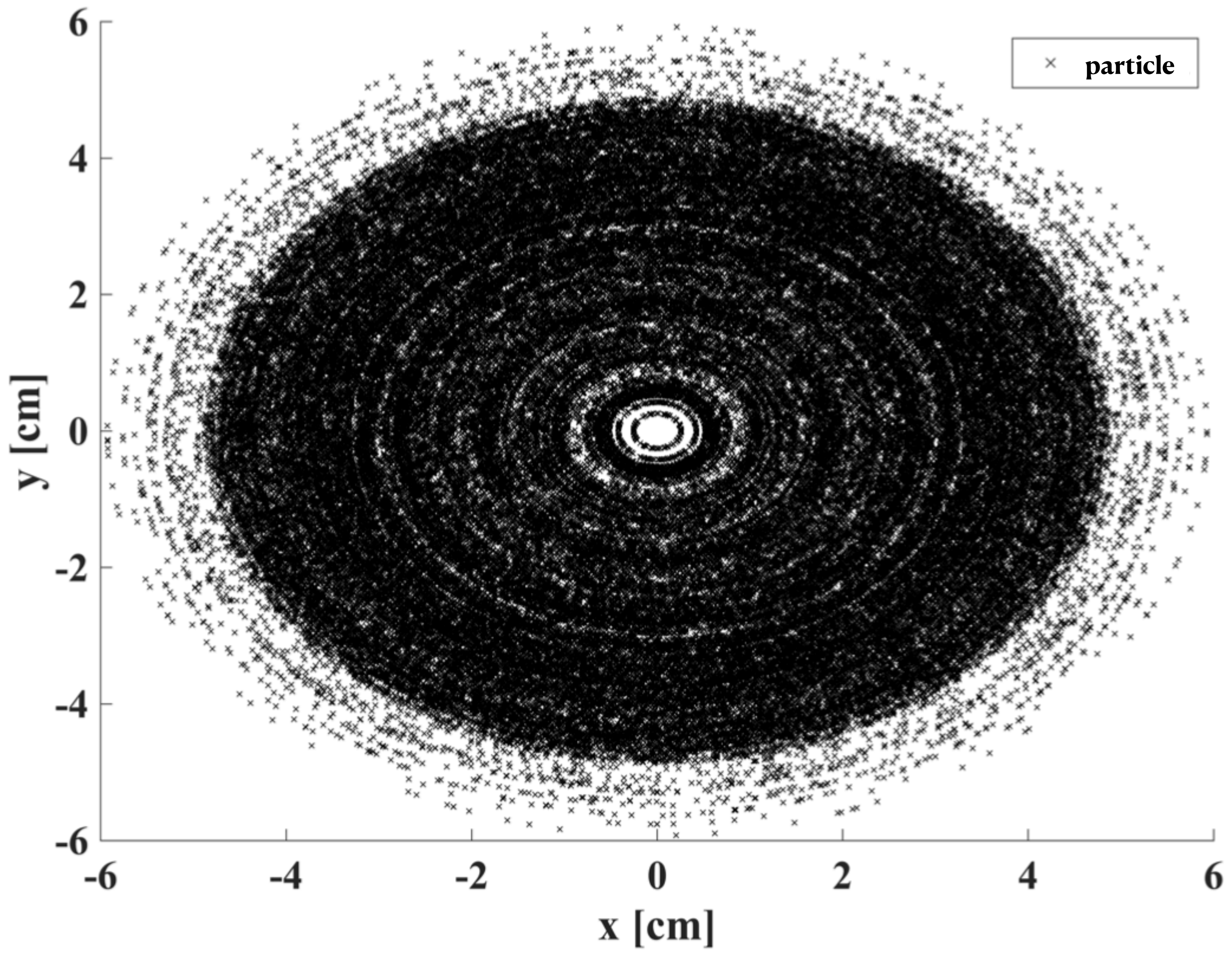
_N=10³_ particles



Rotated 10 times

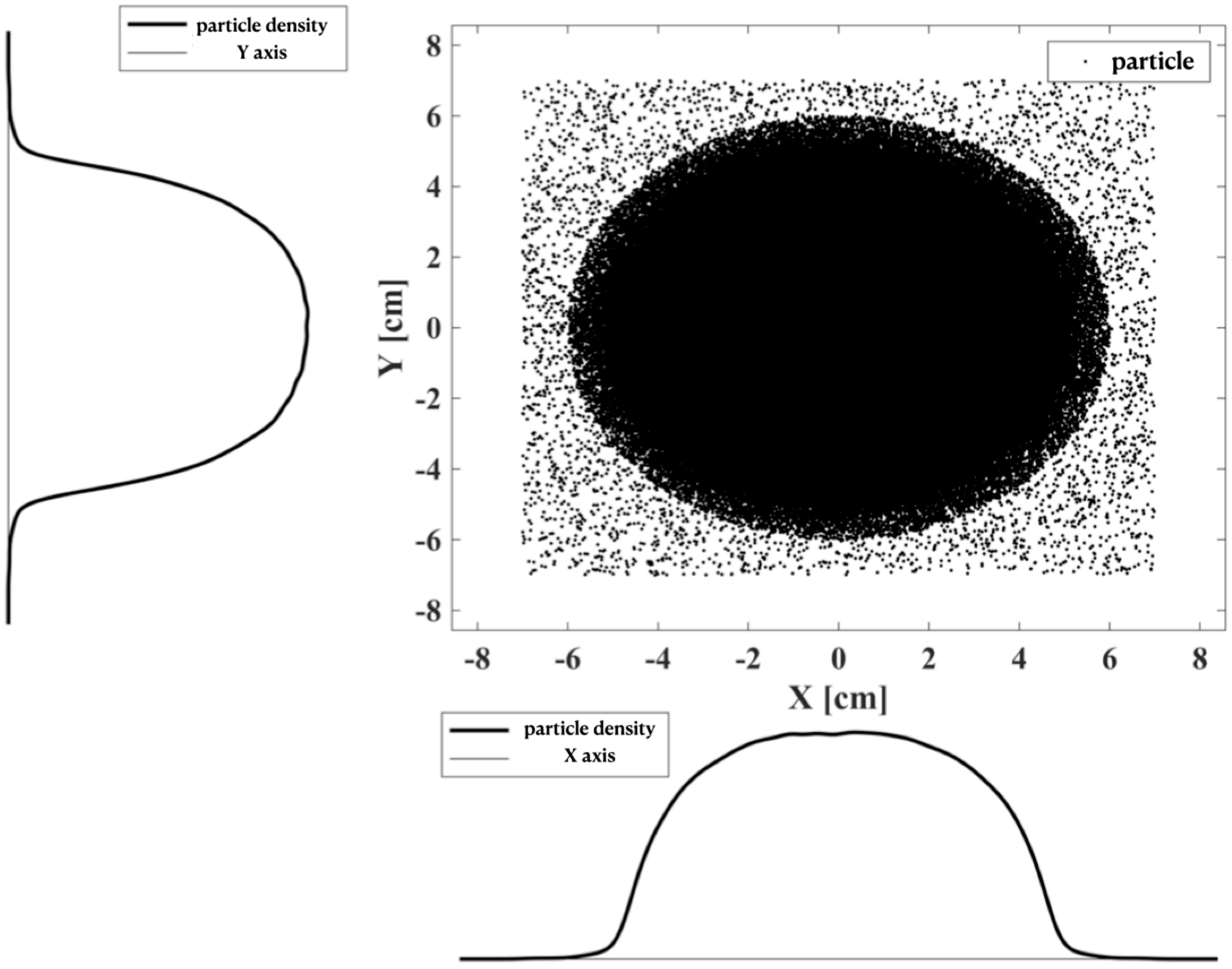


Rotated 100 times

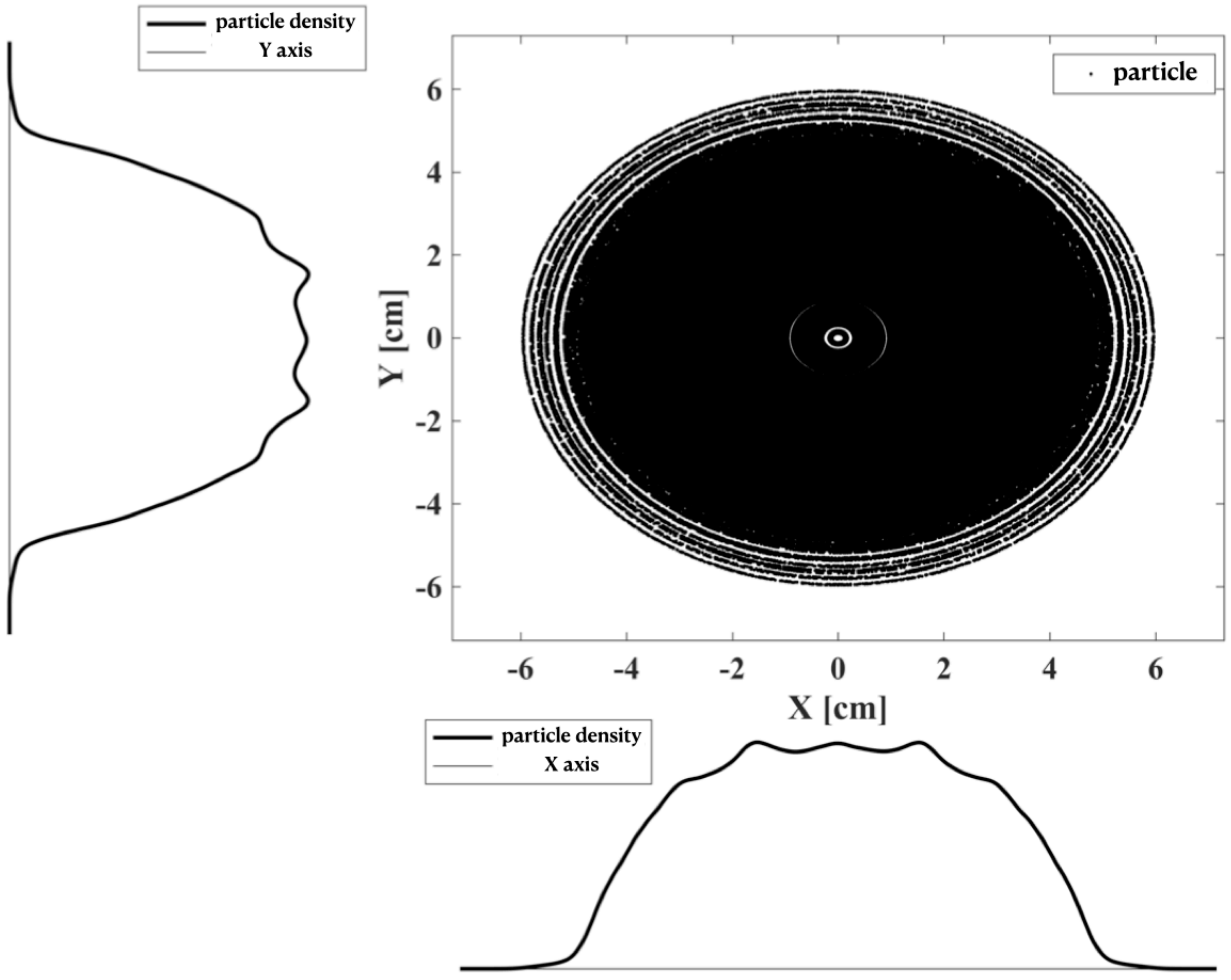


Rotation Artifacts

Analog simulation

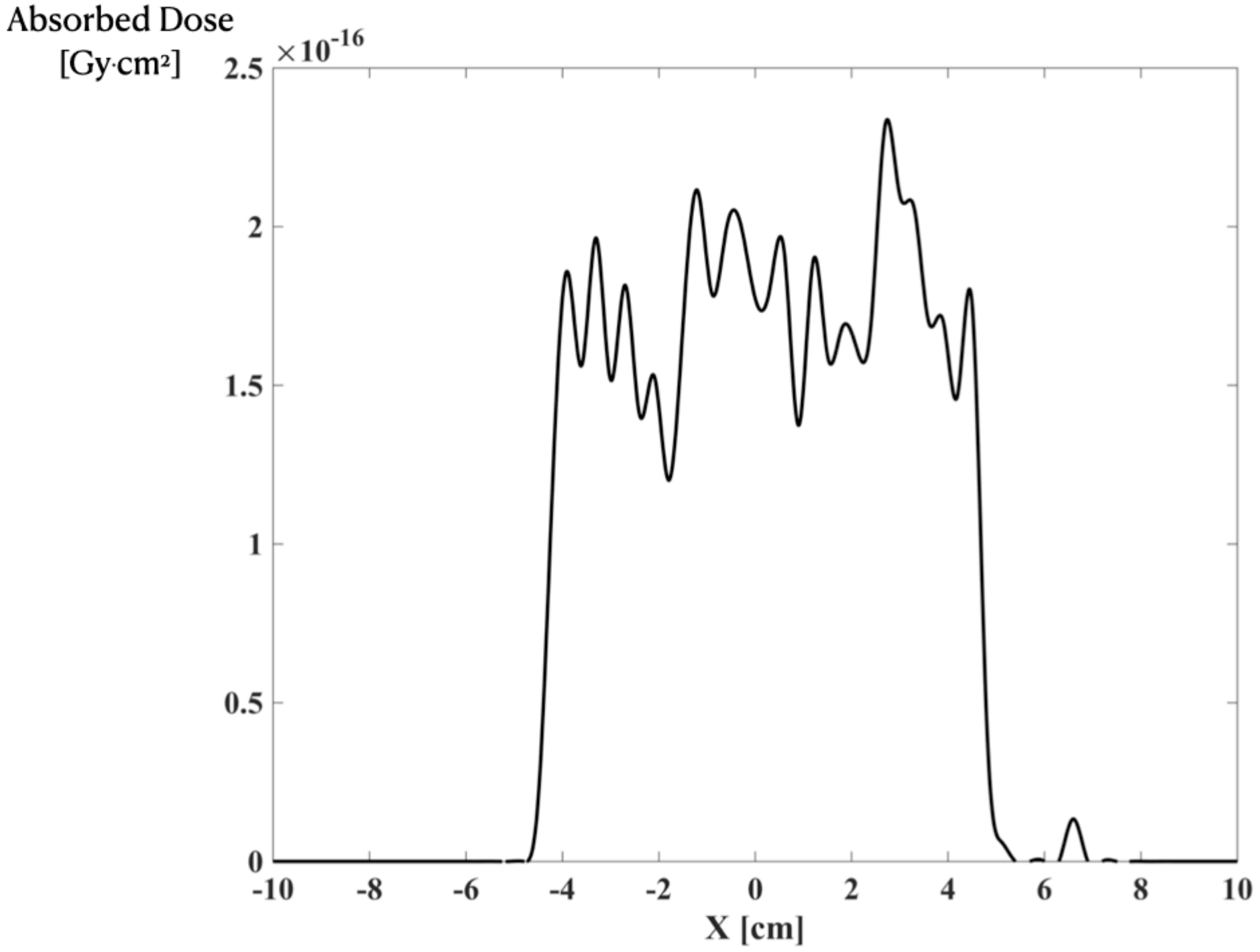


