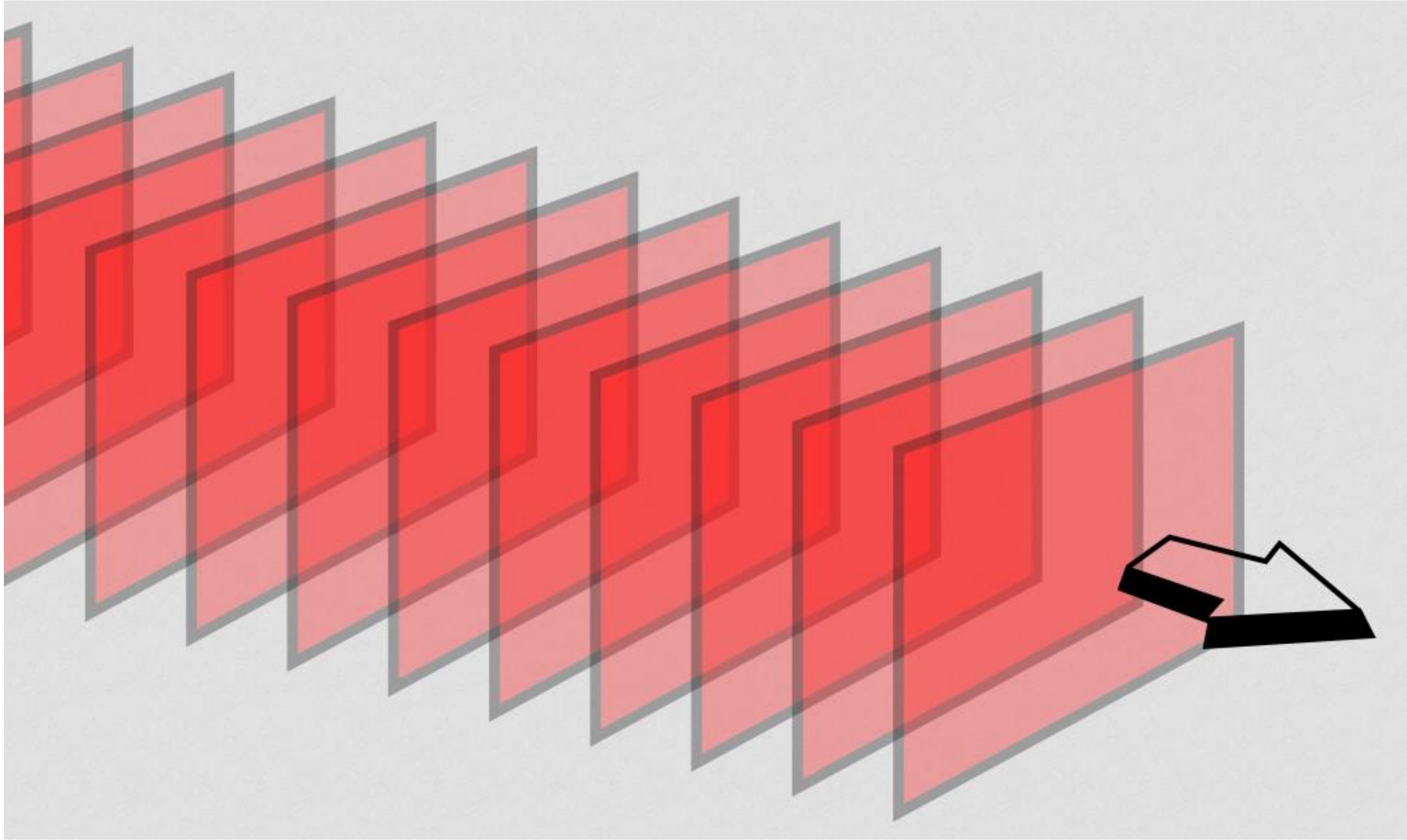
The Gouy phase of long-range Gauss-Bessel beams

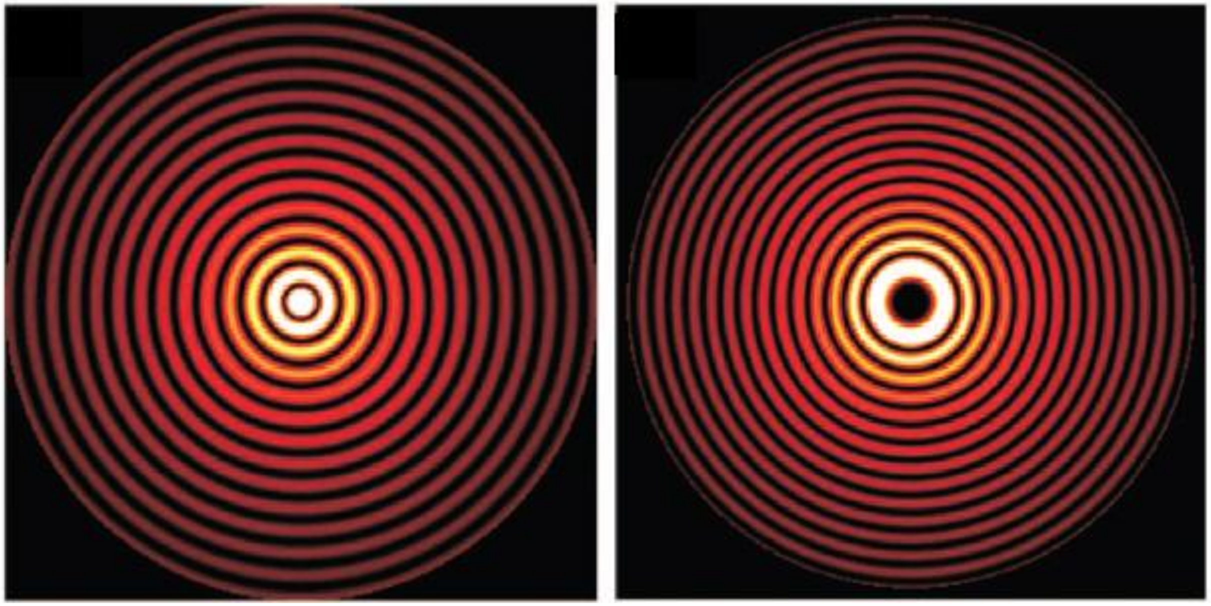
1. (Quasi-)Non-diffracting beams

Plane waves (in rectangular coordinates),



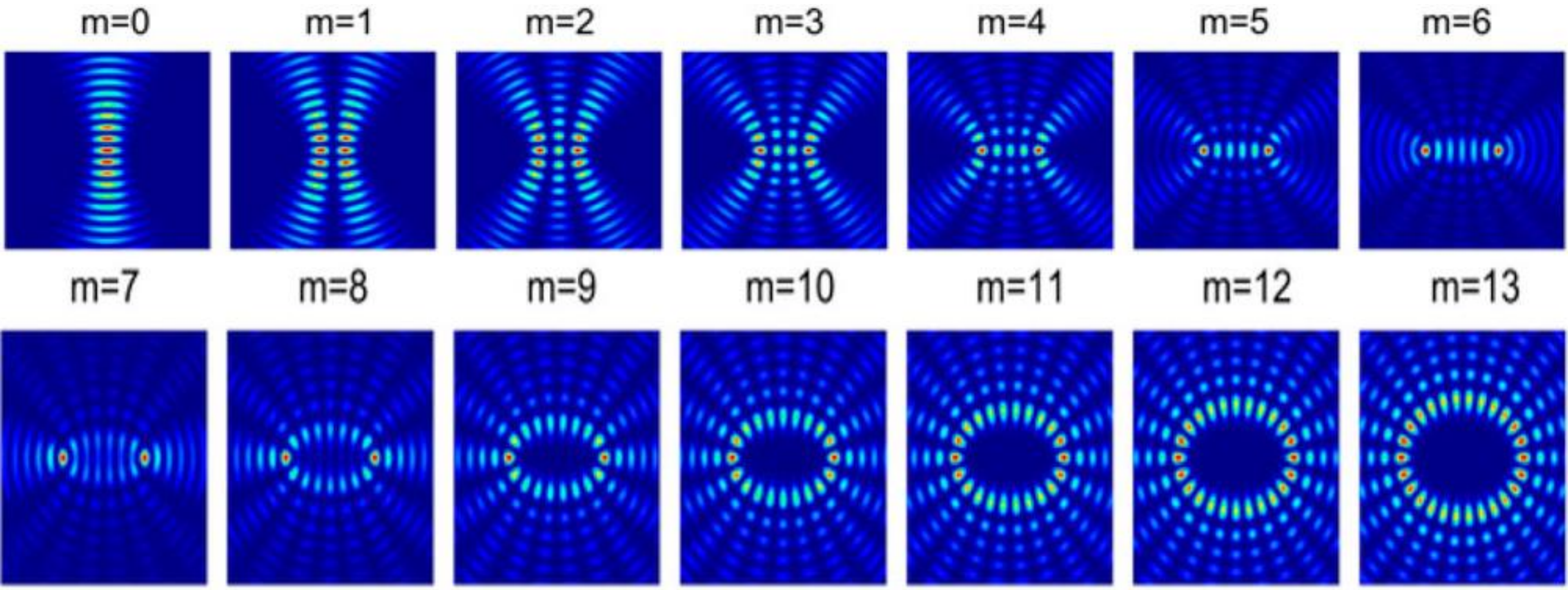
1. (Quasi-)Non-diffracting beams

Plane waves (in rectangular coordinates),
Bessel beams (in circular cylindrical coordinates),



