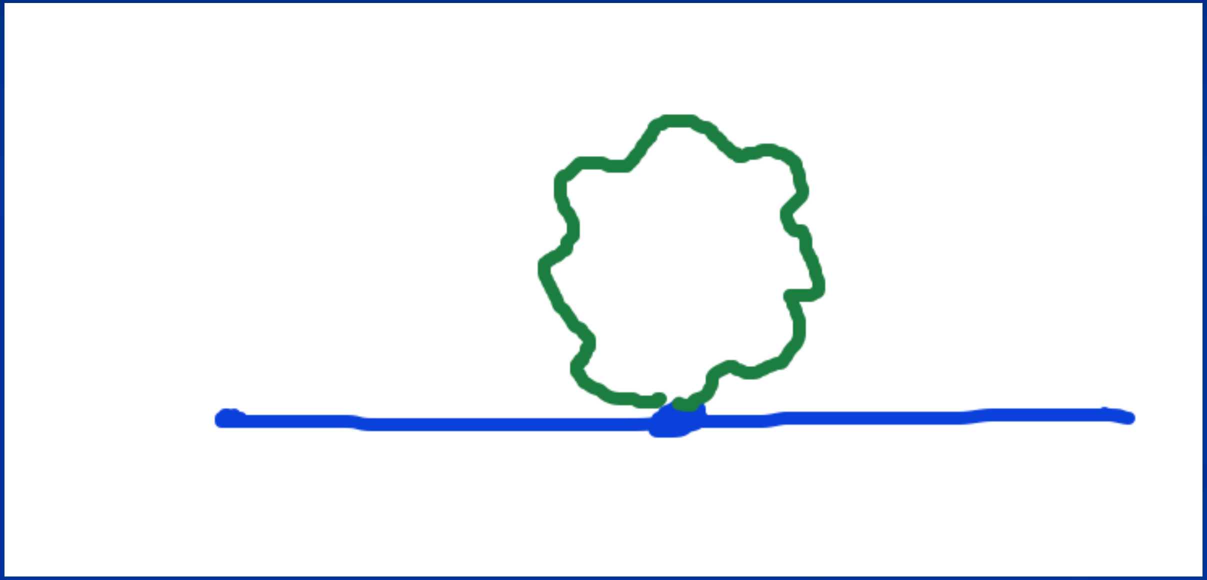
Quantum gravity predictions for particle physics and cosmology

quantum gravity

Graviton fluctuations matter



Quantum gravity needs method to take them into account

Quantum gravity

- Gravity is field theory. Similar to electrodynamics. Metric field.
- Gravity is gauge theory. Similar to QED or QCD. Gauge symmetry: general coordinate transformations (diffeomorphisms)
- Quantum gravity: include metric fluctuations in functional integral

Quantum gravity

- Quantum gravity is similar to other quantum field theories
- Difference: metric is tensor, gauge bosons are vectors
- Difference: Quantum gravity is not perturbatively renormalizable
- no small coupling, effective coupling q^2/M^2

Quantum gravity

Quantum gravity is

non-perturbatively renormalizable

Asymptotic safety : non-perturbative renormalizabilty

Weinberg, Reuter, …

Use functional renormalization !

Flowing couplings

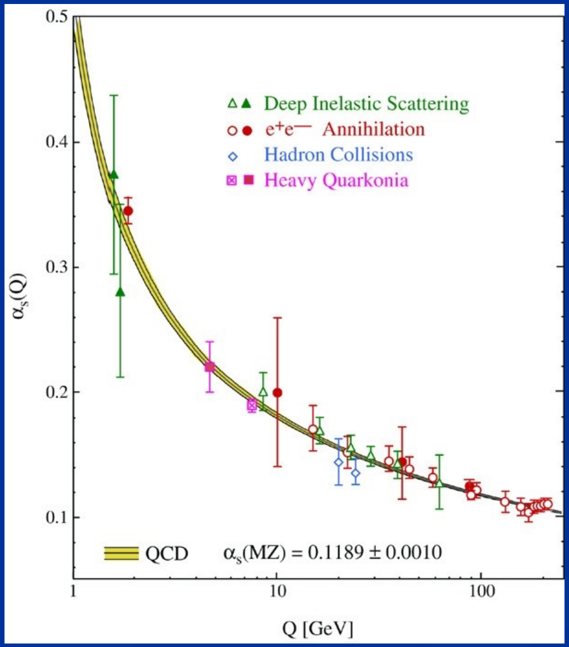
Couplings change with renormalization scale k due to (quantum) fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included.