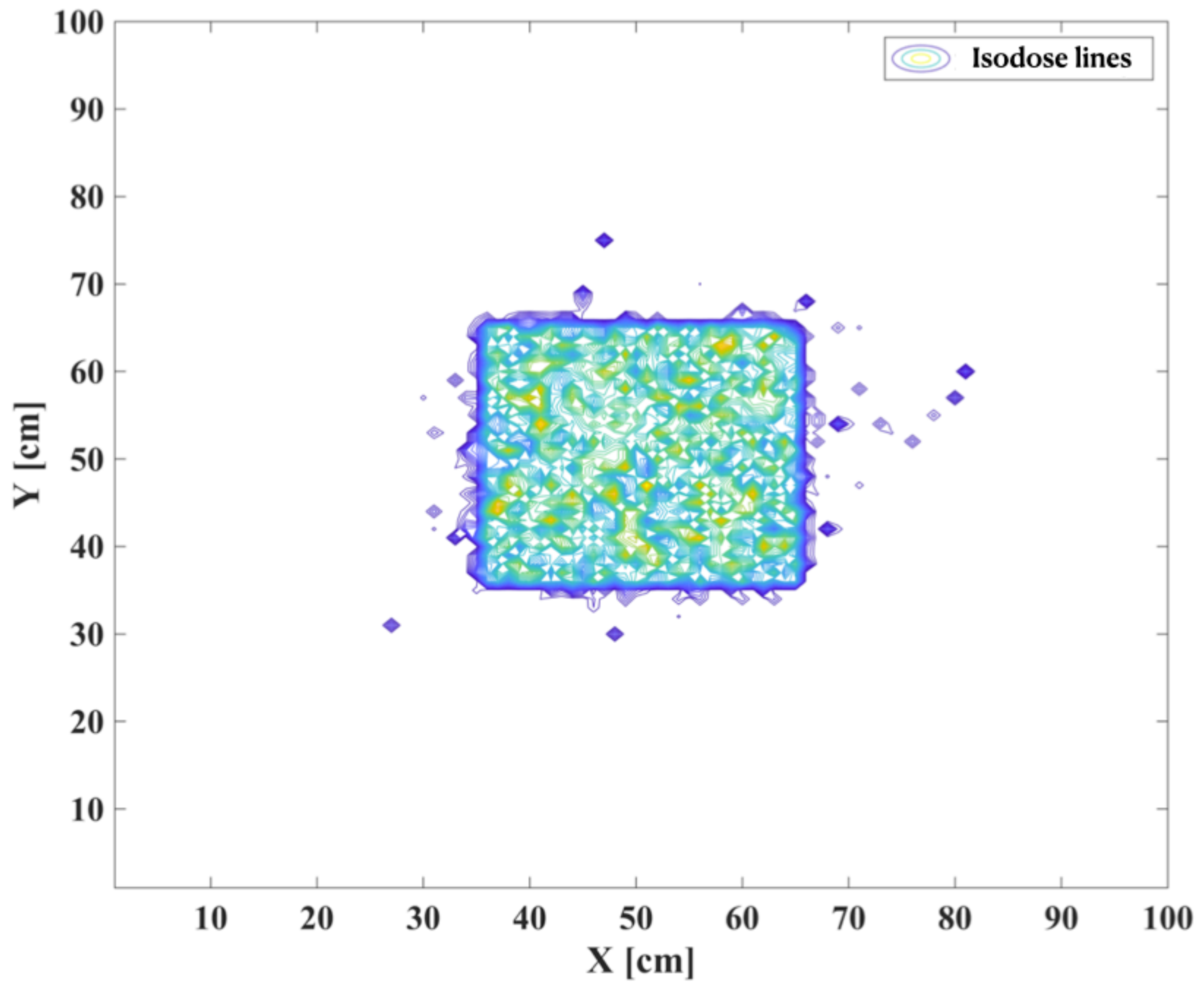
Phase plane rotation



Isodose lines and dose profile artifacts

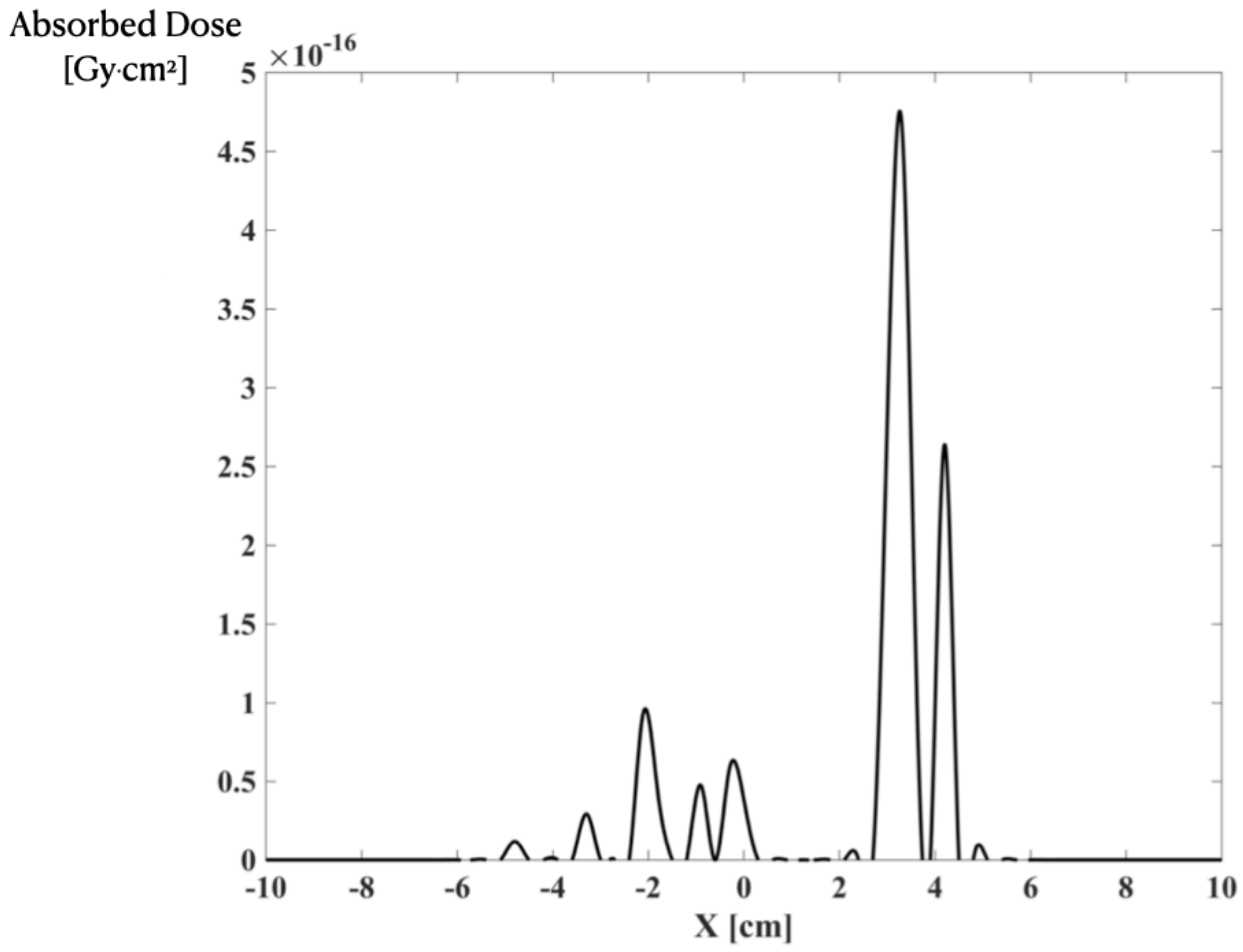
<u>Analog simulation</u>

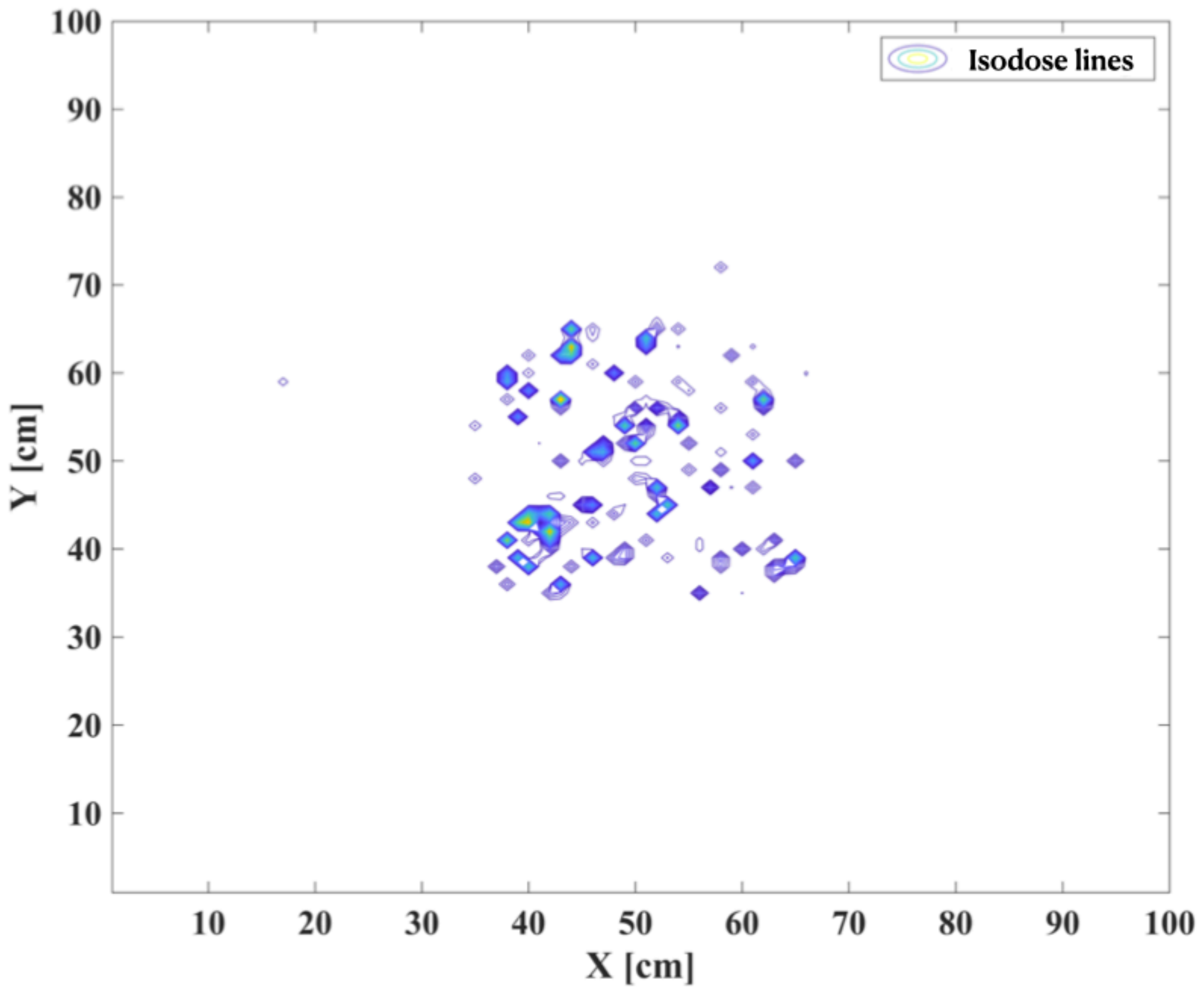




Isodose lines and dose profile artifacts