1. (Quasi-)Non-diffracting beams

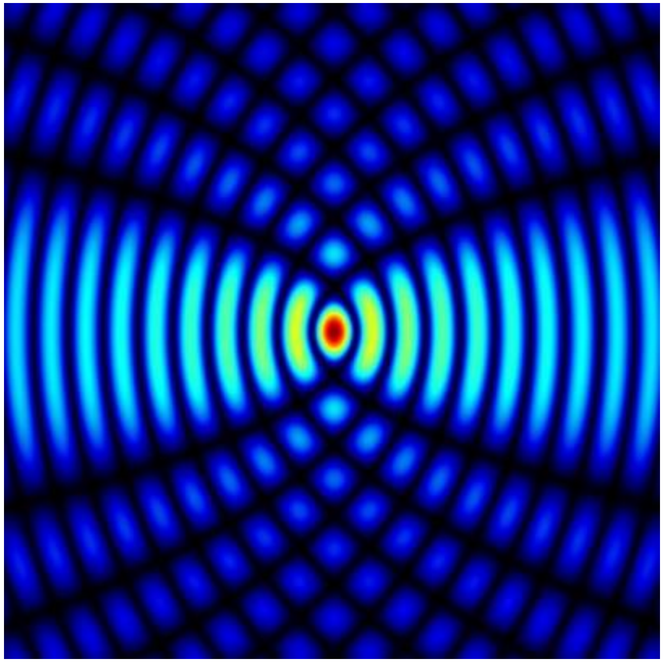
Plane waves (in rectangular coordinates),
Bessel beams (in circular cylindrical coordinates),
Mathieu beams (in elliptic cylindrical coordinates),



1. (Quasi-)Non-diffracting beams

Plane waves (in rectangular coordinates),
Bessel beams (in circular cylindrical coordinates),
Mathieu beams (in elliptic cylindrical coordinates), and
parabolic beams (in parabolic cylindrical coordinates)
are exact solutions of the Helmholtz equation (time-independent wave equation).

$$\nabla^2 A = -k^2 A$$

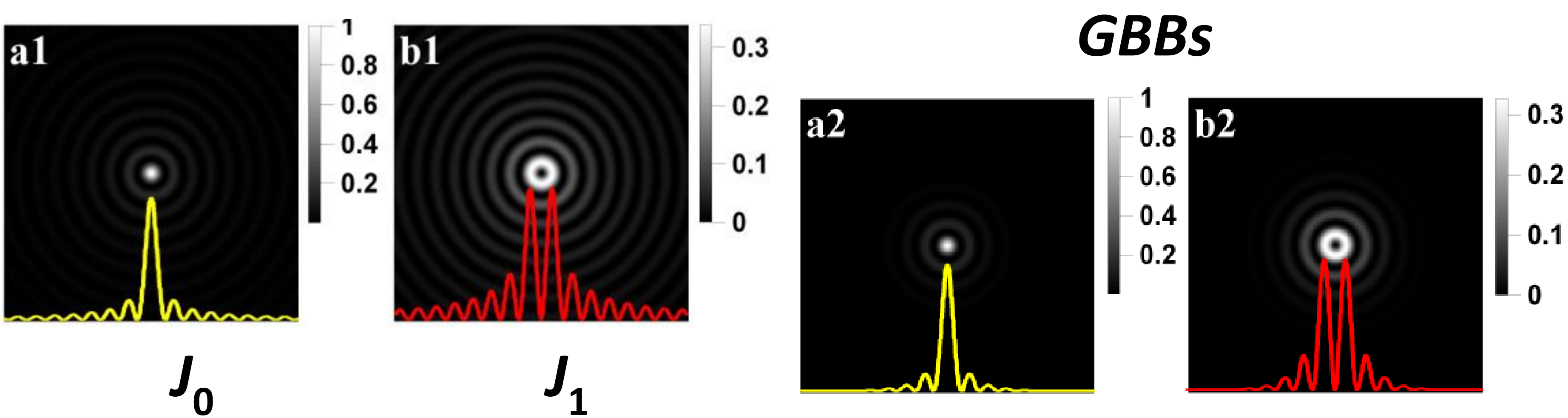


1. (Quasi-)Non-diffracting beams

"Non-diffracting beams" – beams whose central maxima are remarkably resistant to diffractive spreading commonly associated with all wave propagation.;

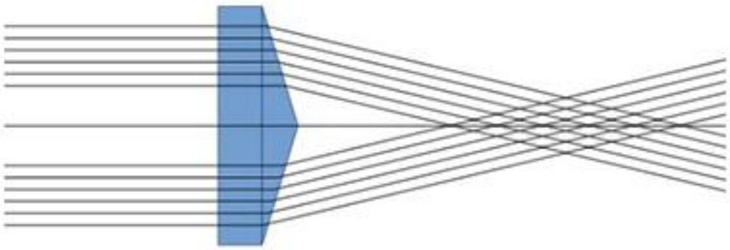
Mathematically, the Bessel beam has an infinite number of rings, and hence, it is carrying infinite power/energy and cannot be generated in the exact sense.;

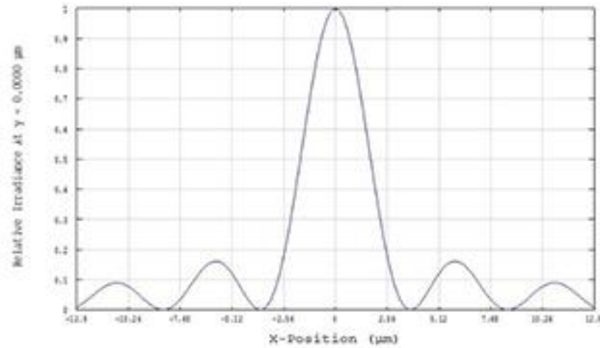
Characteristic for the Bessel beams are radial phase jumps of π resulting in a perfect destructive interference of the signal between the satellite rings of the Gauss-Bessel beams (GBBs) down to the noise level.

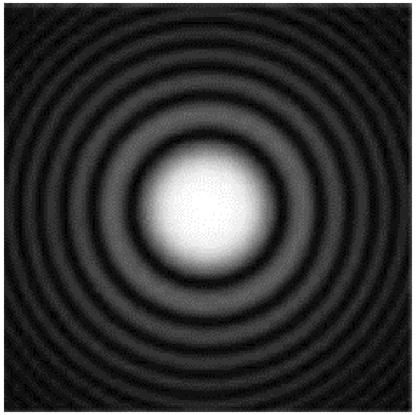


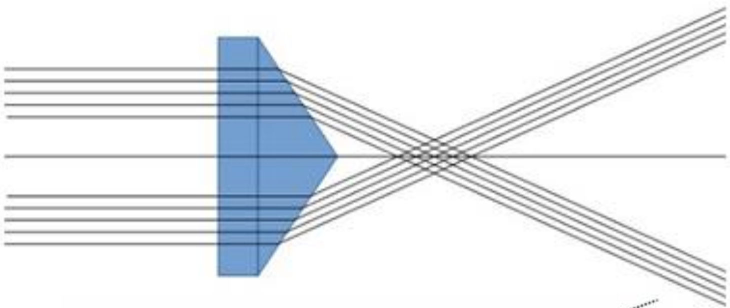
Classical methods for generation of GBBs