Flow of k to zero : all fluctuations included, IR-limit
Flow of k to infinity : UV-limit

Quantum fluctuations induce running couplings

- often k-dependence translates to momentum dependence
- well known in QCD or standard model



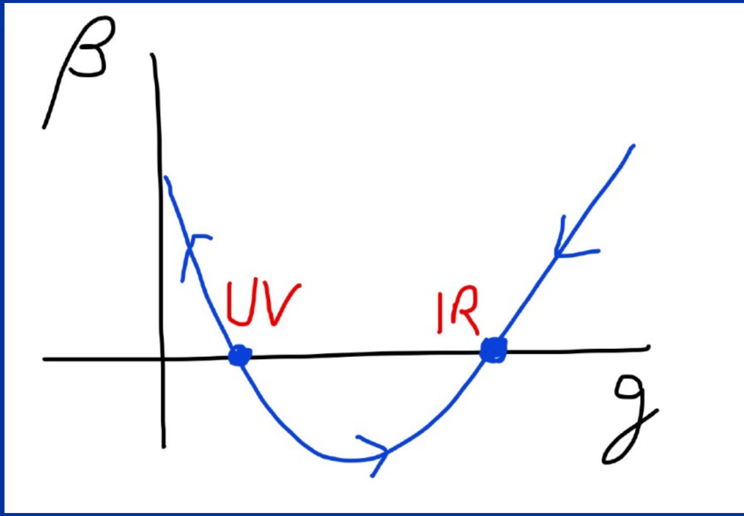
Bethke

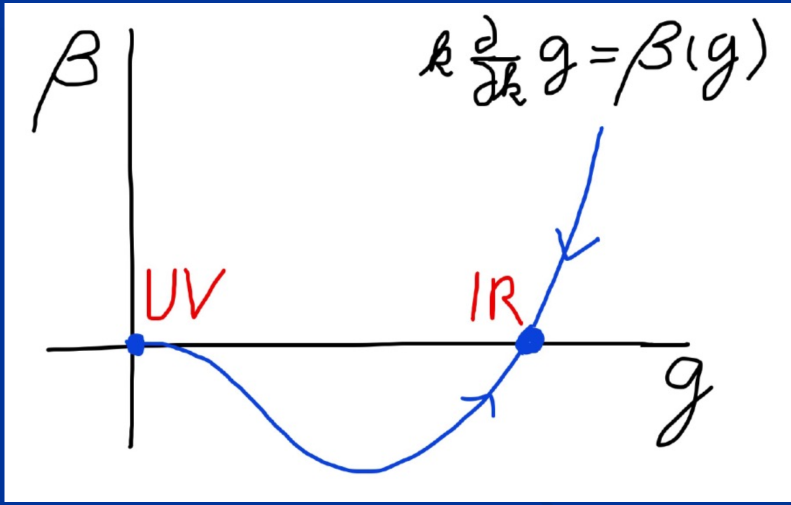
Ultraviolet fixed point

- Flow of couplings stops
- Renormalizable theories have ultraviolet fixed point in the scale dependence of couplings

  (renormalization flow)
- Theory can be extrapolated to arbitrary short distances
- Completeness

Asymptotic safety   Asymptotic freedom





Asymptotically safe gravity

Ultraviolet fixed point exists for quantum field theory for metric (or vierbein).

Quantum gravity predictions for particle physics

Prediction of mass of Higgs boson

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

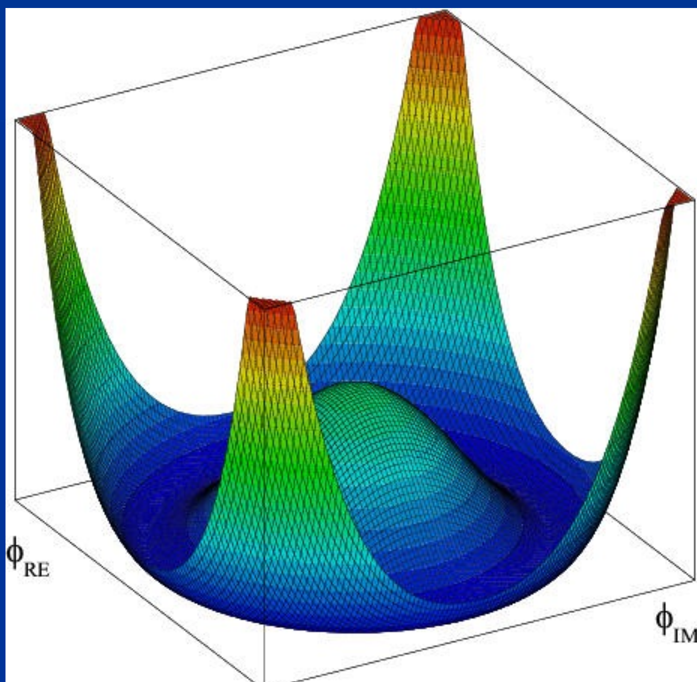
Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

Why can quantum gravity make predictions for particle physics ?

