On the nature of optical rogue waves

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Texas A&M University at Qatar

BPU11 Congress, Belgrade, Serbia, August 2022

Doha, Qatar


Downtown Doha:
A real photo


My place: The Pearl

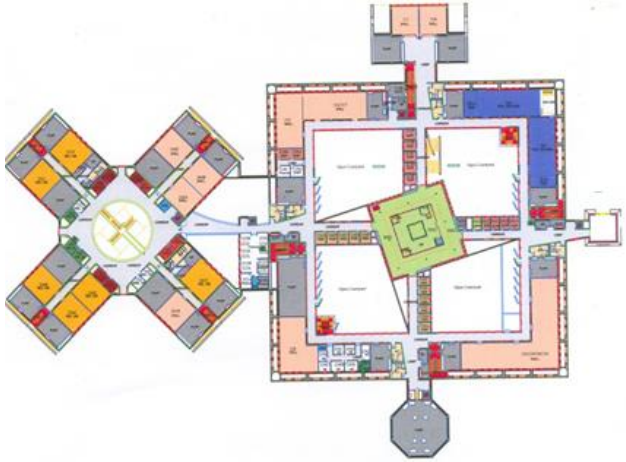

My supermarket:
The City Center Mall


My beach:
My secret

Education city

Member Schools

- VCU
- Weil Cornell
- TAMU
- Carnegie-Mellon
- Georgetown
- Northwestern
- Hamad bin Khalifa Univ.



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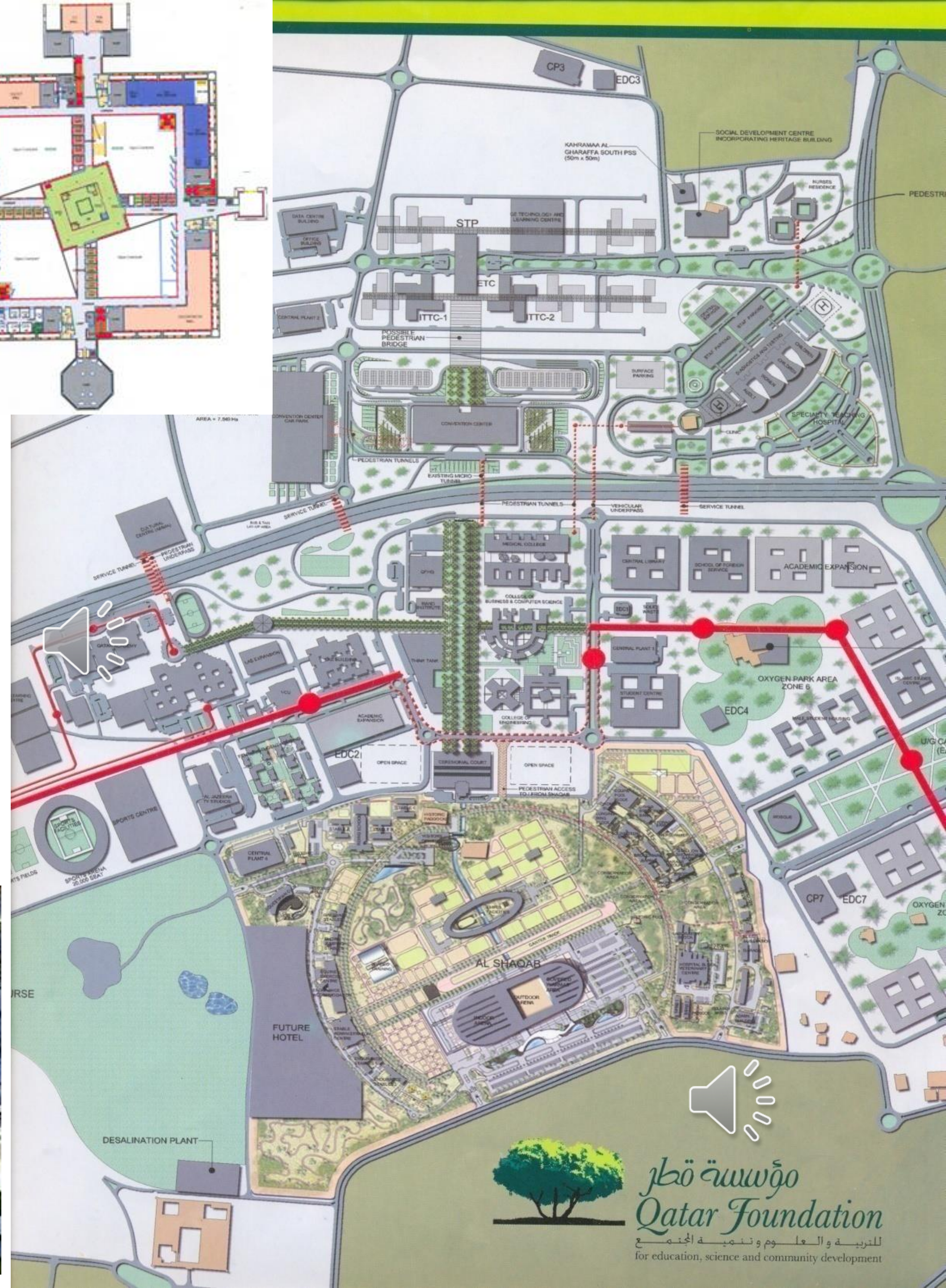

Ed City Stadium

Outline

Abstract: Rogue waves are giant nonlinear waves that suddenly appear and disappear in oceans and optics. We discuss the facts and fictions related to their strange nature, dynamic generation, ingrained instability, and potential applications. We propose the method of mode pruning for suppressing the modulation instability of rogue waves. We demonstrate how to produce Talbot carpets – recurrent images of light and plasma waves – by rogue waves, for possible use in nanolithography.

Nature of optical rogue waves: Facts and fictions

- Conflicting opinions: Are the rogue waves
- Linear *or* nonlinear?
- Random *or* deterministic?
- Numerical *or* physical?
- FACTS:
- Nonlinear, because the cause of RWs is the modulation or Benjamin-Feir instability;
- Deterministic, because modulation instability leads to deterministic homoclinic chaos;
- Physical, because they are observed in many experiments and media. But…

Reminder: What is a rogue wave?

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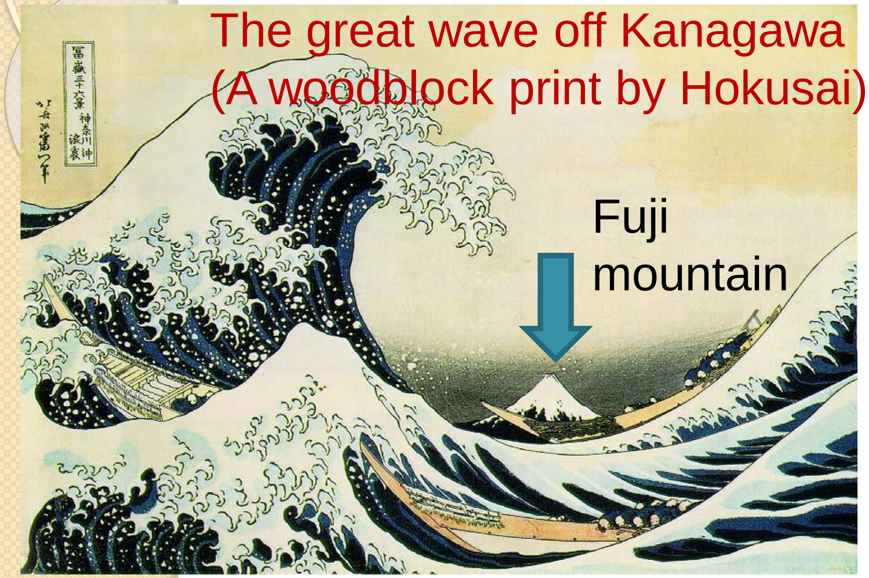

Can be described by the NLSE!

How does optics come into picture?

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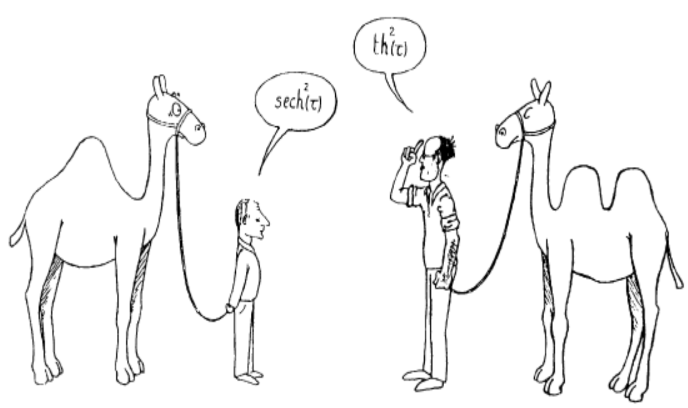
Through the Schrödinger Equation! Paraxial wave equation in optics is equivalent to the SE in QM

$$i\partial_t u = -\frac{1}{2}\partial_x^2 u + \kappa|u|^2 u$$

κ : Nonlinearity strength

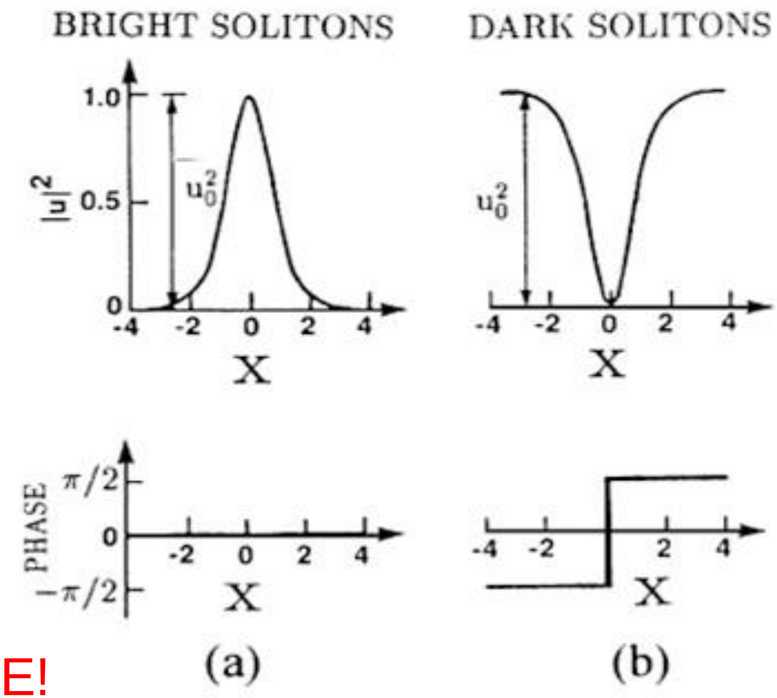
$$\begin{cases} \kappa < 0 & \text{: Bright soliton} \\ \kappa > 0 & \text{: Dark soliton} \end{cases}$$



Do these 'animals' belong to the same soliton family? (the drawing made by Marc Haelterman in 1989).