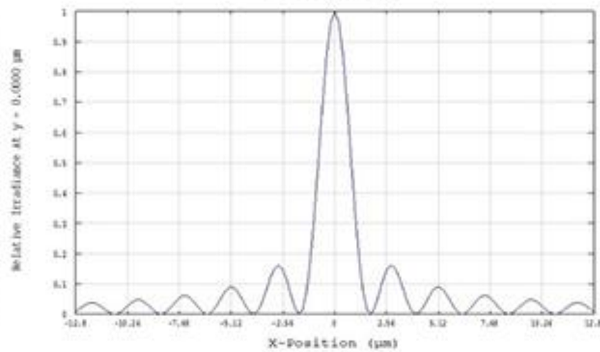
Generation of GBBs with AXICONS

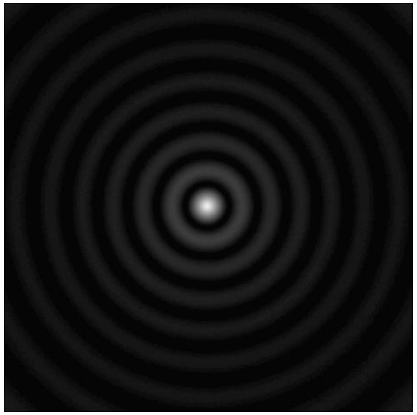


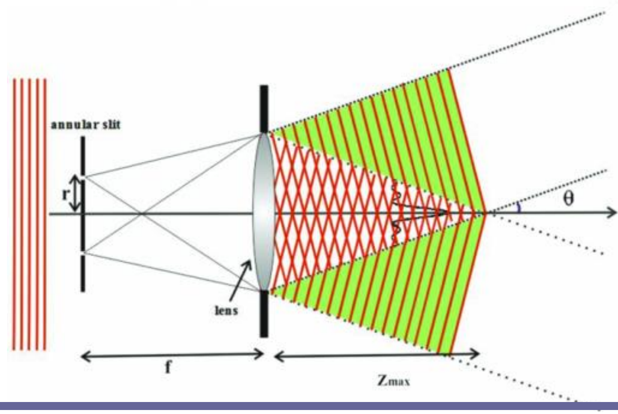








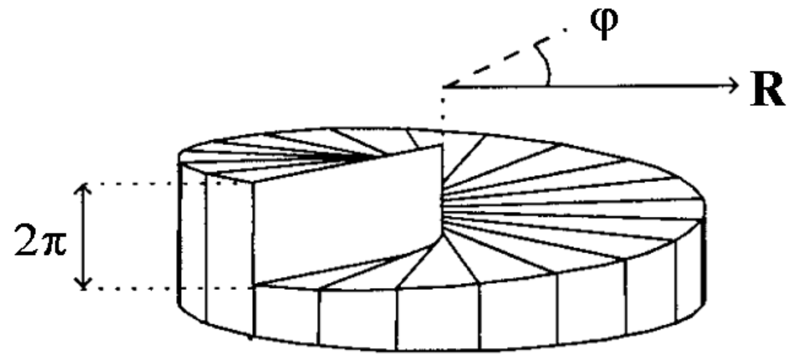




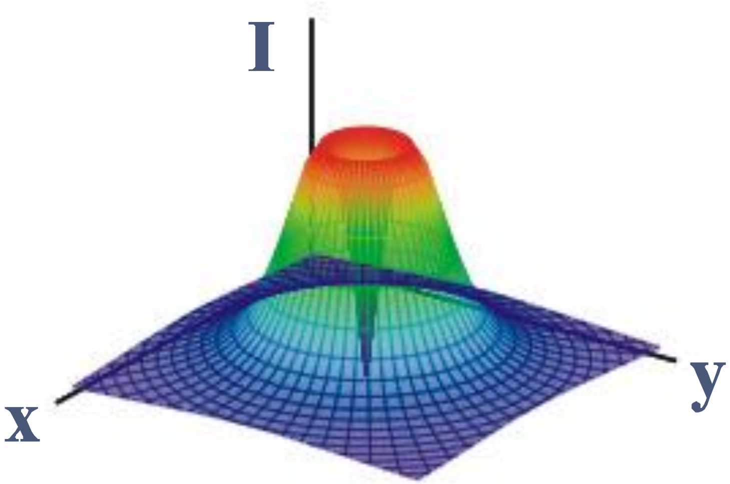
and with ANNULAR SLITS

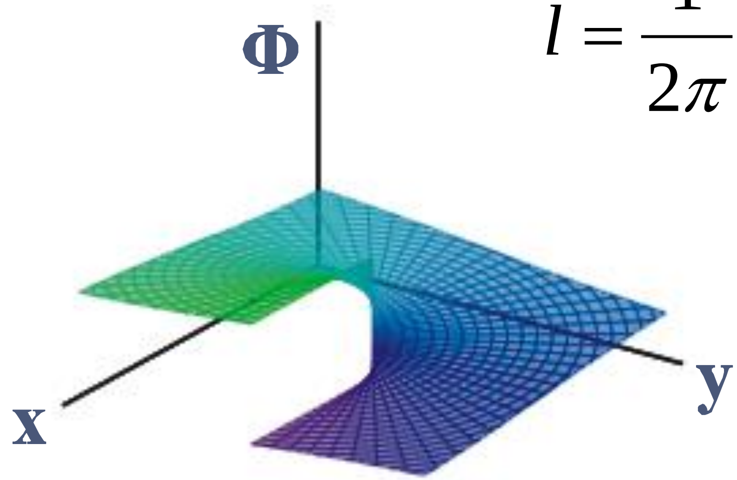
Optical Engineering, 50(7), 078002 (2011).

Optical vortices in brief



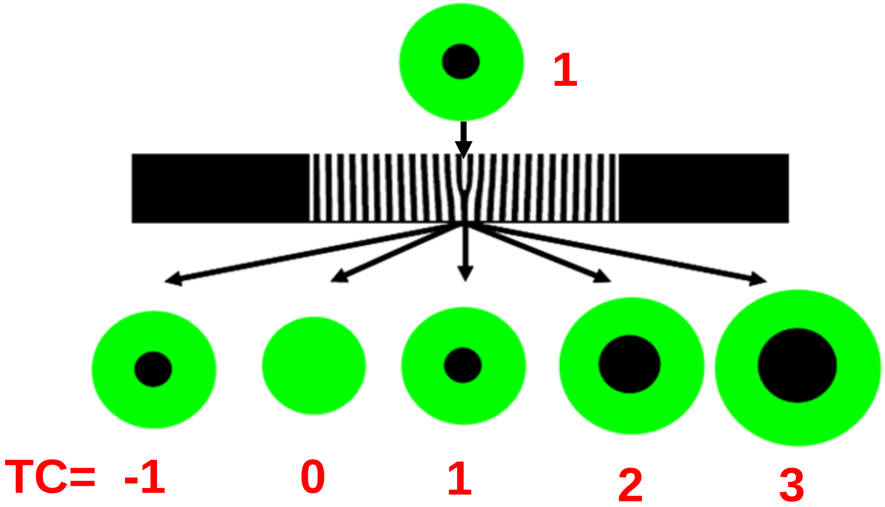
$$l = \frac{1}{2\pi}\oint d\varphi$$

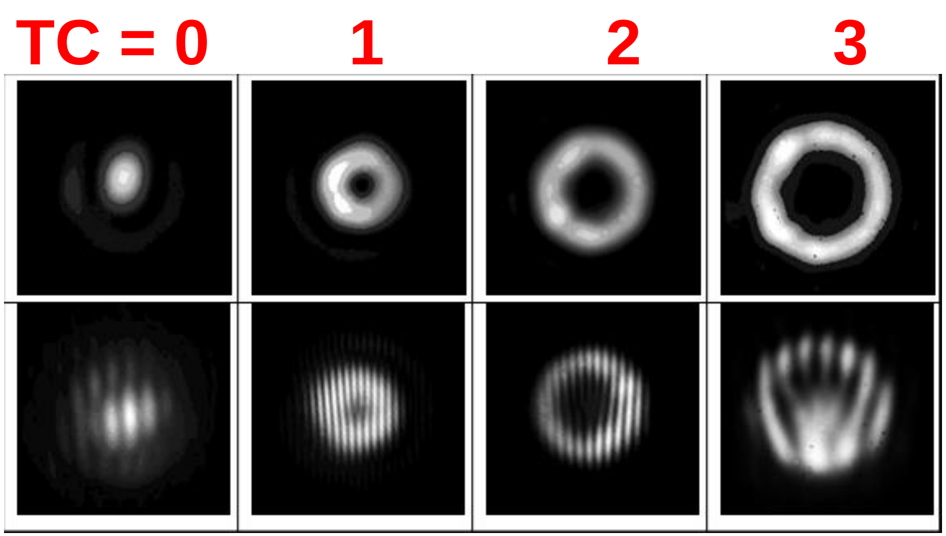




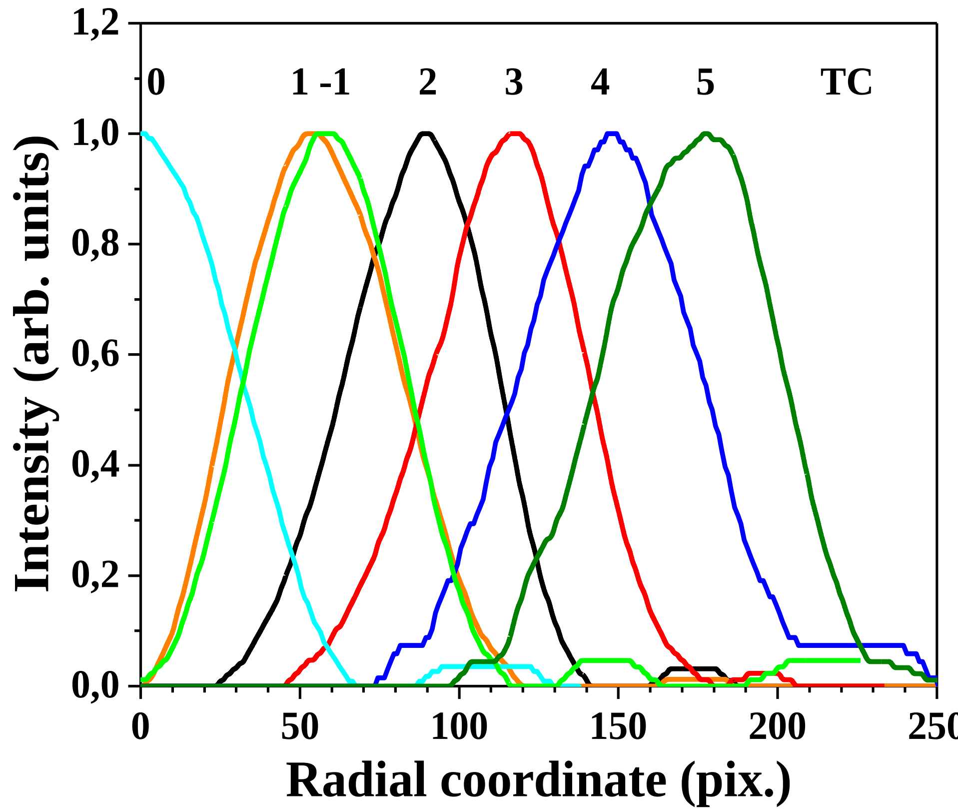
Arithmetics with TCs:

s=in±|m|CGH





Optics Commun. 350, 301–308 (2015)