Effective potential for Higgs scalar



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \tfrac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \tfrac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

Radial mode and Goldstone mode

expand around minimum of potential

$$\varphi = \varphi_0 + \sigma + i\eta$$

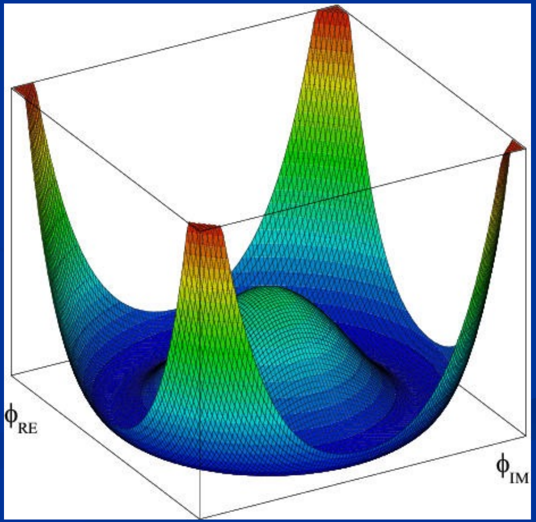
$$\varphi_0, \sigma, \eta : \ \text{real}$$

$$-\mathcal{L}_\varphi = \tfrac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \tfrac{1}{2}\partial^\mu \eta \partial_\mu \eta$$

$$+\tfrac{1}{2} m^2 \sigma^2 + \ldots$$

mass term for radial mode

$$m^2 = 2\lambda \varphi_0^2$$



Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$m^2 = 2\lambda\varphi_0^2$$

Why can quantum gravity make predictions for quartic scalar coupling ?

Mass scales

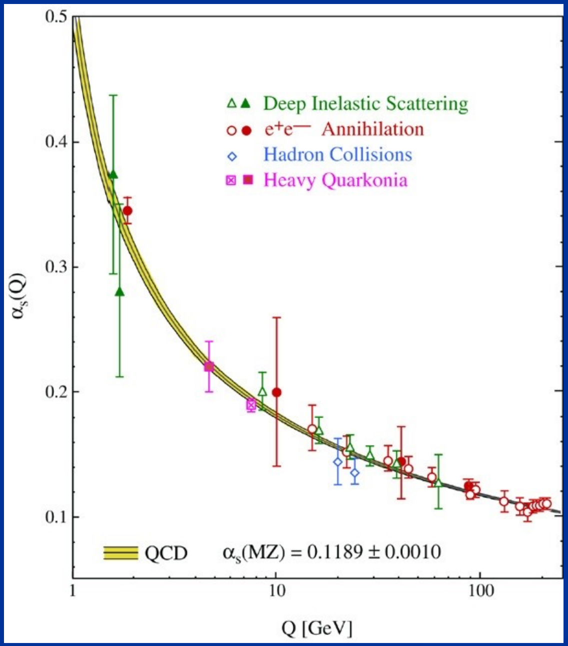
- Fermi scale $\varphi_0 \sim 100$ GeV
- Planck mass M $\sim 10^{18}$ GeV
- Gravity at Fermi scale is very weak : How can it influence the effective potential for the Higgs scalar and the mass of the Higgs boson ?

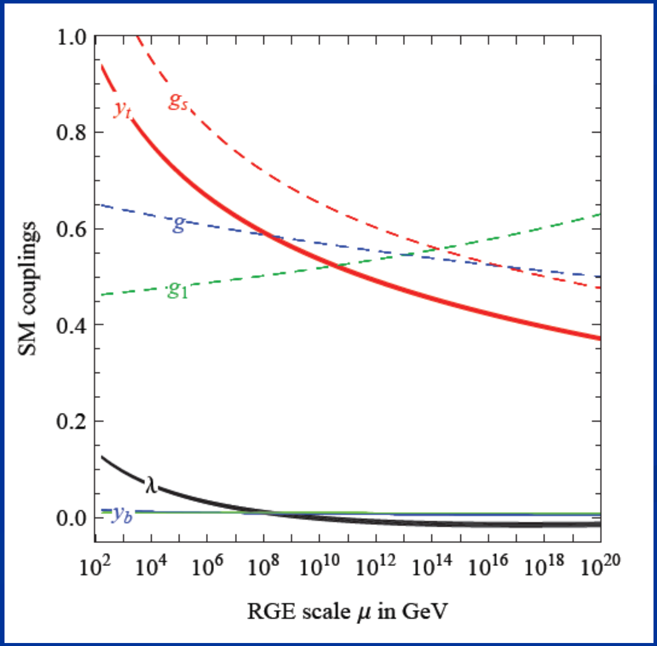
$$\varepsilon = \frac{\varphi_0^2}{M^2} = 5 \cdot 10^{-33}$$