Intensity and Phase of the Soliton

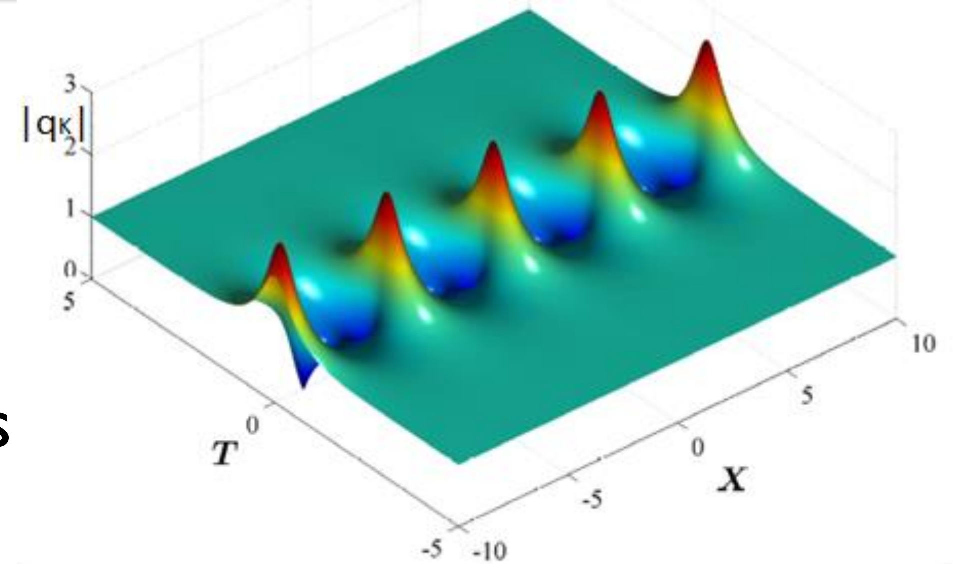
Basic stable solutions

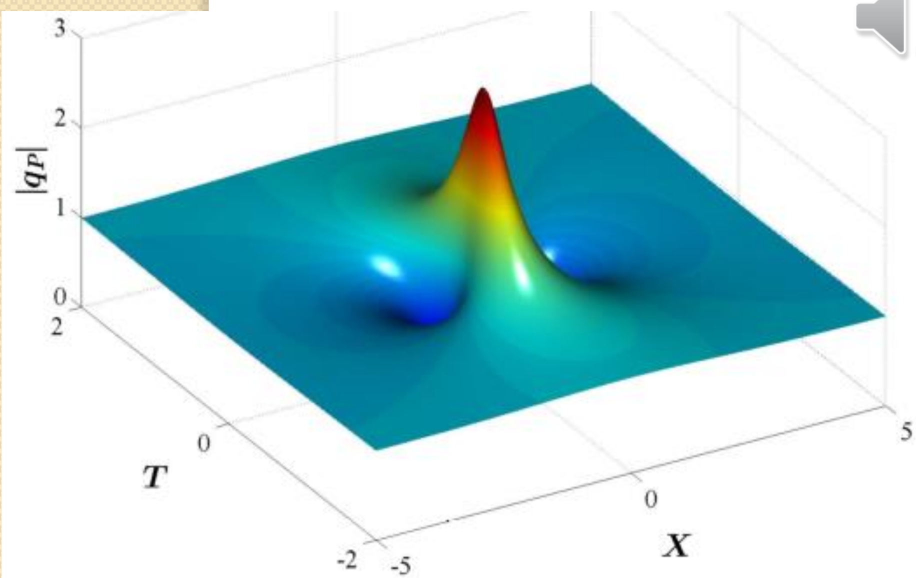


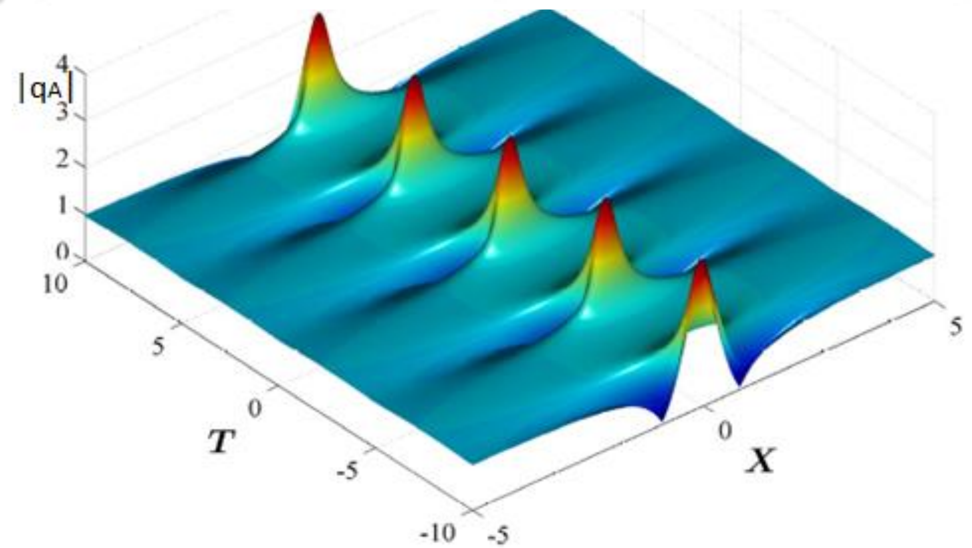
Rogue waves are also solutions of the NLSE!

Basic rogue waves in NLO

- Peregrine soliton
- Kuznetsov-Ma breather
- Akhmediev breather
- Higher-order breathers: RWs







Solutions of NLSE $iq_X + q_{TT} + 2|q|^2 q = 0$

Real rogue waves: 2nd and higher-order solutions

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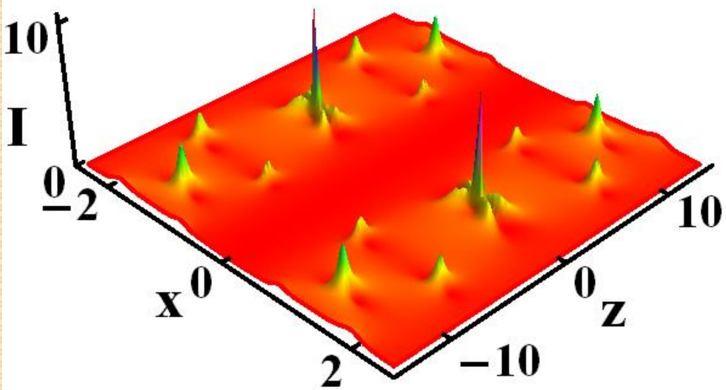
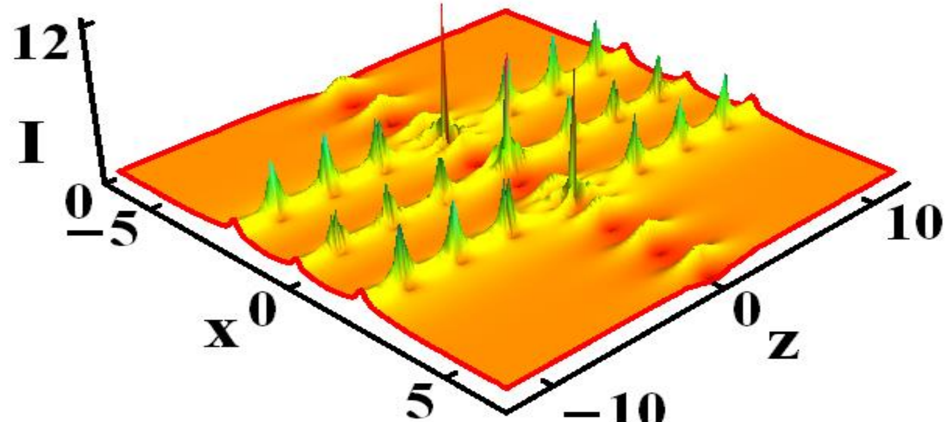
2nd and 3rd order from interacting KMBs, PSs, and ABs

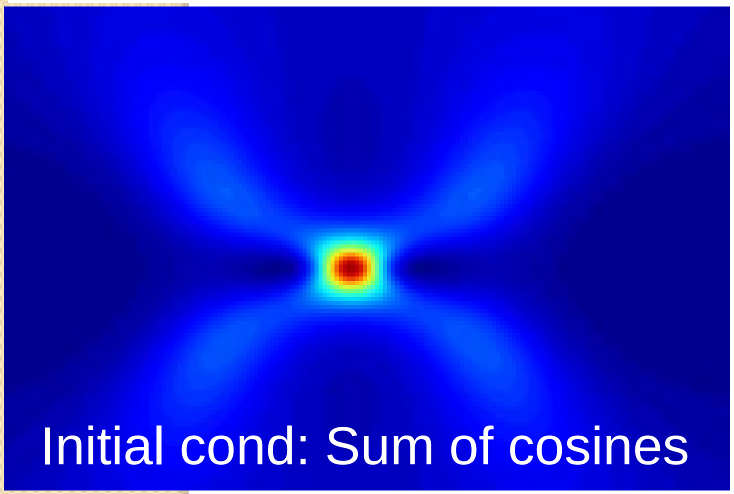
(Roadmap paper on optical RWs, 2016)