$r_0 \gg \omega_0$

$r_0 \gg \omega_0$

Theoretical motivation

Let

$$E(r,\theta) = \exp(il\theta)\, exp\left\{-\frac{(r-r_0)^2}{\omega_0^2}\right\} = \exp(il\theta)\, exp\left\{-\frac{r^2+r_0^2}{\omega_0^2}\right\} exp\left\{\frac{2rr_0}{\omega_0^2}\right\}.$$

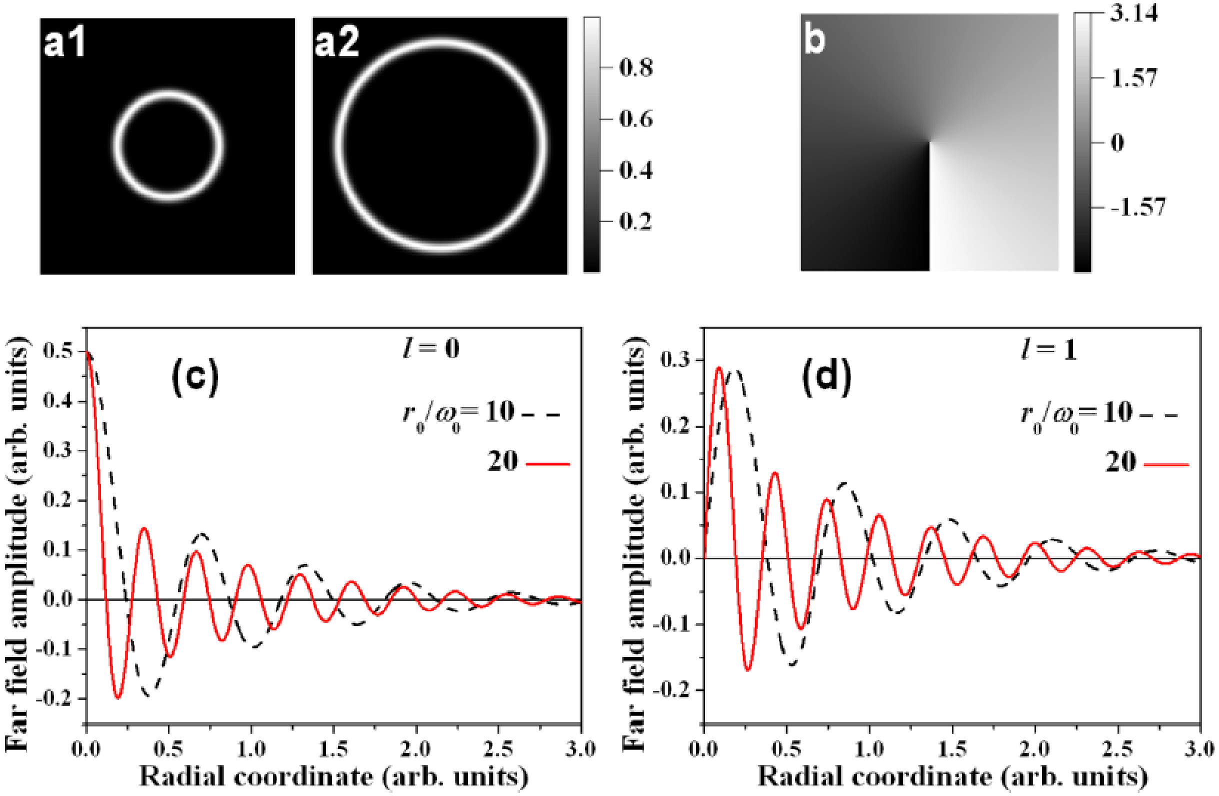
For $r_0 >> w_0$, the electric field amplitude in the (artificial) far field is

$$E'(\rho,\varphi) = F\{E(r,\theta)\} = \frac{\omega_0^2}{2} \exp\{il(\varphi - \pi/2)\}\, exp\left(-\frac{\omega_0^2\rho^2}{4}\right) J_l(r_0\rho).$$

Scientific Reports | (2020) 10:21981 | https://doi.org/10.1038/s41598-020-78613-7 nature research

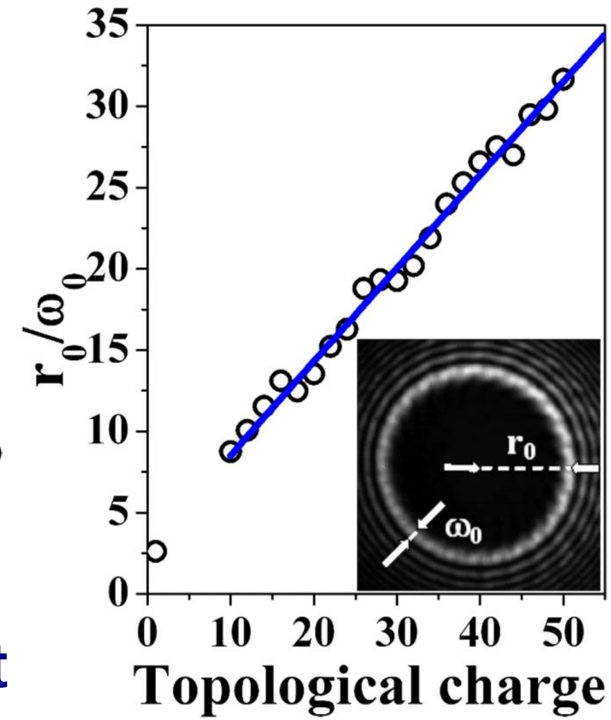
For the Gouy phase we obtain

$$\Phi_G(z) = -\frac{r_0^2 k}{2f^2} z + \frac{r_0^2 k}{2f^2} \frac{z^3}{(z^2 + L^2)} - \arctan\left(\frac{z}{L}\right).$$



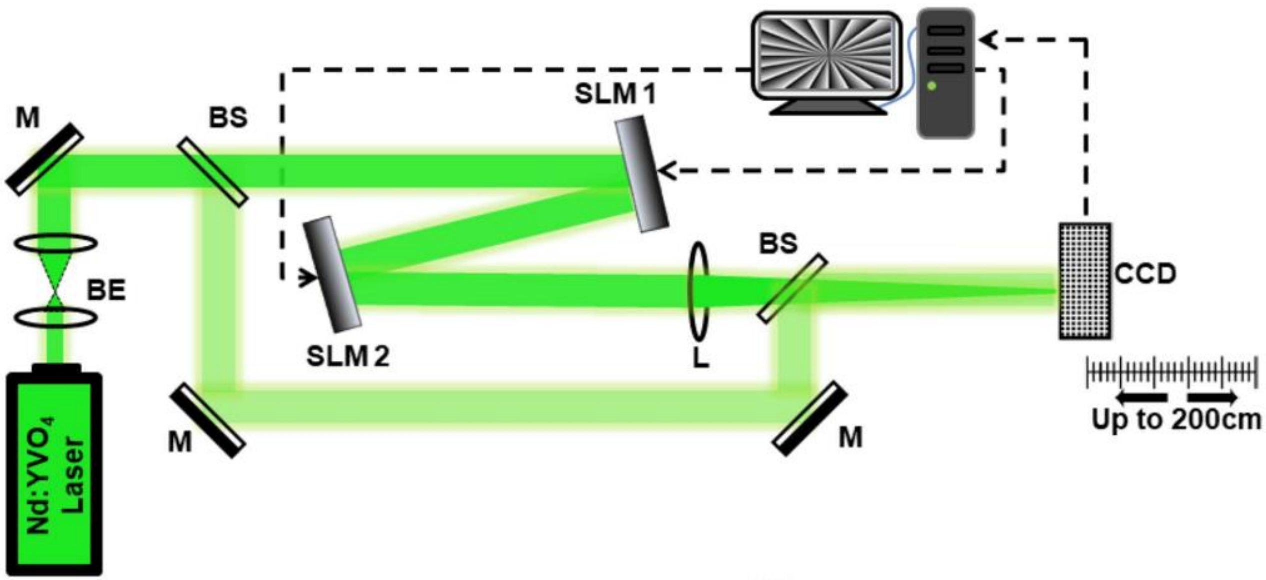
Theory

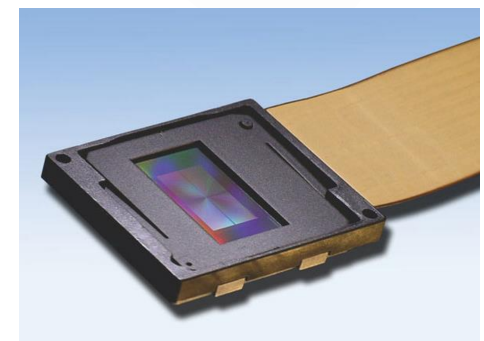
Experiment



Scientific Reports | (2020) 10:21981 | https://doi.org/10.1038/s41598-020-78613-7 nature research