Quantum fluctuations induce running couplings

- running quartic scalar coupling, gauge couplings and Yukawa couplings

Bethke

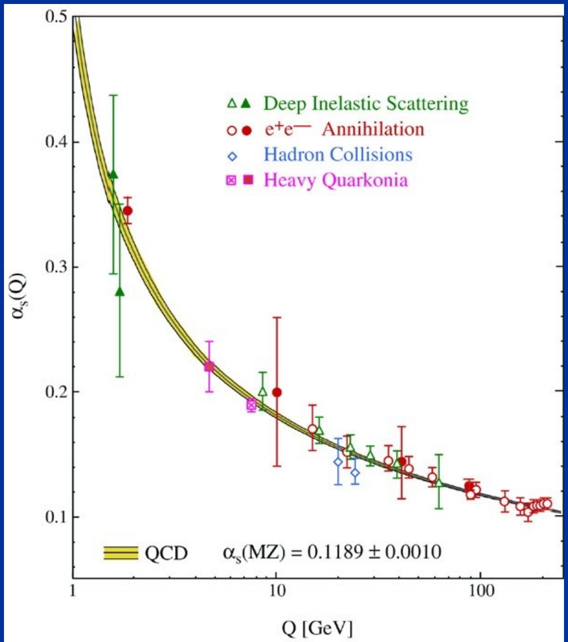


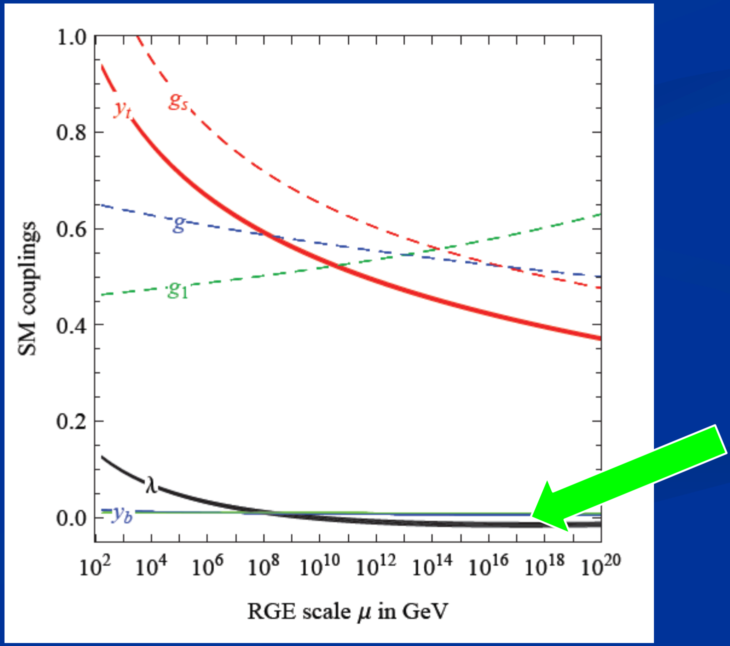


Degrassi et al

Quantum fluctuations induce running couplings

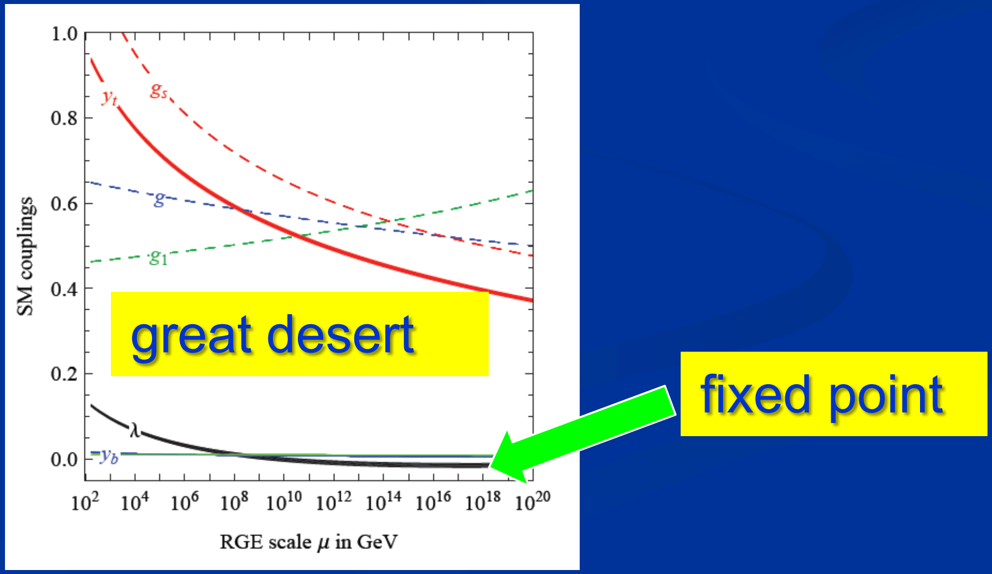
- connect Planck scale and Fermi scale

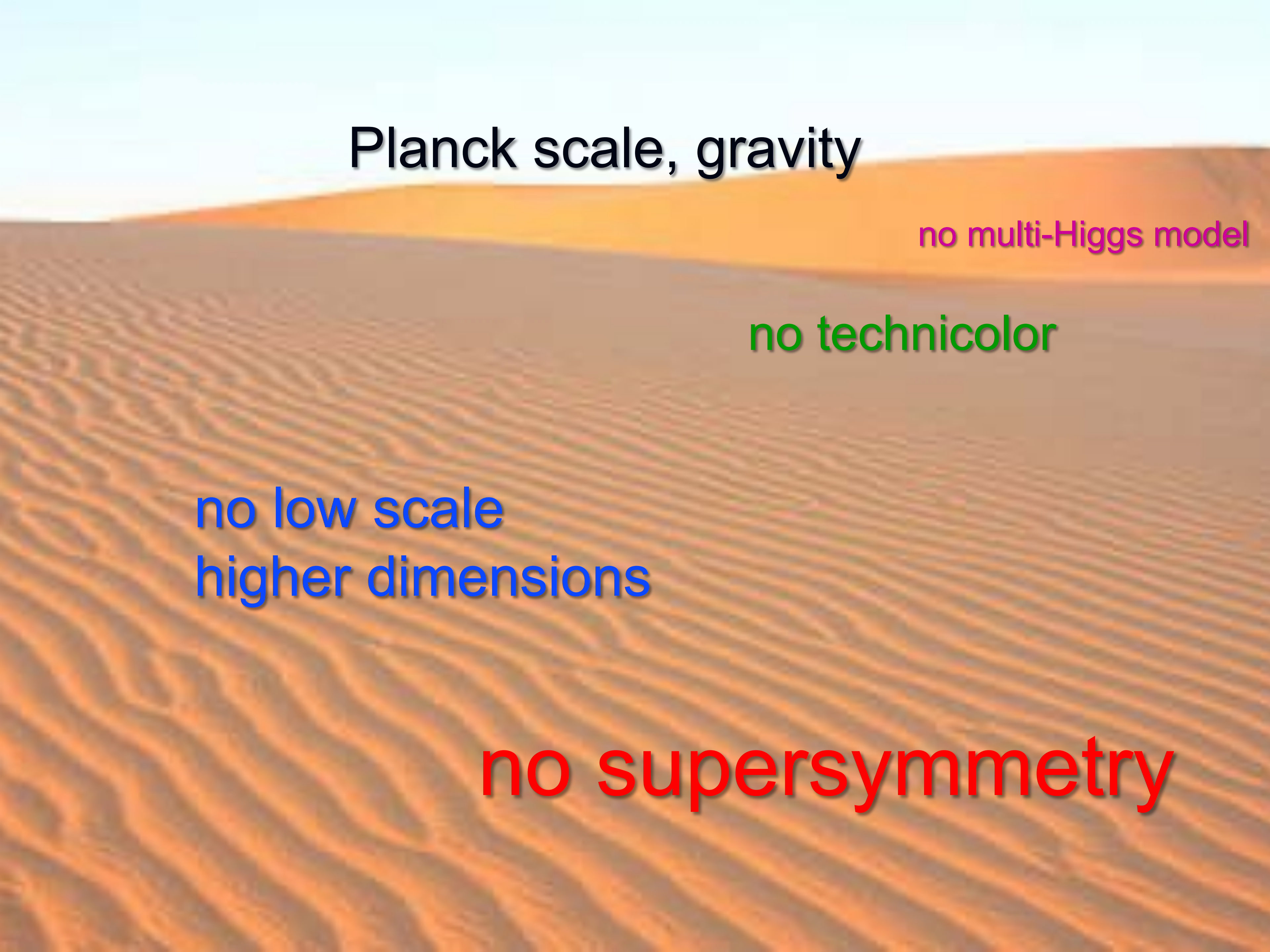




key points

- great desert
  (solution of hierarchy problem at high scale)
- high scale fixed point
- vanishing scalar coupling at fixed point