Weak interaction

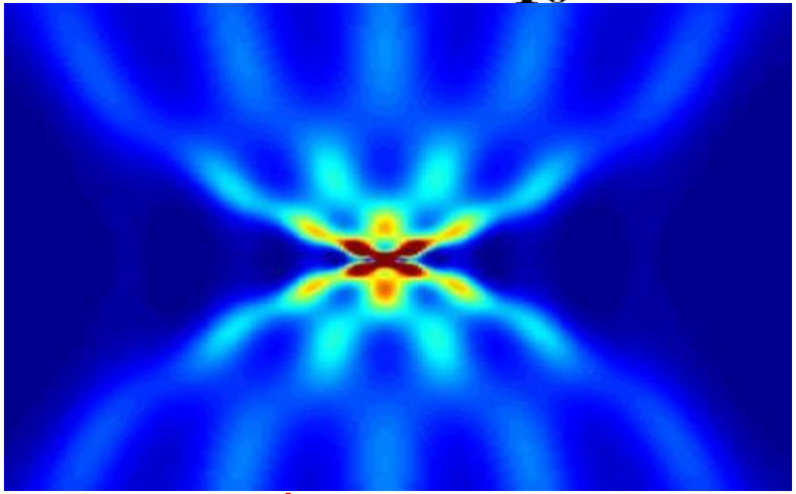


Strong interaction





Dynamically Generated RWs using Darboux Transform



Basic 2nd order RW

5th order RW!

The problem: Modulation Instability

- Basic nonlinear optical process in which a weak perturbation of the background wave produces an exponential growth of spectral sidebands that constructively interfere to build RWs.
- Relevant for the generation of RWs from ABs.
- Key effect: Homoclinic chaos caused by MI of ABs.
- Key question: How observable RWs with MI are?

Major difficulty: How to distinguish RWs from numerical artefacts!

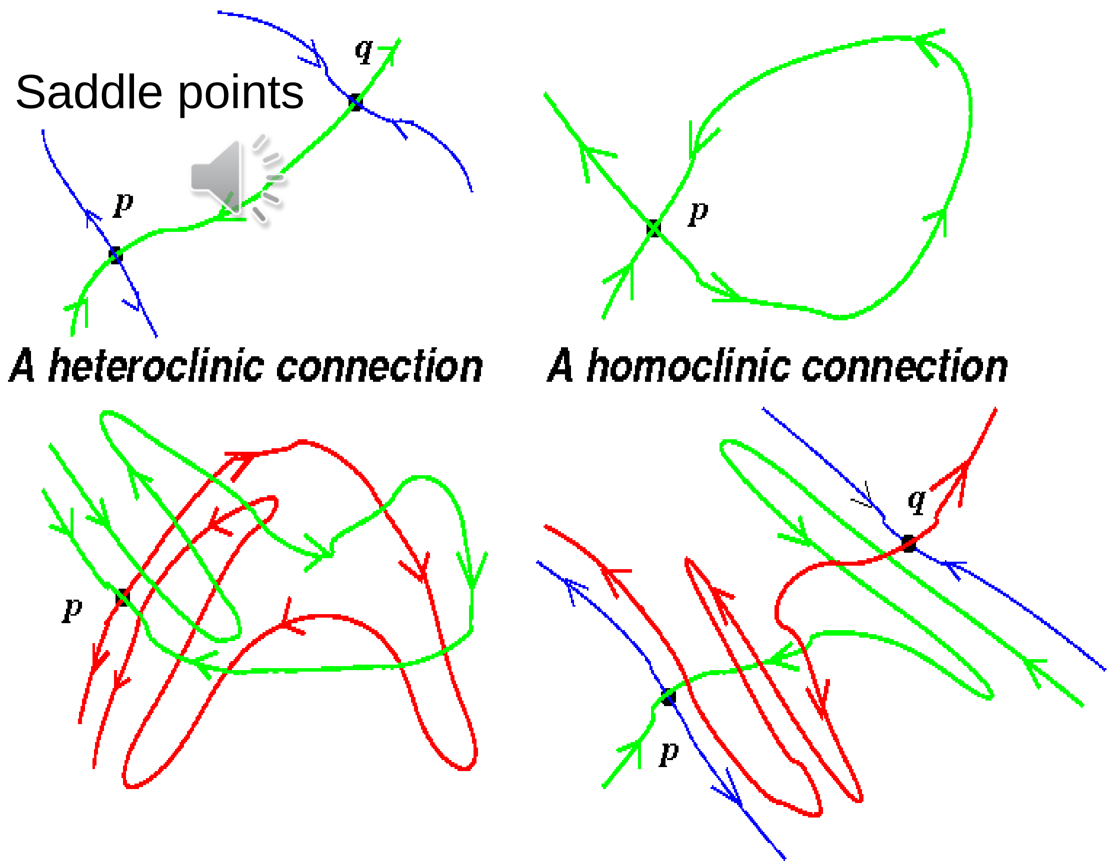
M. J. Ablowitz *et al.*, SIAM J. Appl. Math. 50, 339 (1990)

S.A. Chin et al, Phys. Lett. A380, 3625 (2016)

Homoclinic chaos

- Dynamically, optical RWs represent homoclinic orbits of unstable sideband modes that, due to MI, generate homoclinic chaos.
- Chaos brings sensitive dependence on initial and boundary conditions
- Init'l cond's are found using DT, followed by numerical integration.

Numerical integration leads to problems!