Experimental setup



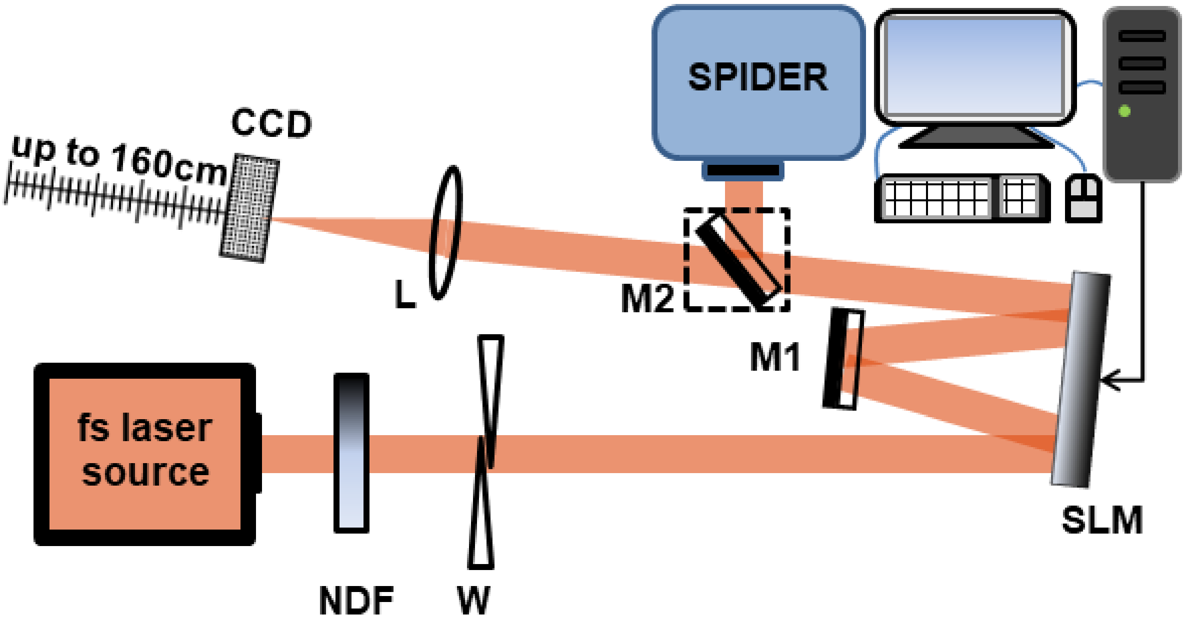


Nd : YVO$_4$ – CW laser @ λ= 532nm;
BE – beam expander;
BS – beam splitters; **M** - flat silver mirrors

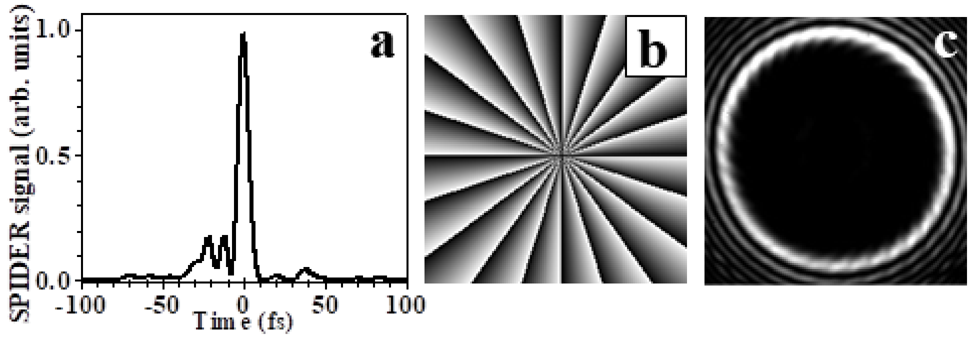
SLM – reflective spatial light modulator;
L – focusing lens (f = 100cm);
CCD camera place on a rail.

Scientific Reports | (2020) 10:21981 | https://doi.org/10.1038/s41598-020-78613-7 nature research

What is the case in the fields of femtosecond pulses?



SPIDER – commercial device performing spectral phase interferometry for direct electric-field reconstruction;

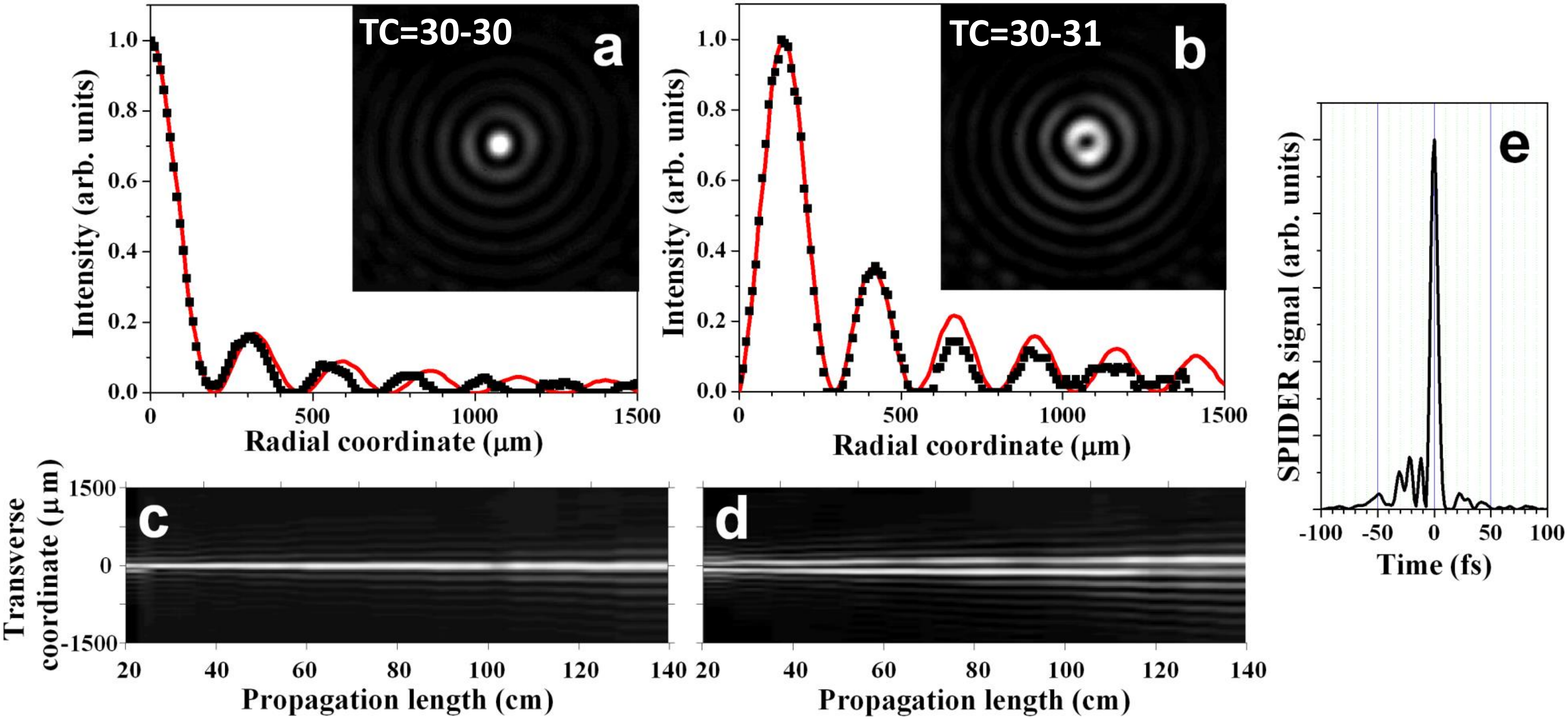


τ = ~7.5fs

TC=30

Optics Express, vol. 29, 10997-11008 (2021)

What is the case in the fields of femtosecond pulses?