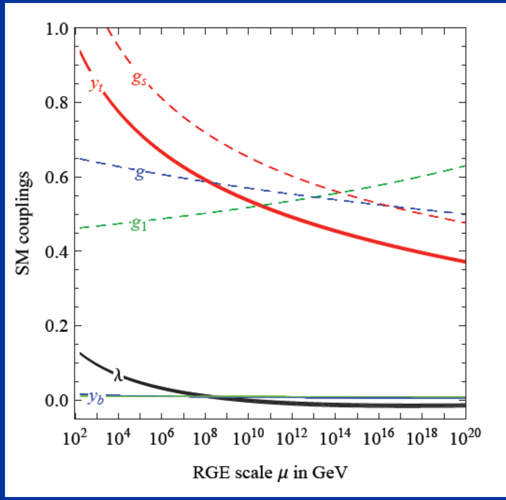



Planck scale, gravity
no multi-Higgs model
no technicolor
no low scale
higher dimensions
no supersymmetry

Essential point for quantum gravity prediction of Higgs boson mass:

Initial value of quartic scalar coupling near Planck mass is predicted by quantum gravity

Extrapolate perturbatively to Fermi scale



Results in prediction for ratio Higgs boson mass over W- boson mass, or Higgs boson mass over top quark mass

Near Planck mass gravity is not weak !