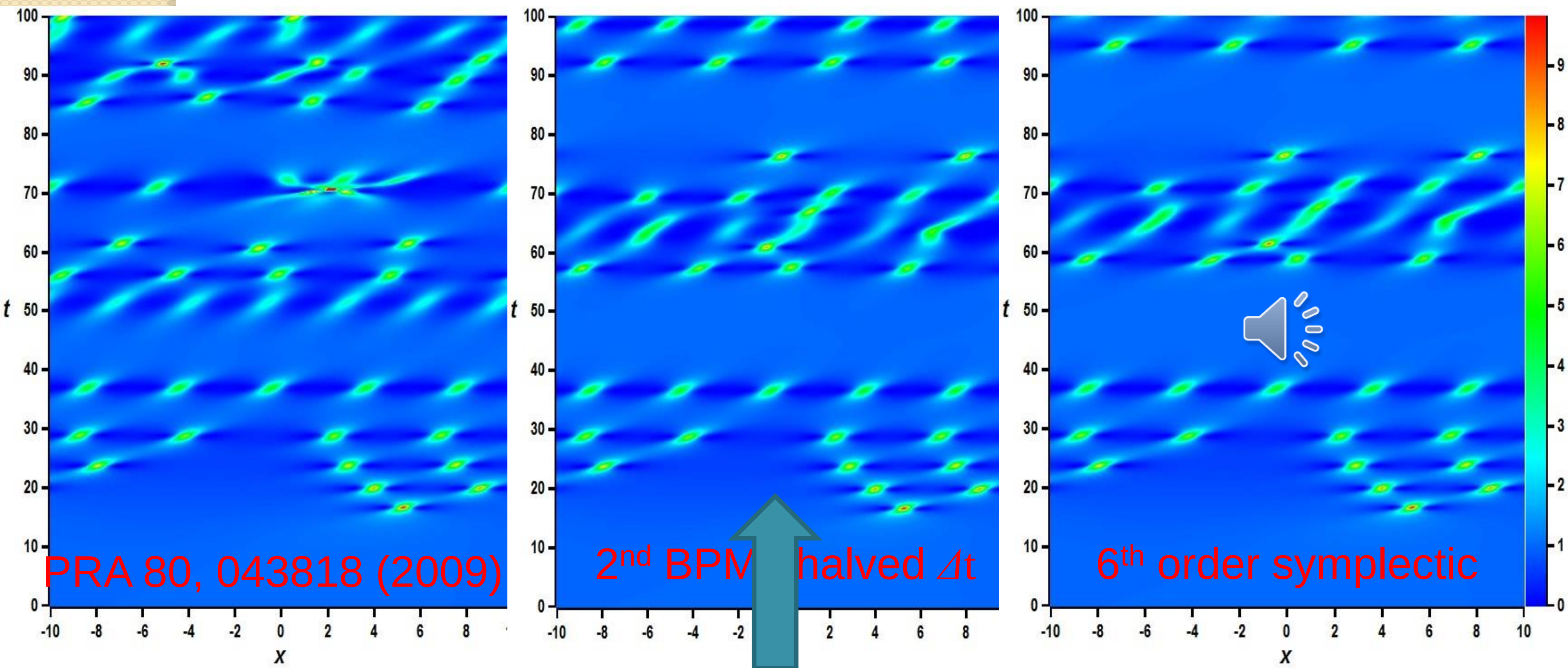


A heteroclinic connection *A homoclinic connection*

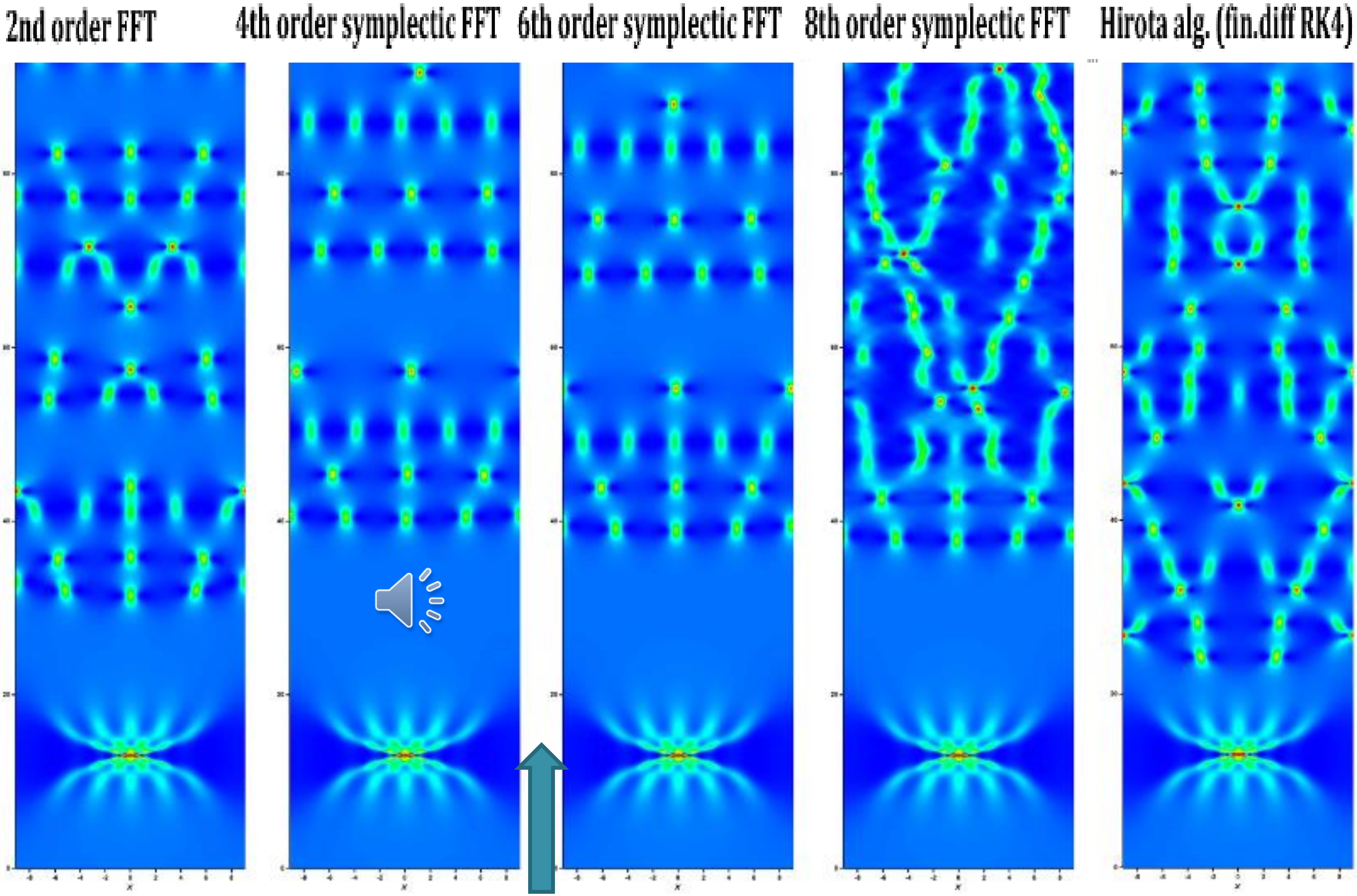
A homoclinic intersection *A heteroclinic intersection*

Numerical Instabilities and Chaos

- Round-off errors grow
- Instabilities develop in numerical solutions
- Spurious RWs appear — Belic *et al.*, NODY 108, 1655 (2022)
- Are the RWs seen part of the model or numerics?

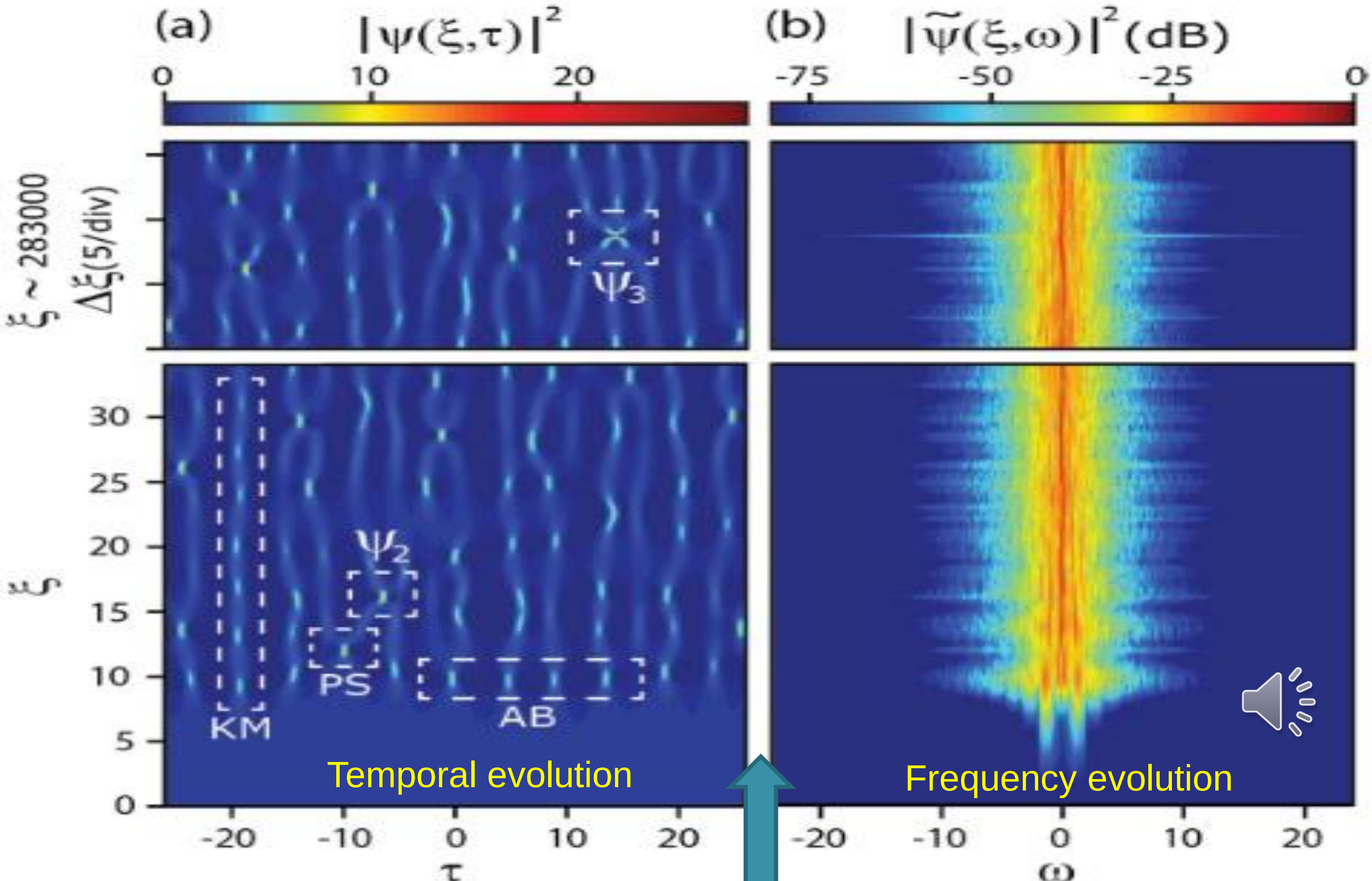


Different algorithms lead to different chaos!



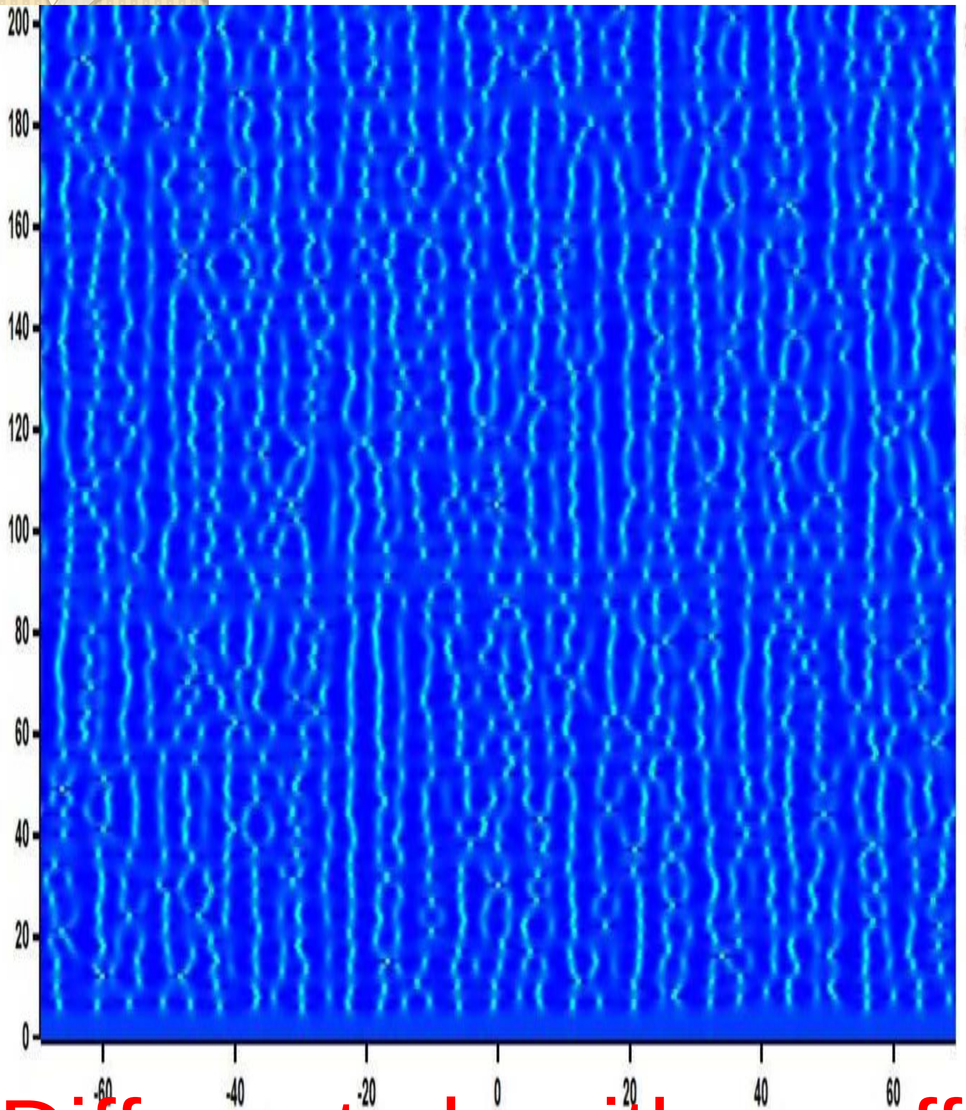
How to trust RW generation from noise?