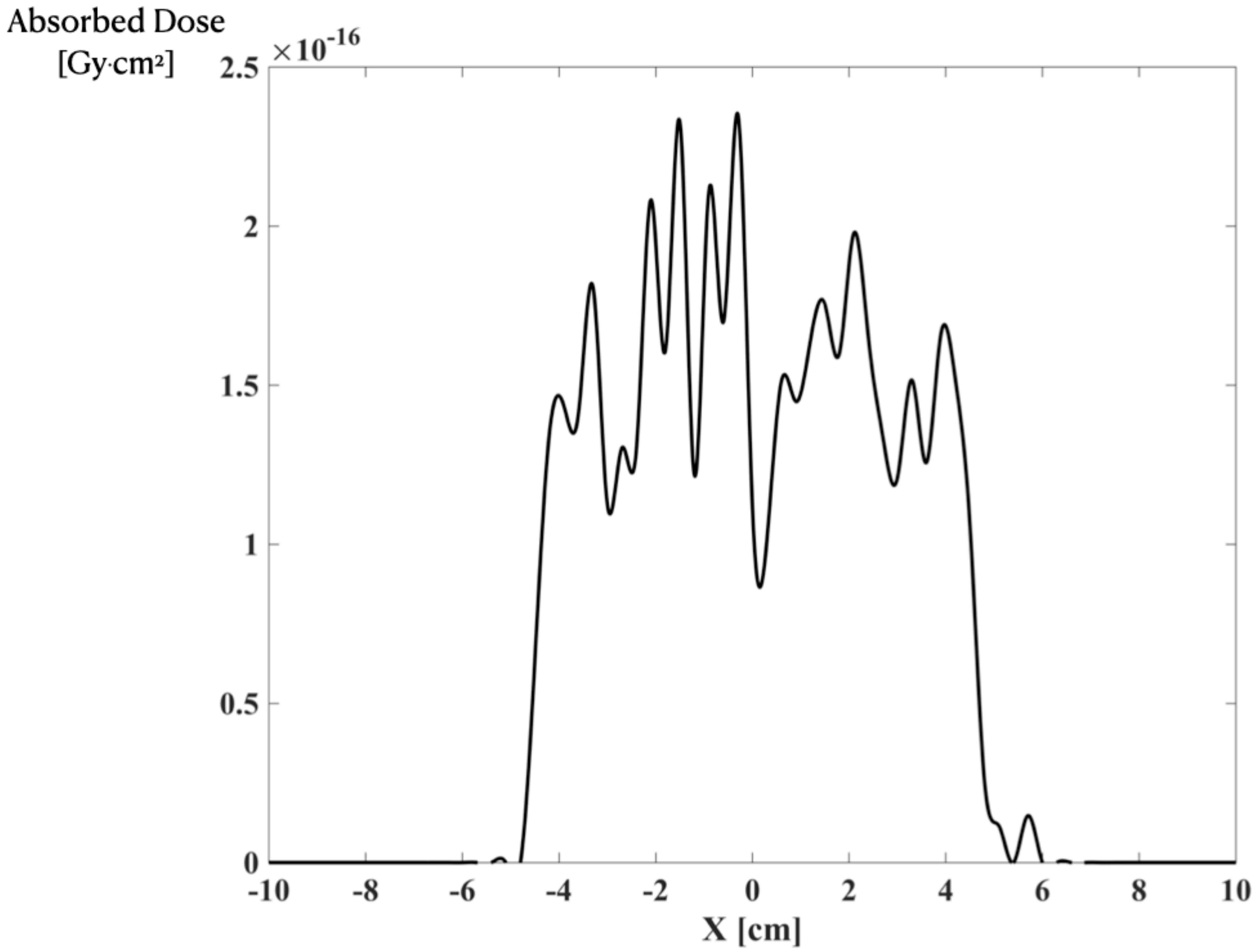
<u>Particle recycling</u>

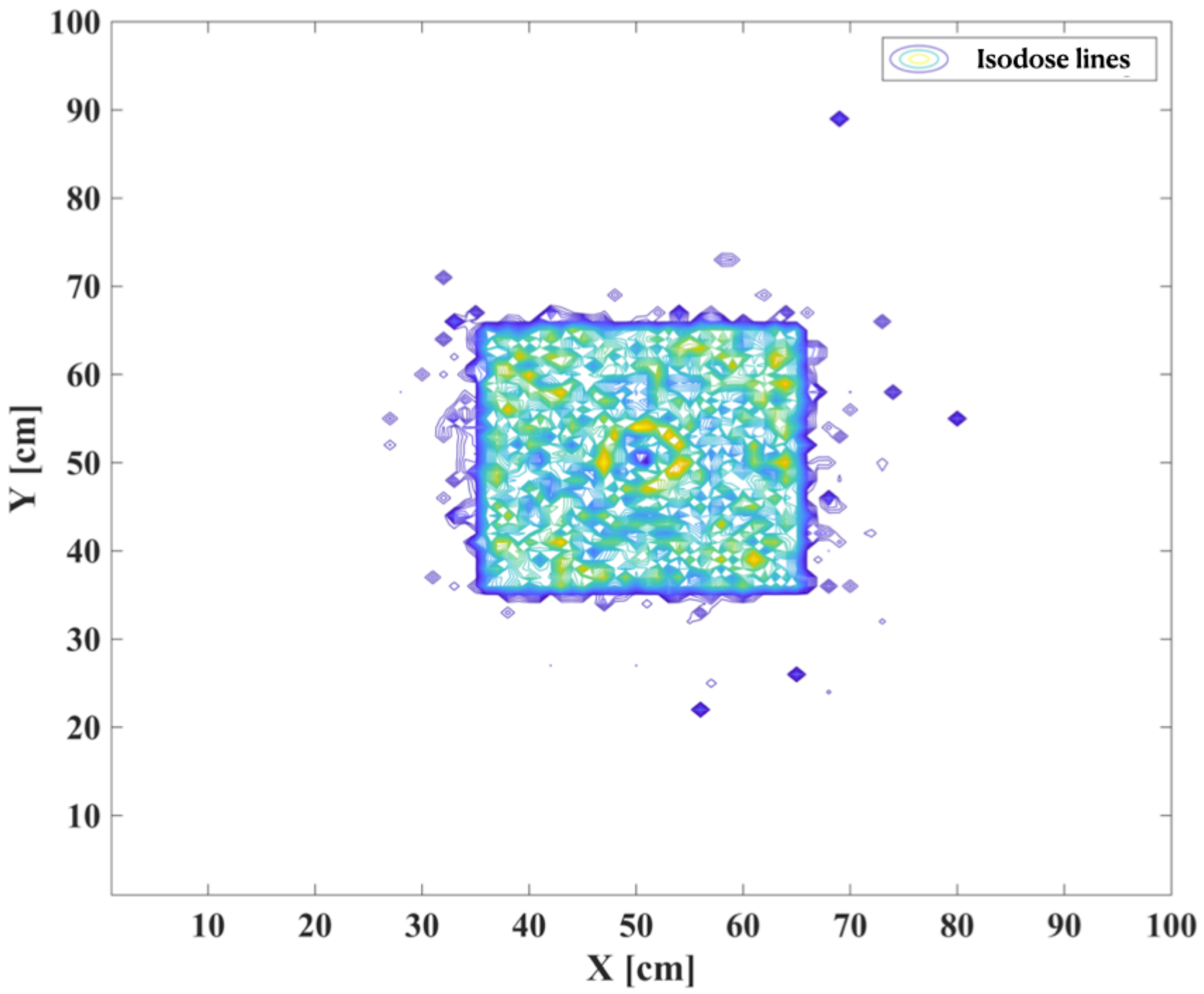




Isodose lines and dose profile artifacts

<u>Phase plane rotation</u>

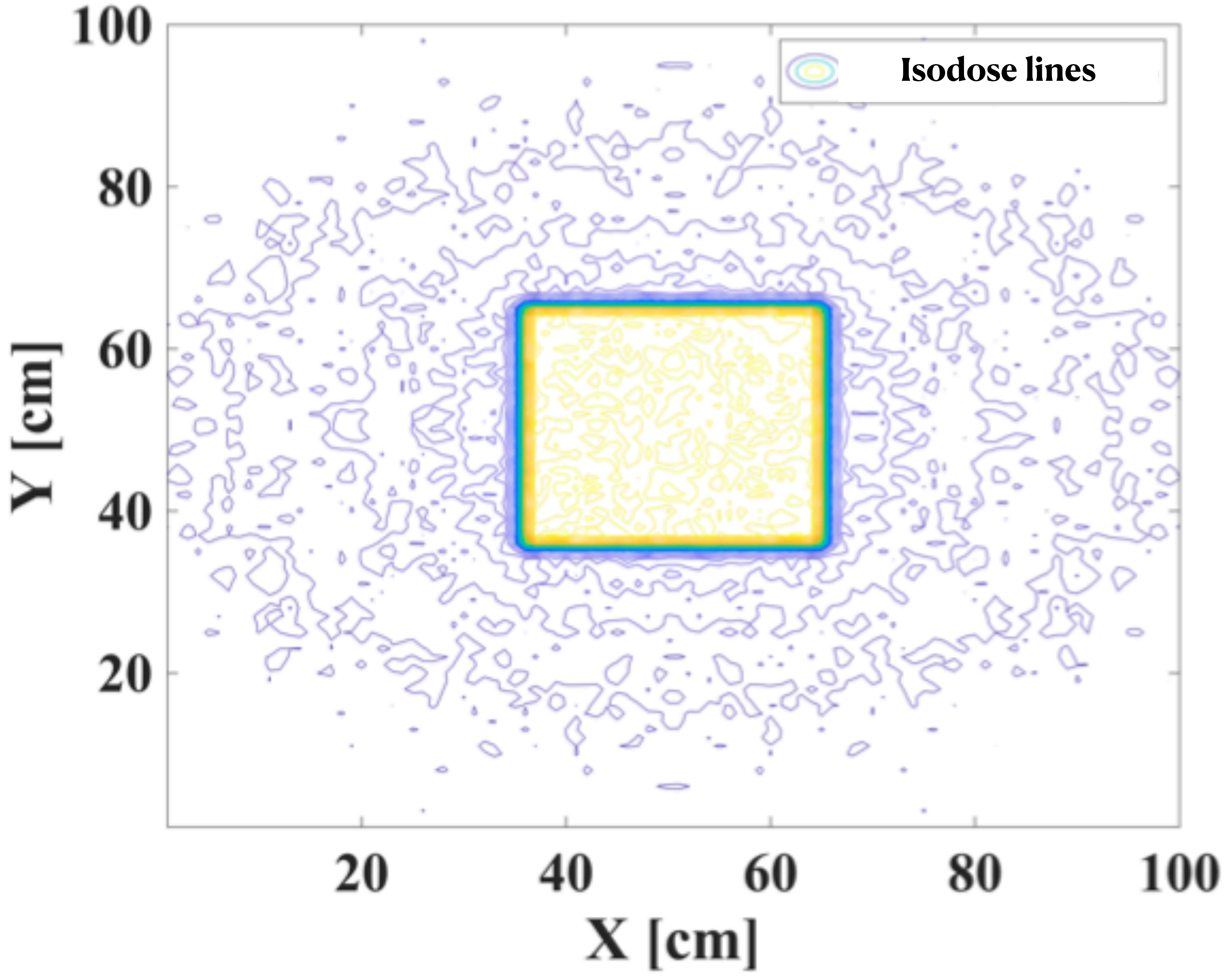




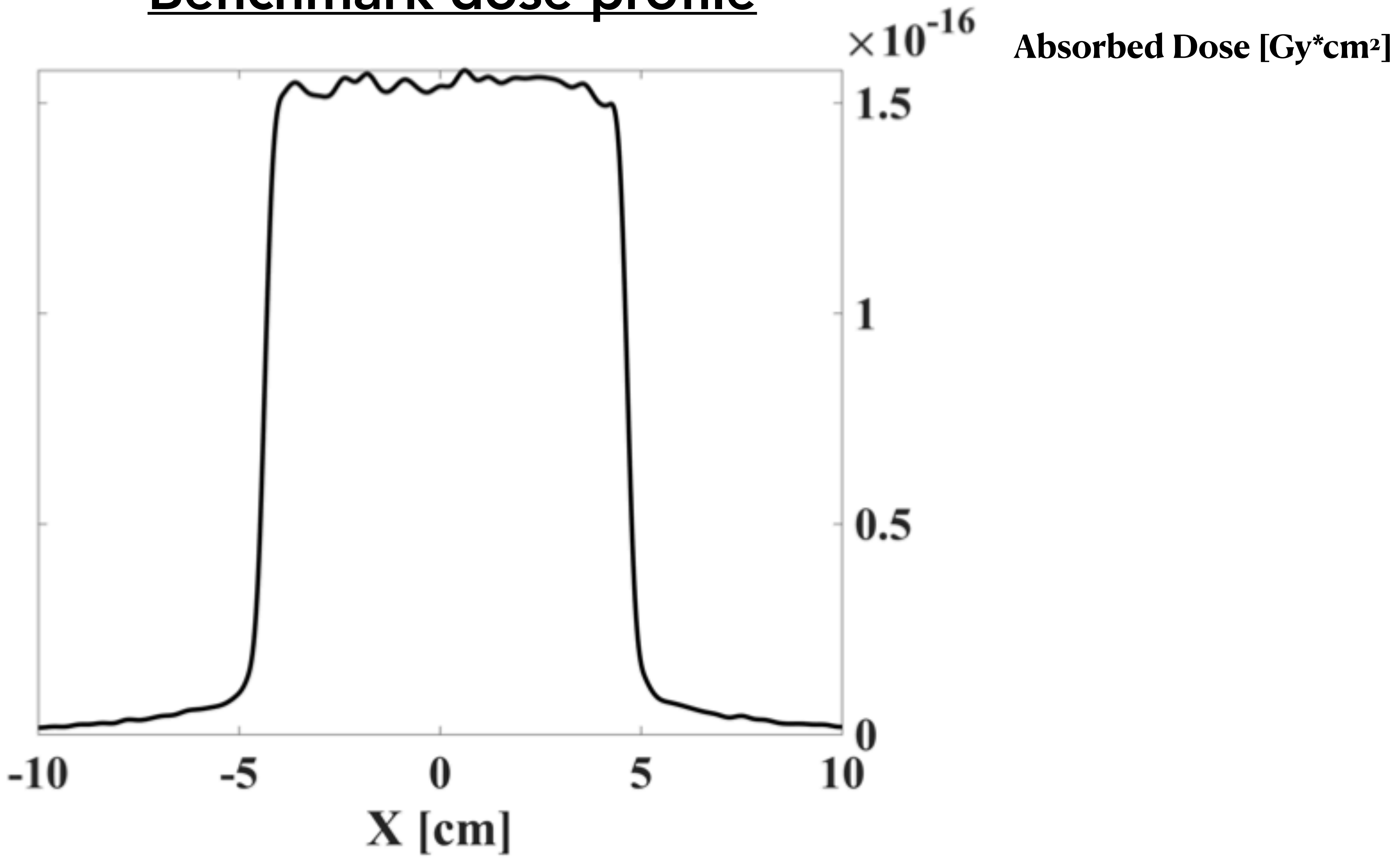