Predictive power !

Renormalization

How do couplings or physical laws change with scale k ?

Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included.

Consider first only fluctuations of metric or graviton :

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced anomalous dimension

A > 0

for constant A :

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

$$k \to 0 \quad \Rightarrow \quad \lambda \to 0$$

Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

The quartic scalar coupling λ has a
fixed point at λ=0

For A>0 it flows towards the fixed point as k is lowered:
irrelevant coupling

For a UV – complete theory irrelevant couplings are predicted to assume the fixed point value

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_\mathrm{p}^2} \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]$$

$$\partial_t = k \partial_k$$

running Planck mass : $M_\mathrm{p}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$

dimensionless squared Planck mass $\tilde{M}_\mathrm{p}^2 = \frac{M_\mathrm{p}^2}{k^2}$

for length scales smaller than the Planck length:
metric fluctuations dominate, constant A

Strength of gravity

$$g_{grav} = \frac{l_p^2}{2l^2} = \frac{k^2}{2M^2}$$

l_p : Planck length
M : Planck mass

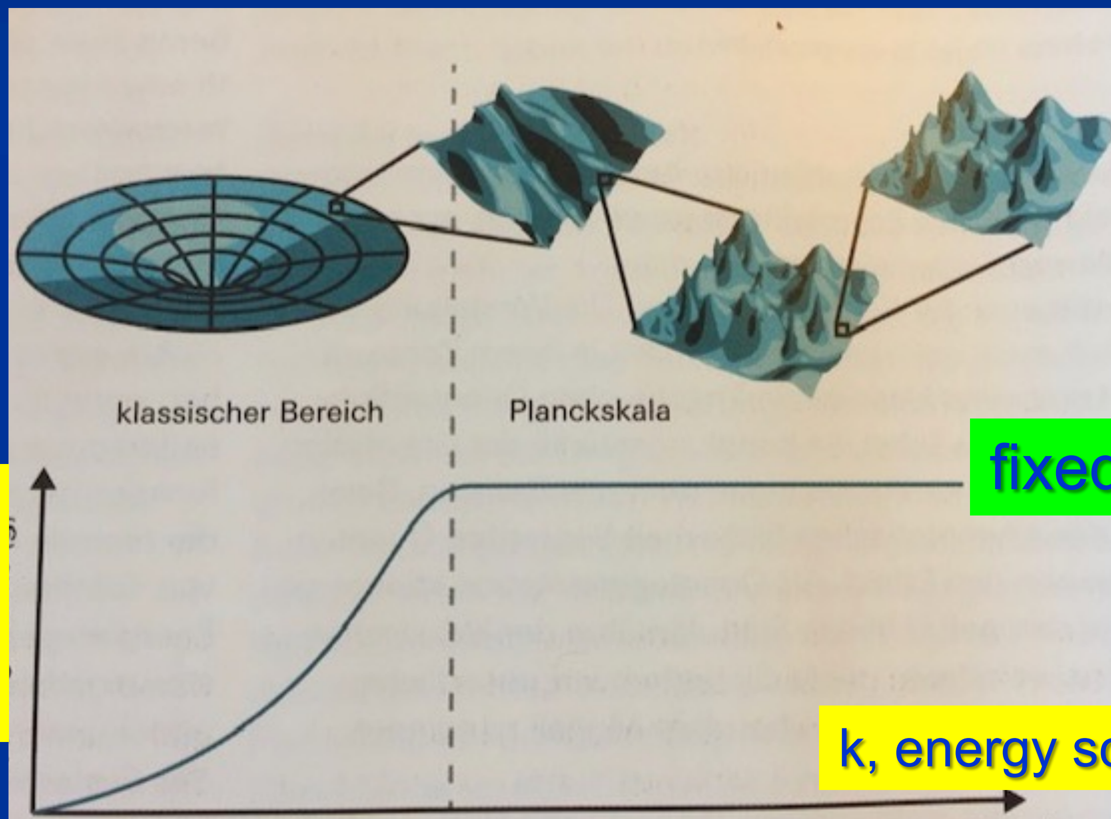
running gravitational coupling

$$g_{grav} = \frac{k^2}{2M^2(k)} = w^{-1}(k)$$

Strength of gravity

classical gravity

quantum gravity



fixed point

strength of gravity g_{grav}

k, energy scale

inverse distance

A. Eichhorn, CW, Spektrum der Wissenschaft

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

Enhanced predictivity for UV – fixed point

- Free parameters of a theory correspond to relevant parameters for small deviations from fixed point.
- If the number of relevant parameters at the UV-fixed point is smaller than the number of free parameters (renormalizable couplings) in the standard model:
- Relations between standard model parameters become predictable !