Optics Express, vol. 29, 10997-11008 (2021)

The Gouy phase revisited

Gaussian beam with an amplitude

$$\mathbf{E}(r,z) = E_0\,\hat{\mathbf{x}}\,\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(-i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right)$$

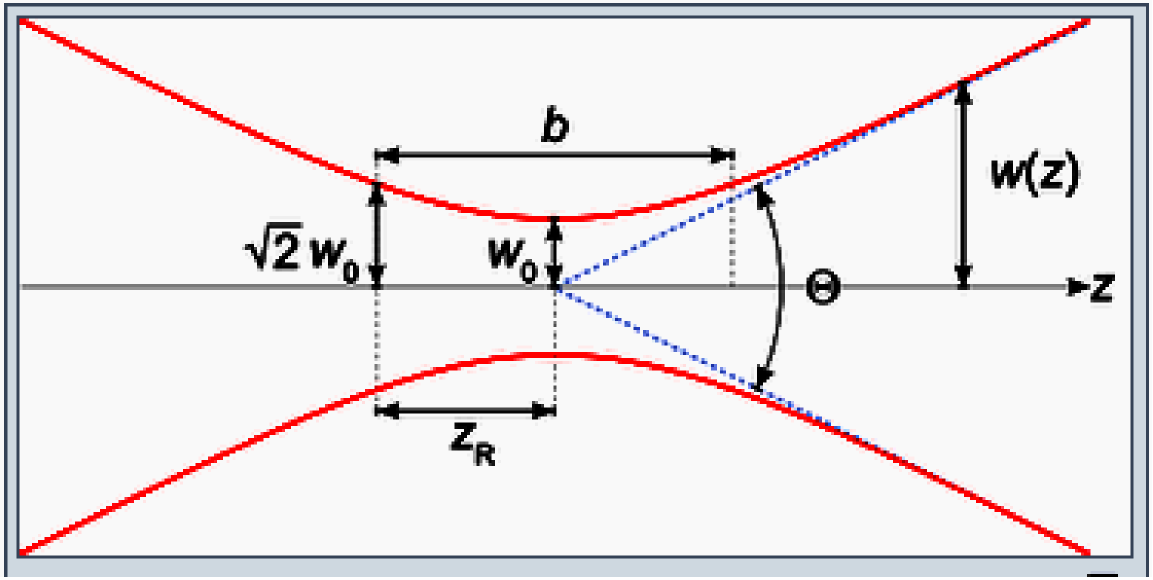
is a solution of the paraxial Helmholtz equation, where $\psi(z)$ is an additional phase term – the Gouy phase.

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2},$$

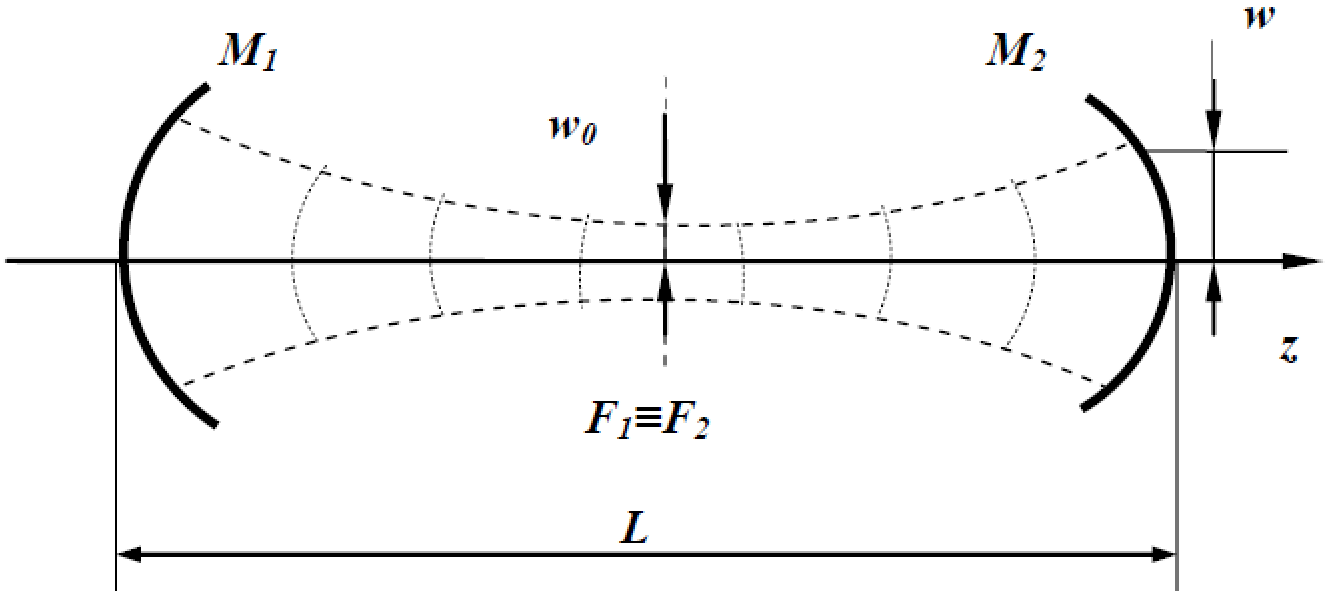
$$z_R = \frac{\pi w_0^2 n}{\lambda}$$

$$\frac{1}{R(z)} = \frac{z}{z^2 + z_R^2}$$

$$\psi(z) = \arctan\left(\frac{z}{z_R}\right)$$



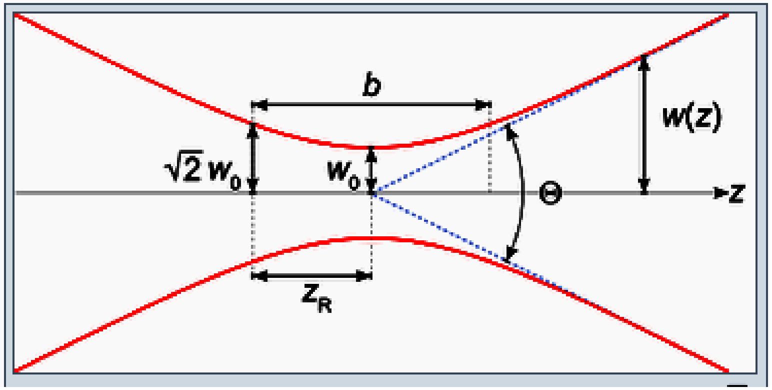
The Gouy phase revisited



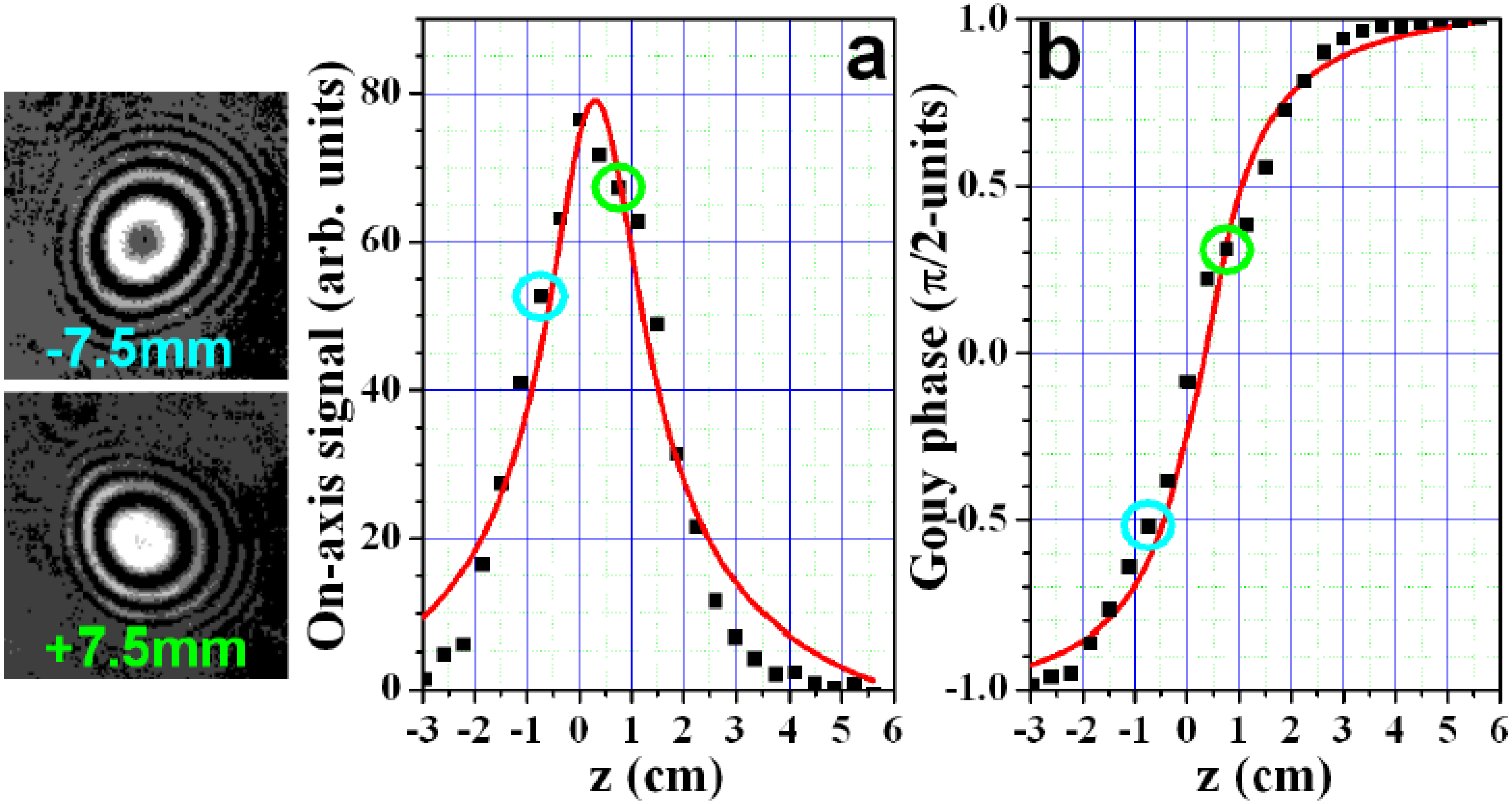
Longitudinal field distribution in a symmetrical confocal cavity.