Toenger et al., Sci. Reps. 5,10380 (2015)

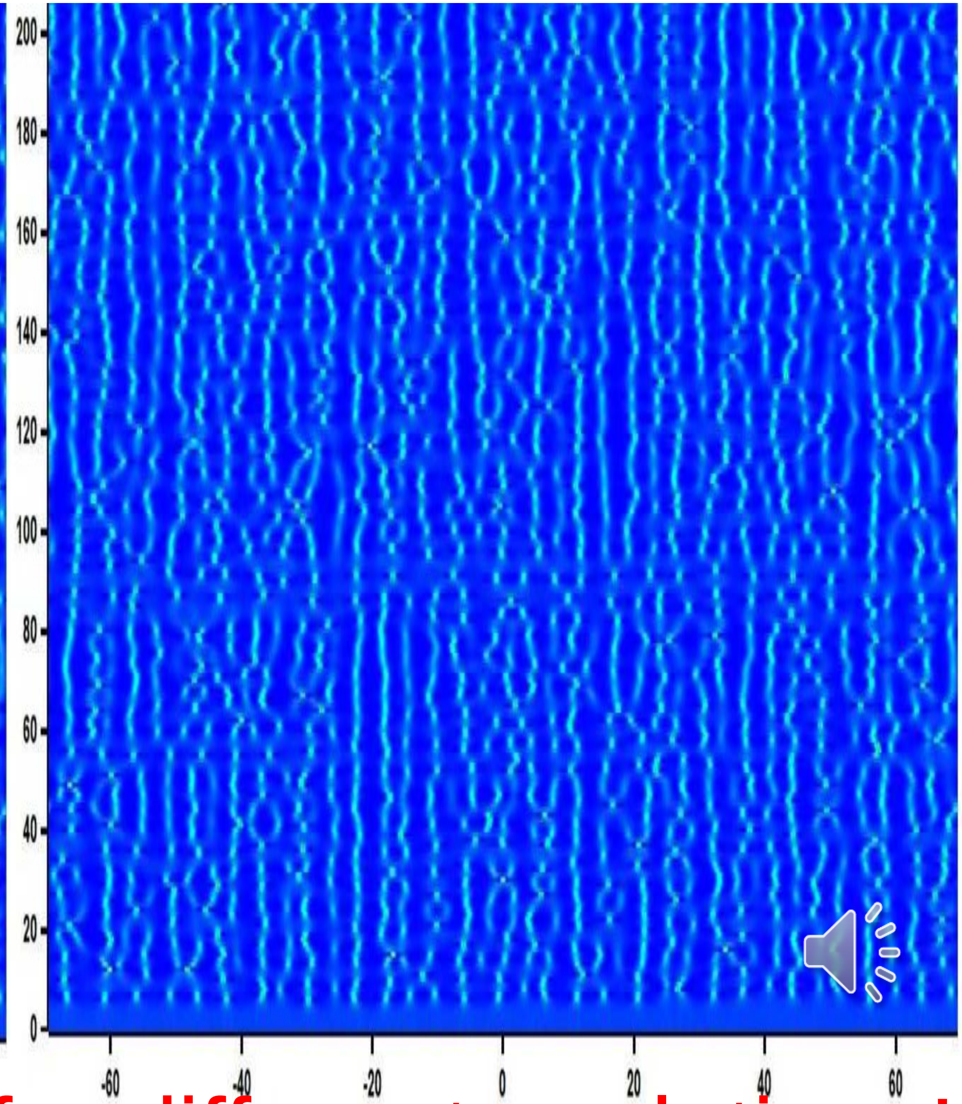


Our RW generation from white noise

2nd-order beam propagation method



4th-order symplectic algorithm



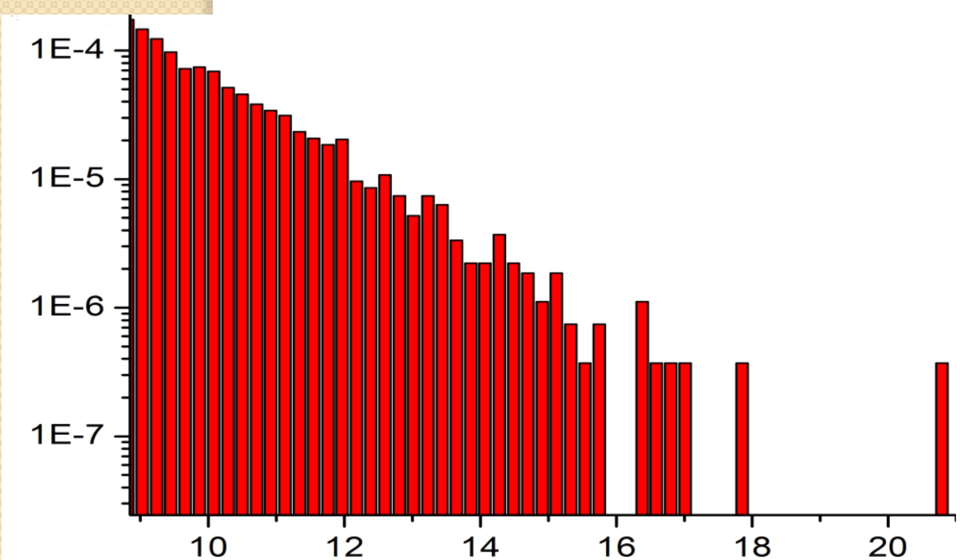
Different algorithms offer different evolutions!

Statistics of high intensity peaks

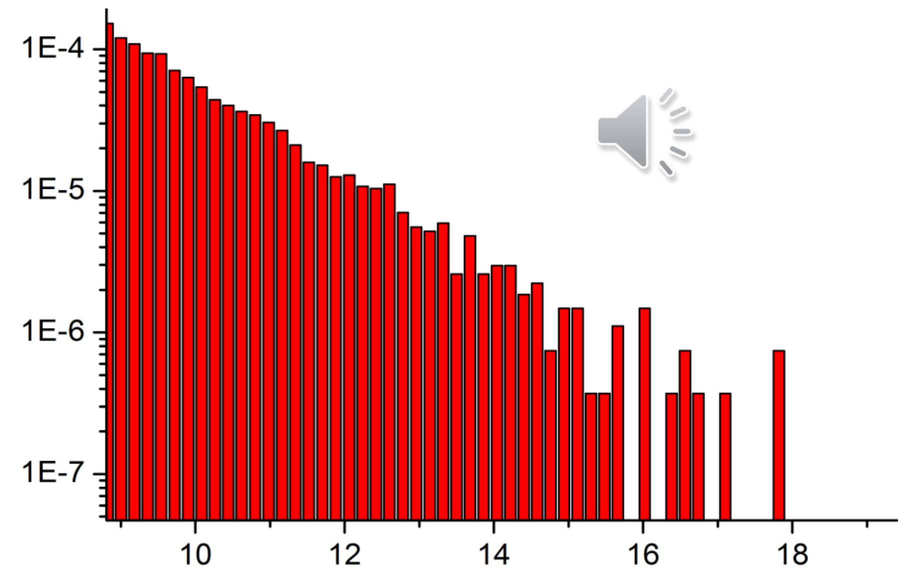
Belic *et al.*, NODY 108, 1655 (2022)

- The number of peaks, the maximum of intensity, and the slope of distributions, among other things, are different.
- The intensity scale starts from the intensity of Peregrine soliton I_{PS}=9.
- 6.5x10^6x2048 grid points and more than 10^6 peaks above I_{TH}=2.
- There are 2346 RW peaks on the left and 2643 on the right.

Different algorithms lead to different statistics!