Benchmarking with analog simulation

- $N_{REF} = 1 \cdot 10^9$ particles in a rectangular field

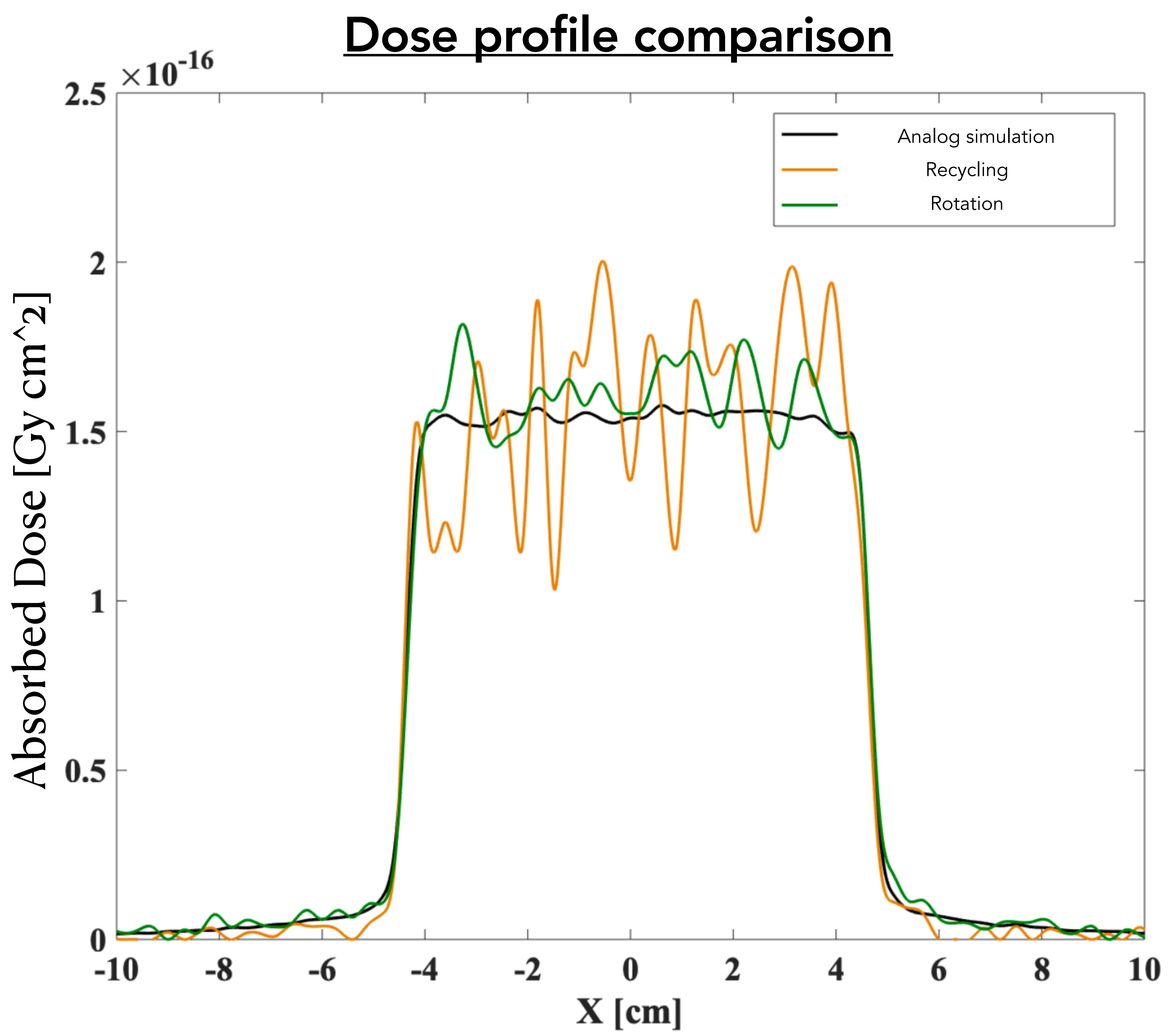
<u>Benchmark isodose lines</u>

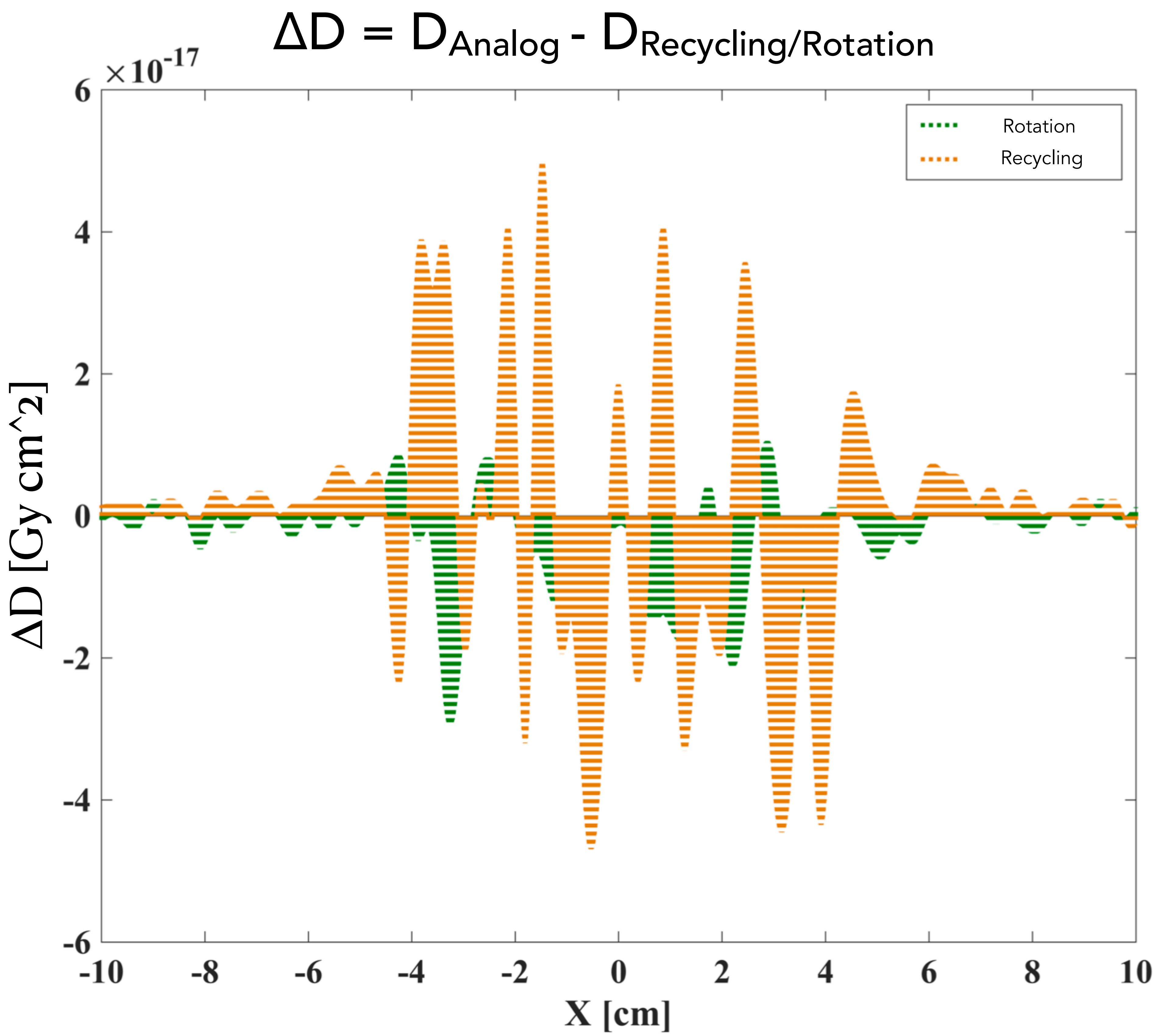


<u>Benchmark dose profile</u>



Benchmarking with recycling and analog simulation





Disscusing efficiency

Simulation	*N*	T_{rot} or T_{rec} *[s]*	T_s *[s]*	*Σ T [s]*	σ^2	*ε [s⁻¹]*
S1 rotation	10^3	268	2005	2273	0.88	5.02E-04
S1 recycling		149	1964	2133	0.90	5.21E-04
S2 rotation	10^4	1117	6695	7812	0.61	2.09E-04
S2 recycling		534	6890	7424	0.62	3.02E-04
S3 rotation	10^5	2558	21111	23669	0.31	1.36E-04
S3 recycling		1314	19698	21012	0.32	1.50E-04

$$\epsilon = \frac{1}{T \cdot \sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{D_i >= D_{max/2}}^{n} \left(\frac{\Delta D_i}{D_i}\right)$$

**Counting the contribution of voxels in which the dose is greater than or equal to half the maximum dose in the phantom.*

Conclusions

- Method proven succesful for variance reduction
- Once processed phase space file could be used in multiple different simulations
- Package independent solution - interoperability between different output formats
- Extending the concept beyond radiation dosimetry simulations

Thank you for your time!

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