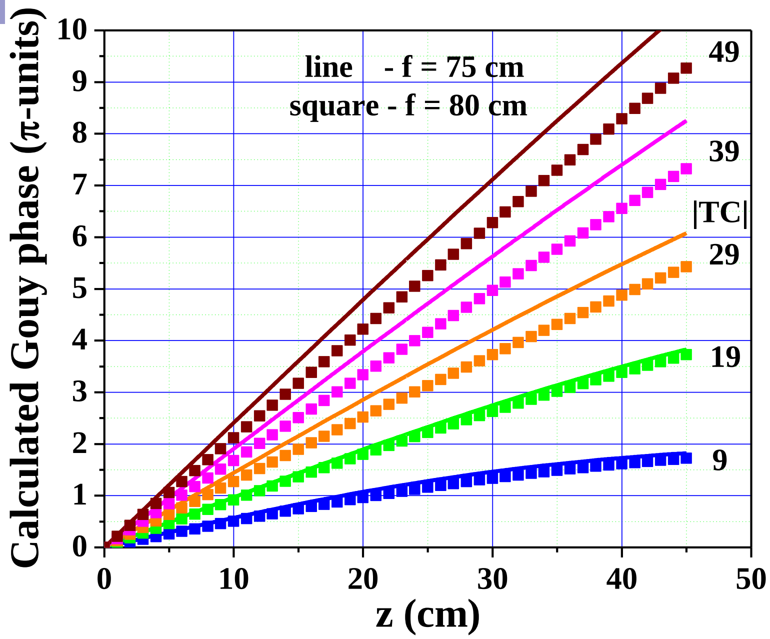
$$\frac{1}{R(z)} = \frac{z}{z^2 + z_R^2}$$

$$\psi(z) = \arctan\left(\frac{z}{z_R}\right)$$

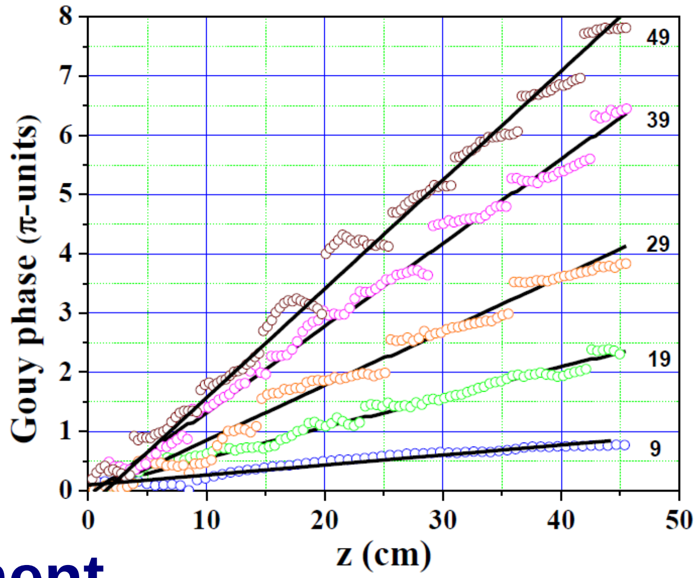


Gouy phase of a Gaussian beam





Gouy phase of GBBs

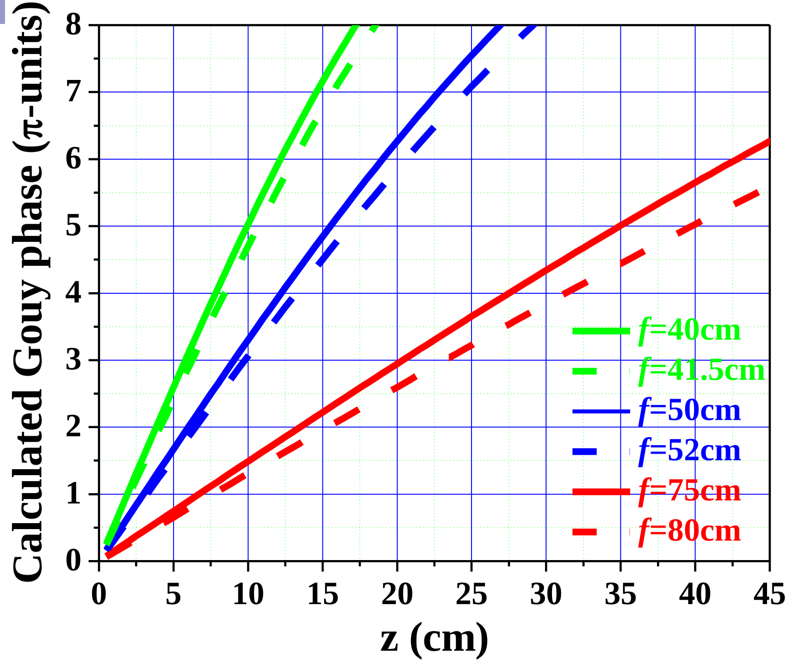


Theory **Experiment**

Experiment

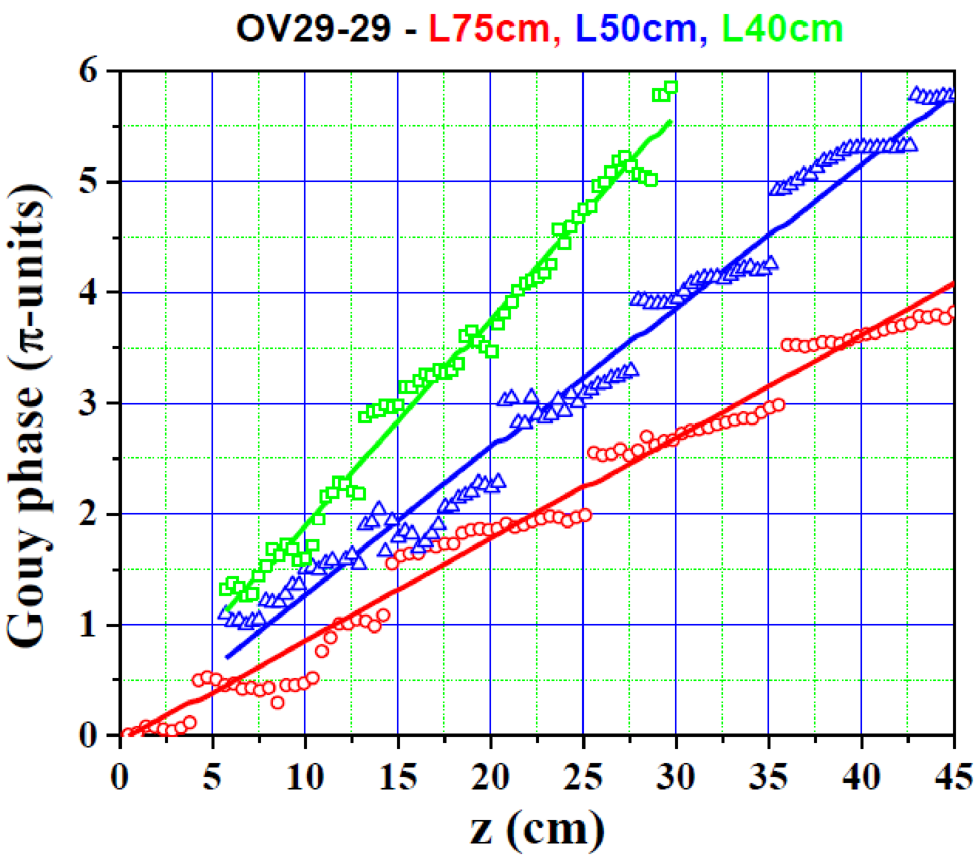
|TC|	r_0/ω_0	**Divergence half angle** (μrad)	**Slope** (lens focal length *f*=75 cm)
9	8.7	85	0.02(1)π/cm
19	13.5	67	0.04(3)π/cm
29	19.3	65	0.09(2)π/cm
39	26.5	49	0.14(2)π/cm
49	31.7	52	0.18(4)π/cm
Gaussian beam	-----	-----	0.38(4)π/cm (within the Rayleigh range)