2nd-order beam propagation method

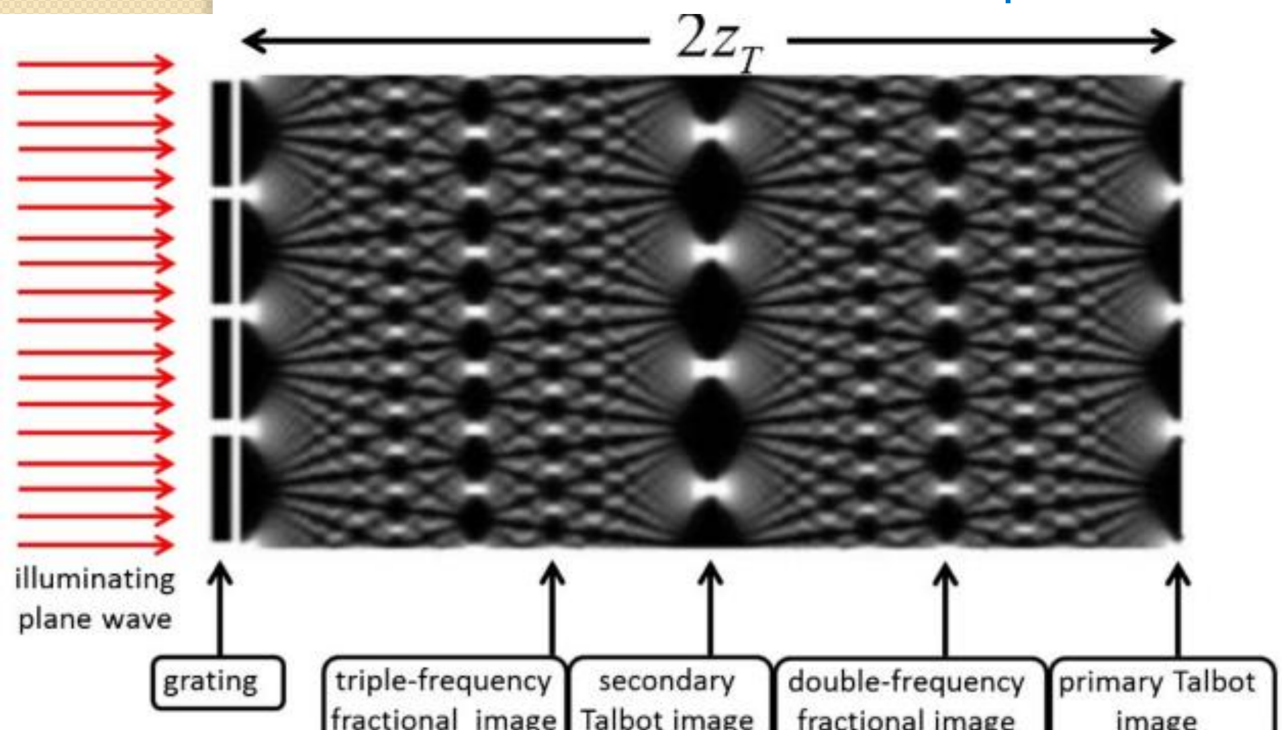


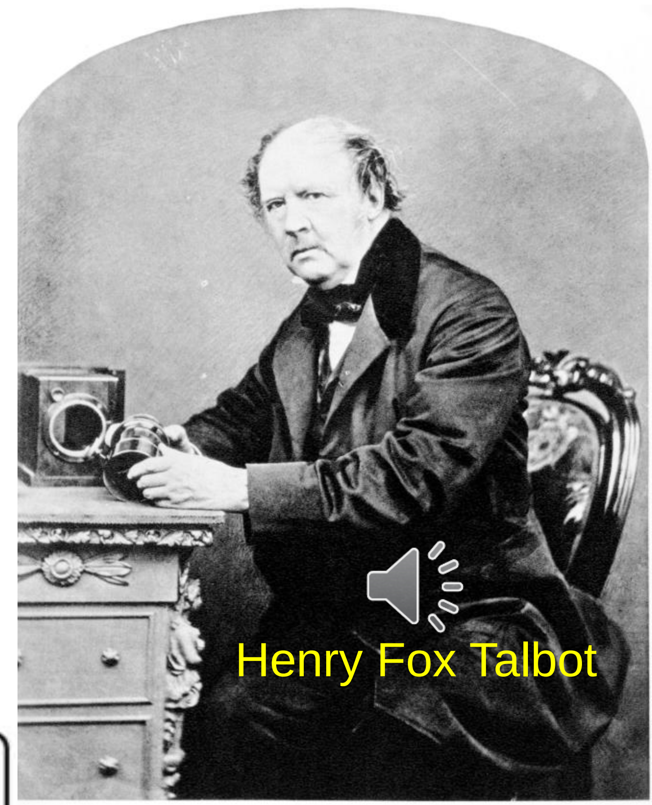
4th-order symplectic algorithm

Fallout: Talbot Effect of RWs

Wikipedia: **Talbot effect** is a near-field diffraction effect observed in 1836 by Henry Fox Talbot. When a plane wave is incident upon a periodic diffraction grating, the image of the grating is repeated at regular distances away from the grating plane. The regular distance is called the Talbot length, and the repeated images are called self-images or Talbot images. At half the Talbot length, a self-image also occurs, but phase-shifted by half a period. At smaller regular fractions of the Talbot length, sub-images can also be observed. The overall image is known as the Talbot carpet.

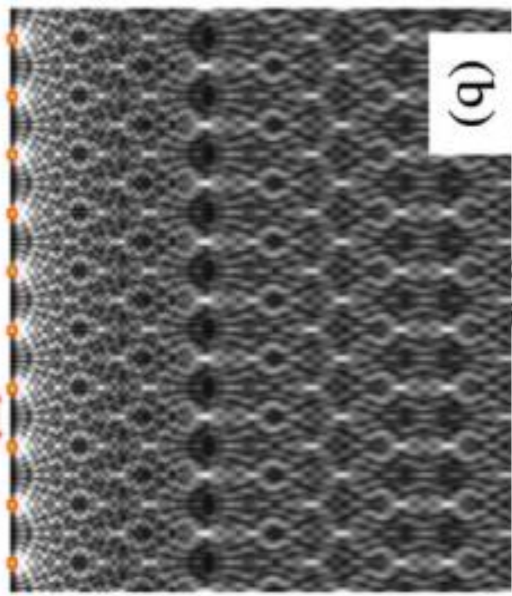
Talbot carpet

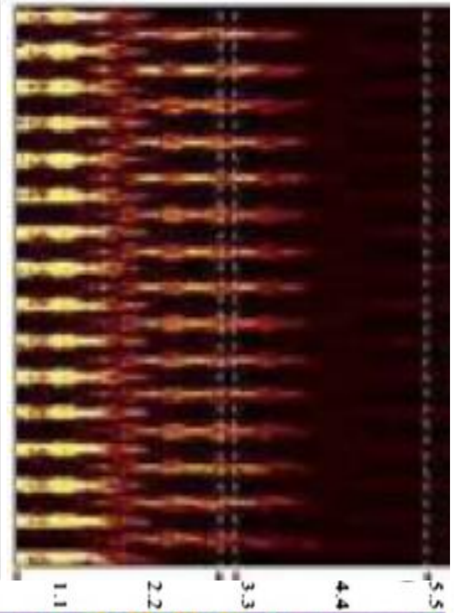


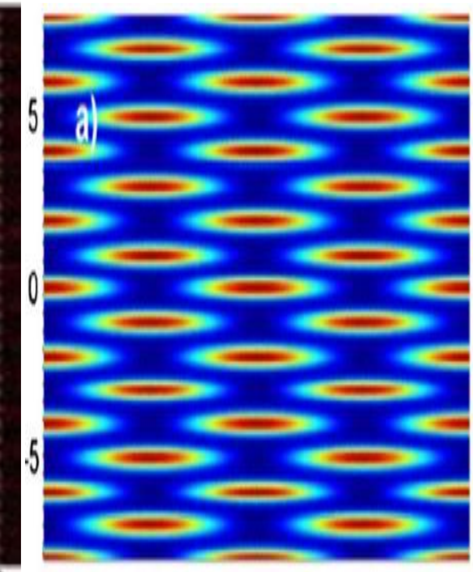


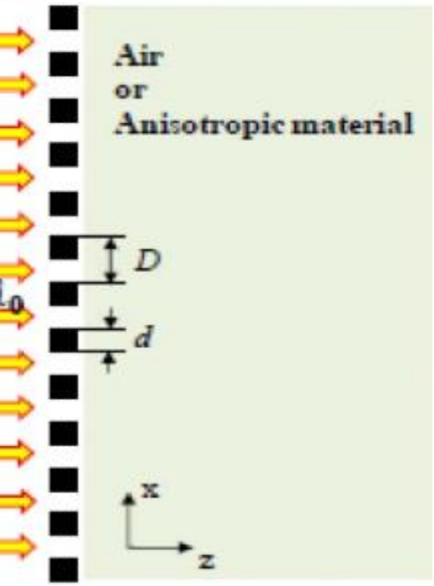
Current research on Talbot carpets

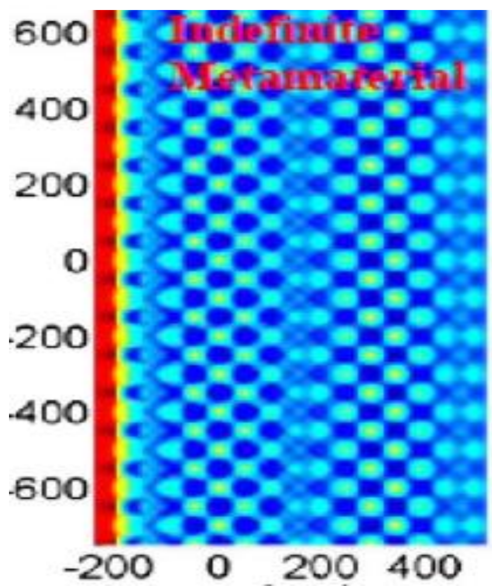
SPP from drilled nanoholes | SPP on silver | SPP experimental | Discrete Talbot

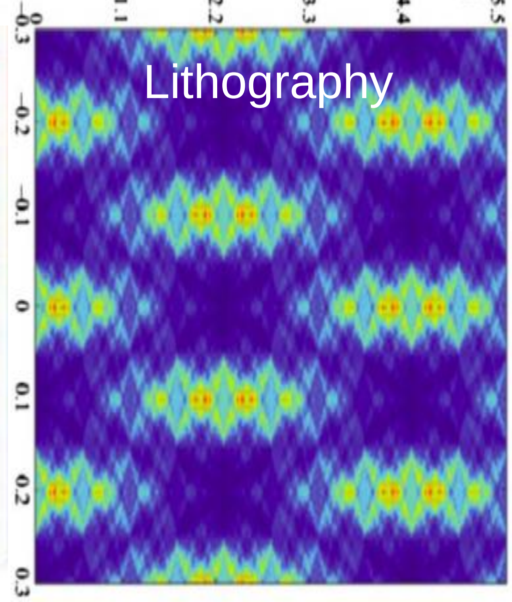


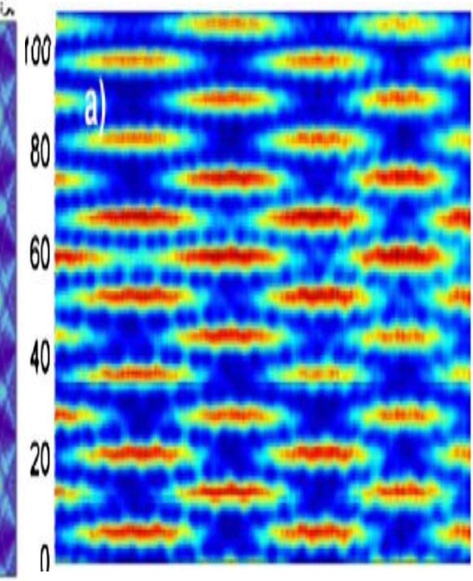




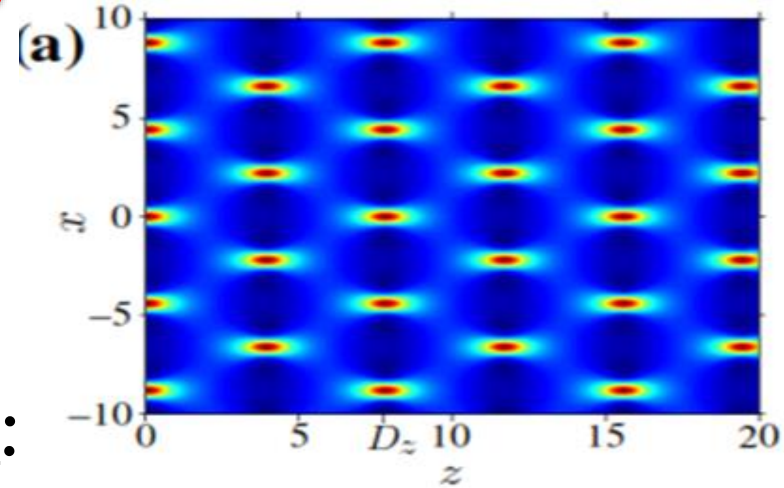








Bring the two together: Talbot Carpets by RWs



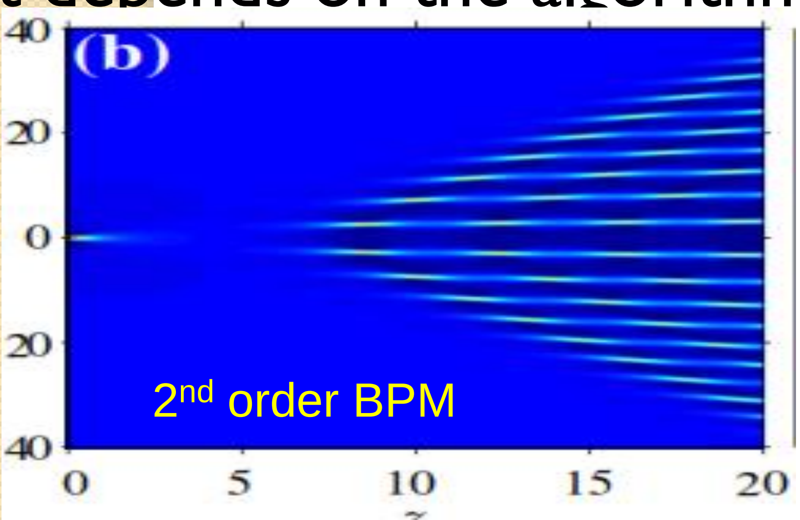
- An expected phenomenon seen:

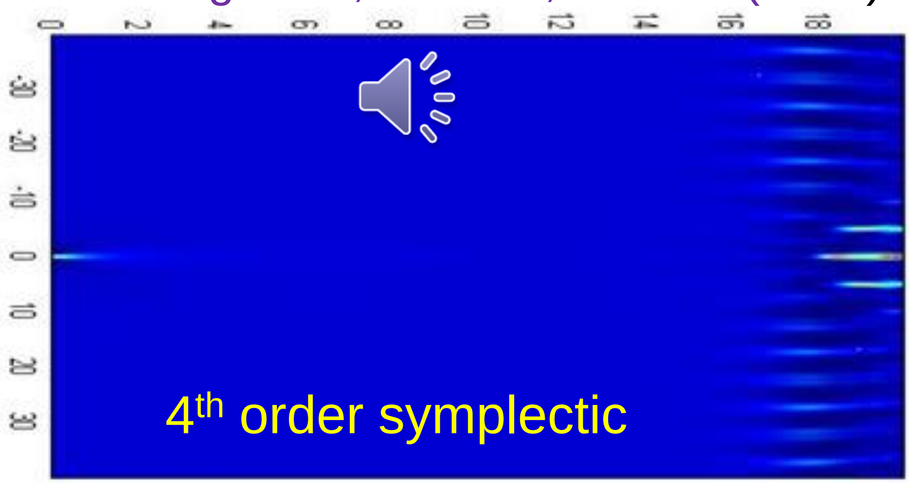
Stable doubly-periodic Akhmediev Breather as a Talbot carpet

- An unexpected phenomenon seen:

Unstable propagation of the Peregrine wave: It depends on the algorithm!

Zhang *et al*., PRE 89, 032902 (2014)

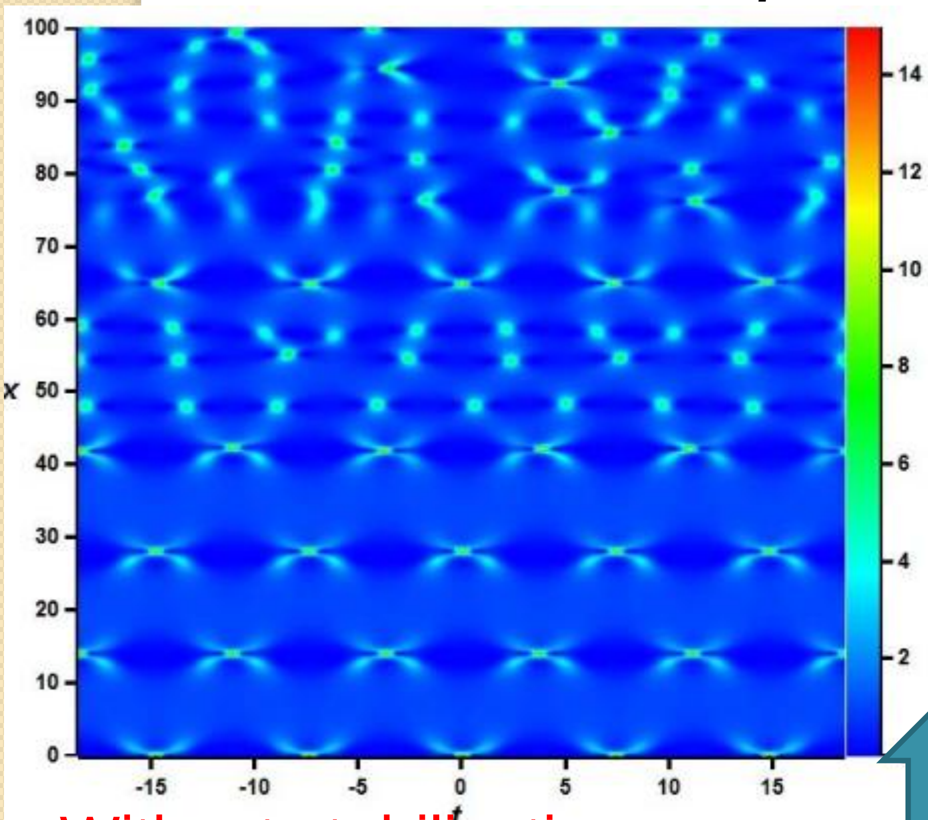




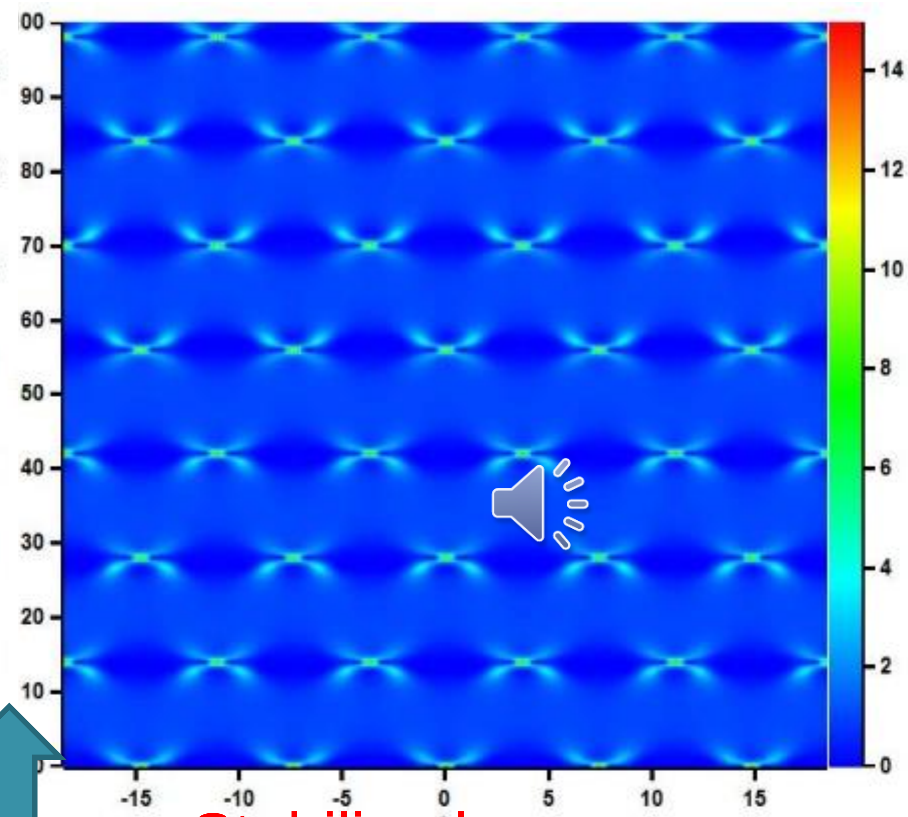
RW Carpets: How to suppress MI

- Stabilization by mode pruning
- Perfect carpets from "random" RWs
- *Second order RW carpets*