Gouy phase of GBBs

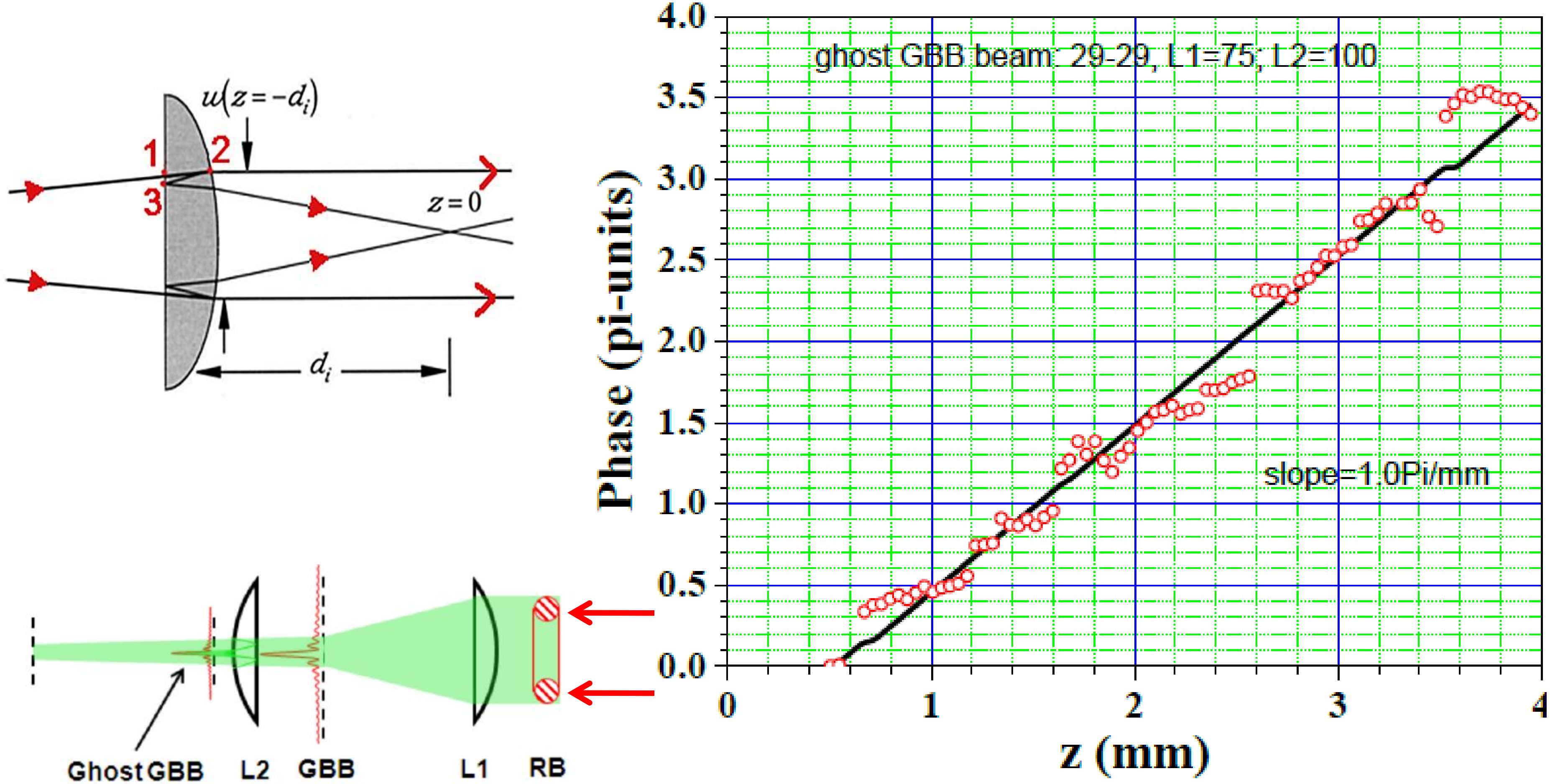


Theory

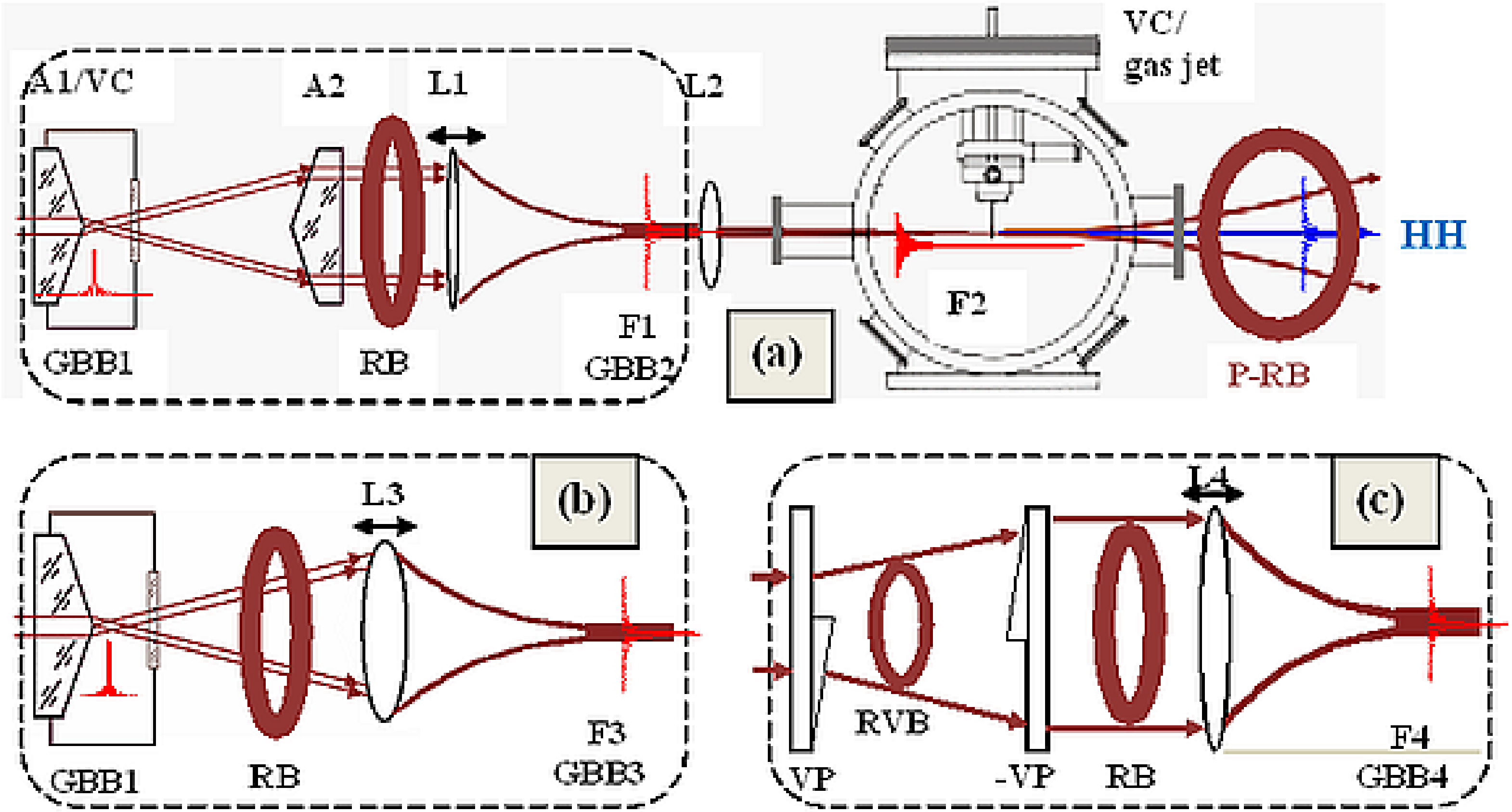
Experiment



J. Peatross and M. V. Pack, *Am. J. of Phys*. 69, 1169 (2001).



What next? … HH generation with femtosecond GBBs…



Emission wavelength – 1300 nm ; **Pulse energy** - 1.5mJ or higher; **Mean power** - 1.5W or higher
Pulse length ~ 60 fs or less ; **Pulse repetition rate** – 1kHz

My sincere gratitude for the fruitful cooperation goes to

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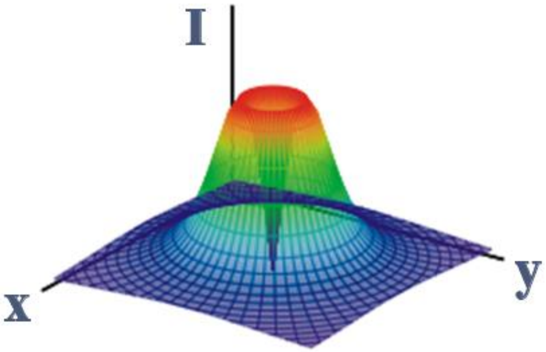
and to the colleagues from the Department of Quantum Electronics, especially to



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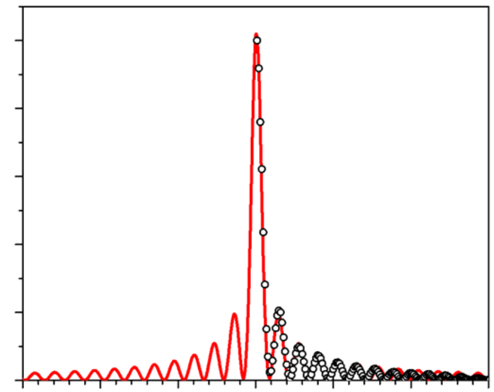

THANK YOU FOR YOUR ATTENTION!!!