Nikolic *et al.*, NODY 97, 1215 (2019)



Without stabilization



Stabilized

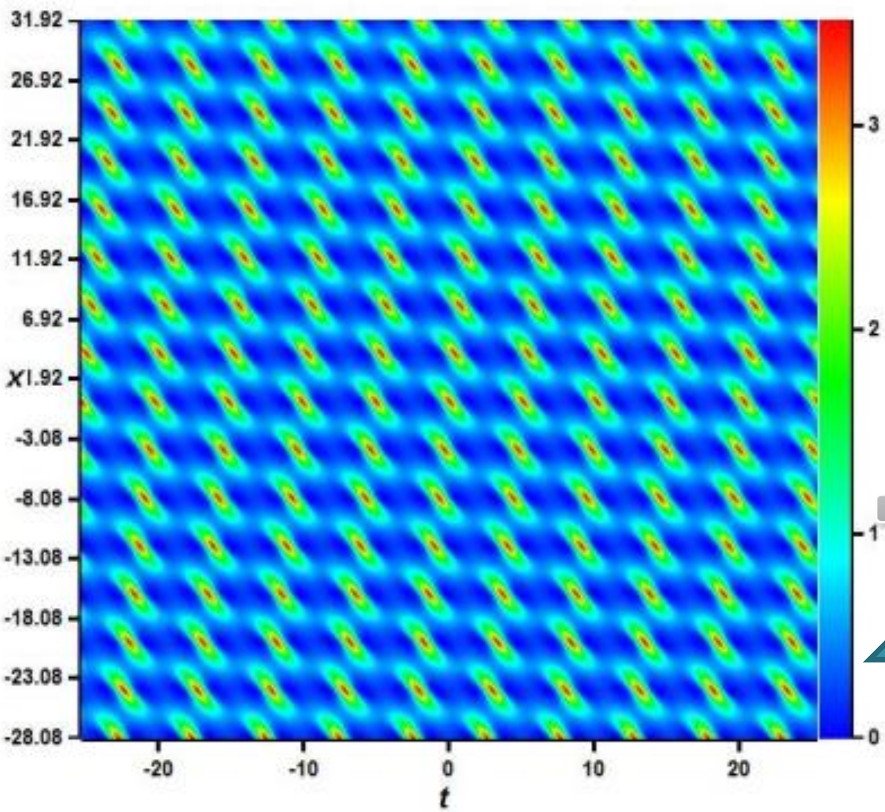
Carpets elsewhere: Hirota

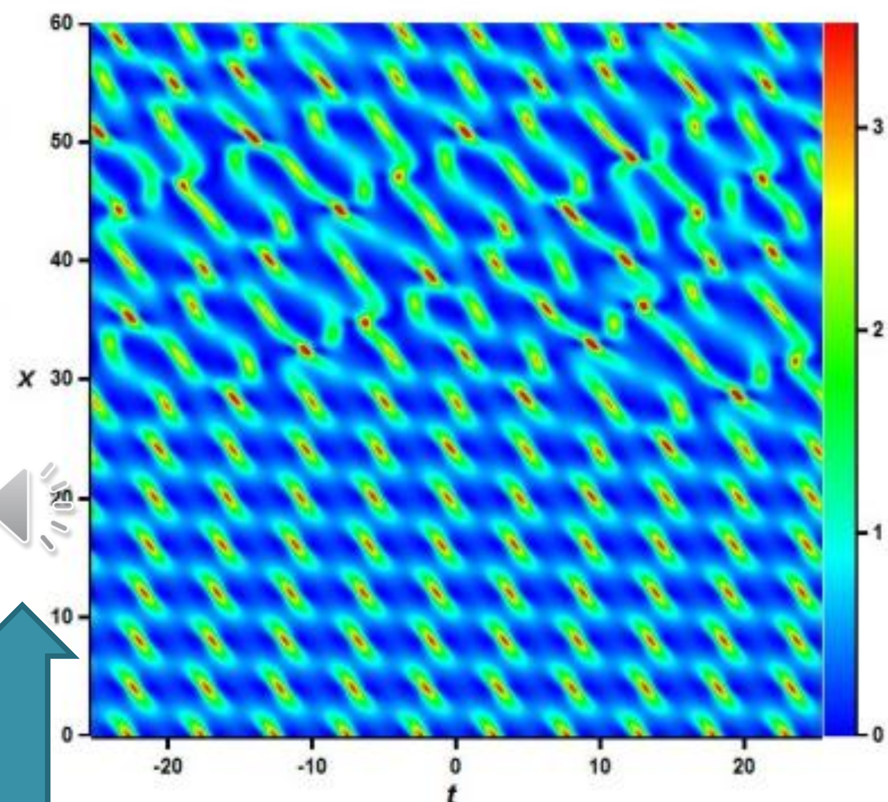
Hirota Equation: $$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+|\psi|^2\psi-i\alpha\left(\frac{\partial^3\psi}{\partial x^3}+6|\psi|^2\frac{\partial\psi}{\partial x}\right)=0$$

Nikolic *et al*., NODY 97, 1215 (2019)

- With stabilization

Without stabilization

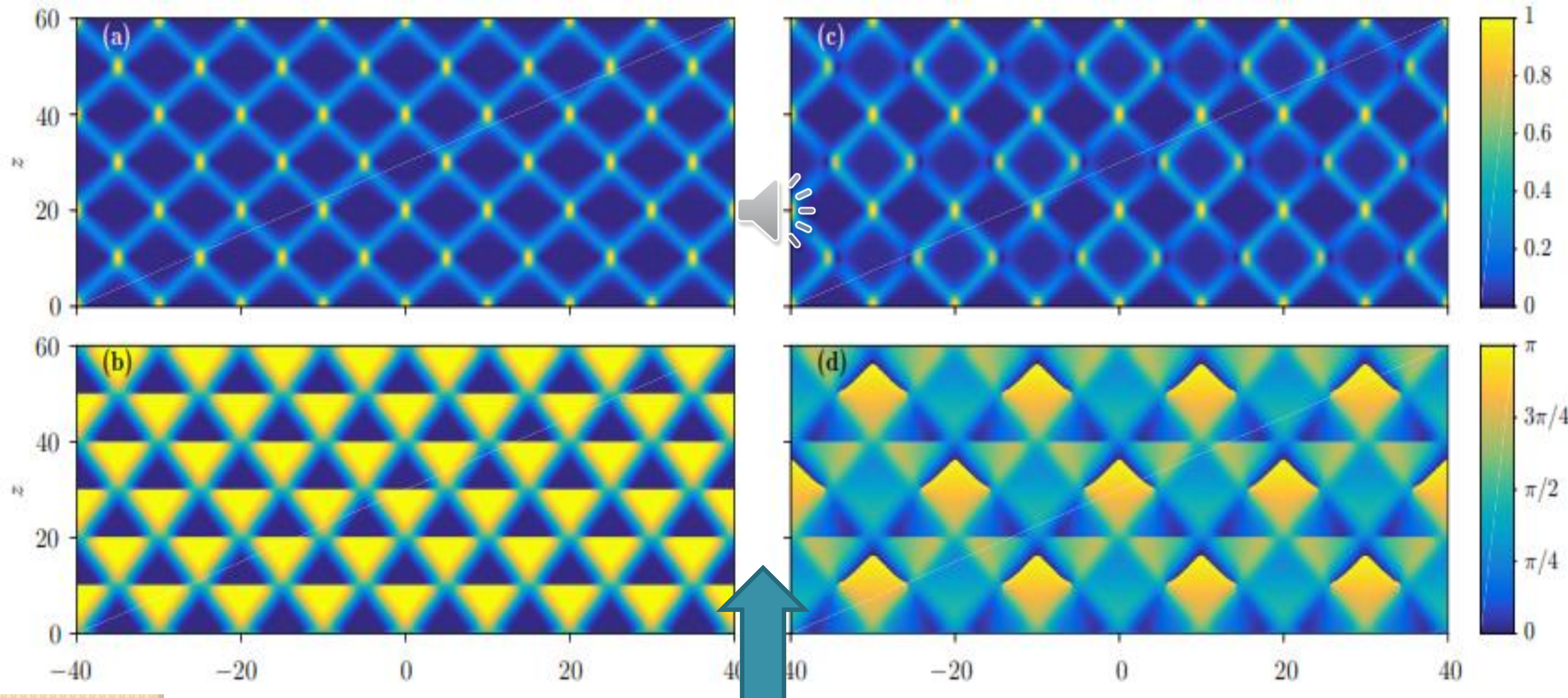




Carpets in fractional Schrȍdinger

$$i\frac{\partial\psi(x,z)}{\partial z}-\frac{1}{2}\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2}\psi(x,z)=0$$

Zhang *et al.*, SREPS 6, 23645 (2016)



Summary

- Utilized the simple NLSE as the paraxial WE in optics
- Generated RWs as finite-background solutions of NLSE
- Established RWs as NL, deterministic and physical in nature
- Driven RWs to HC chaos by modulation instability
- Shown that statistics of RWs depend on numerical algorithm
- Formed Talbot carpets by RWs
- Stabilized carpets by mode pruning

Basic question: Are these effects real or just numerical artefacts?