Conclusion (1)

- Quantum gravity is a renormalizable quantum field theory, realized by UV - fixed point of running couplings or flowing effective action

- Quantum gravity is predictive :

  Mass of the Higgs boson (and more …?)

  Properties of inflation

  Properties of dark energy

Quantum gravity predictions for cosmology

Scaling solutions

- At fixed point: all (infinitely many) dimensionless couplings take fixed values
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

Scaling solutions are restrictive

- Scaling solutions are particular solutions of non-linear differential equations
- In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

Scaling solutions and cosmology

- Cosmology involves scalar potentials over large range of field values
- Inflaton potential
- Higgs potential for Higgs inflation
- Cosmon potential for dynamical dark energy or quintessence

Quantum gravity :
these potentials are not arbitrary

Dilaton quantum gravity

quantum gravity coupled to a scalar field

Henz, Pawlowski, Rodigast, Yamada, Reichert, Eichhorn, Pauly, Laporte, Pereira, Saueressig, Wang…

Scaling potential in standard model

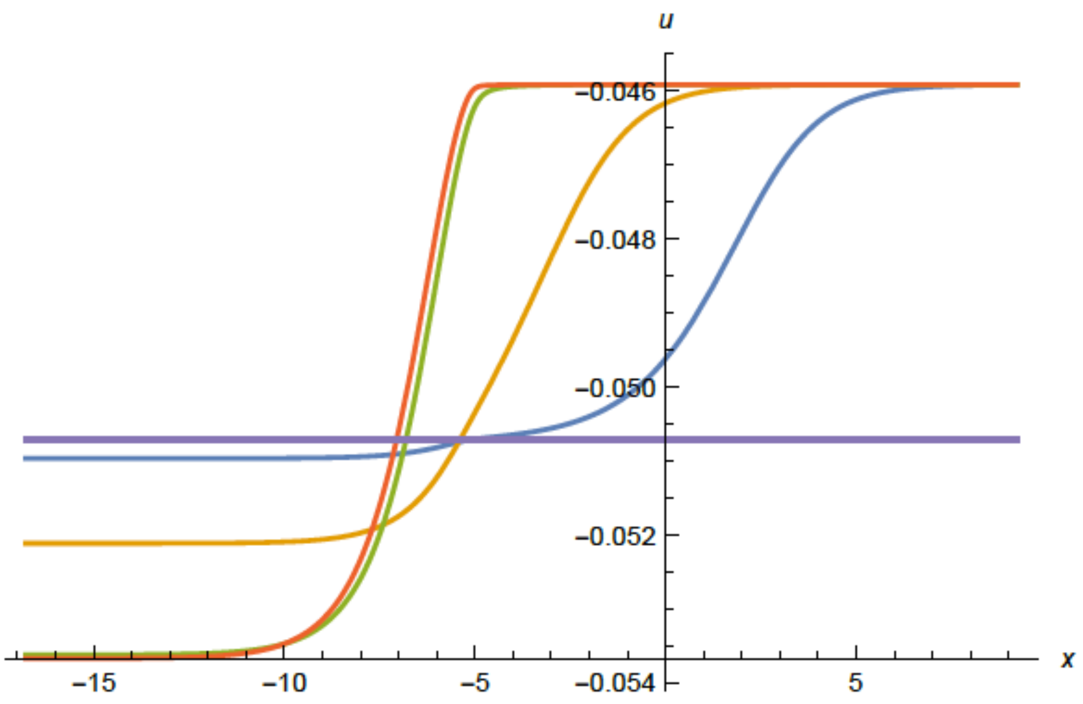


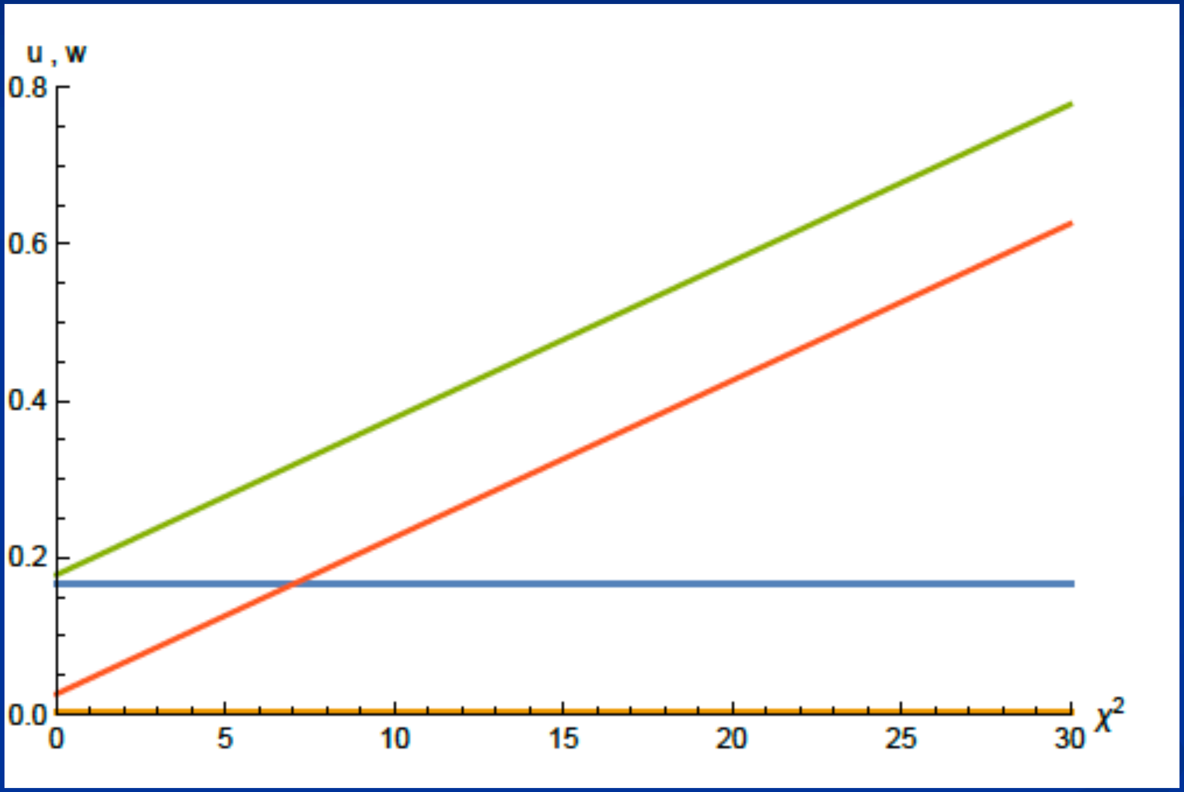
FIG. 19. Effective potential u as function of $x = \ln \tilde{\rho}$ for $\xi_\infty = 0.1$ (blue), 1.0 (orange), 10^3 (green) and 10^4 (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model, $N_S = 4$, $N_V = 12$, $N_F = 45$.

u : dimensionless scalar potential

$u = U/k^4$

x : logarithm of scalar field value

Scaling solution : flat potential



squared scalar field value χ^2

Derivative expansion of effective action

$$\Gamma = \int_x \sqrt{g}\left\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\right\}$$

variable gravity

Coefficient of curvature scalar in standard model

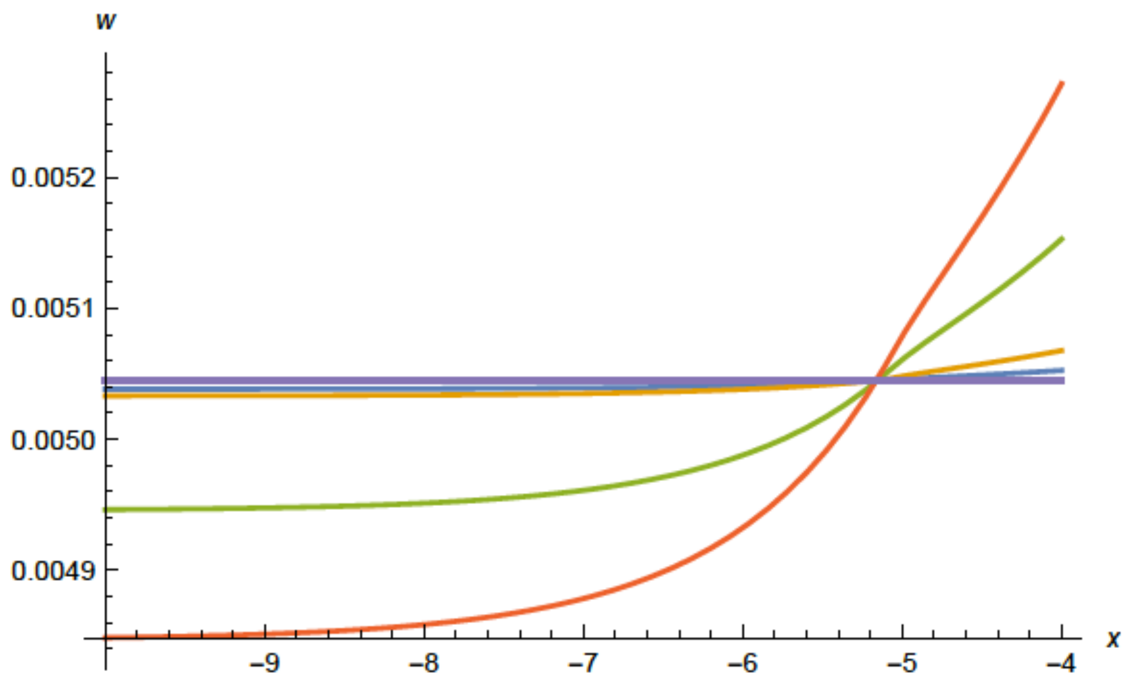


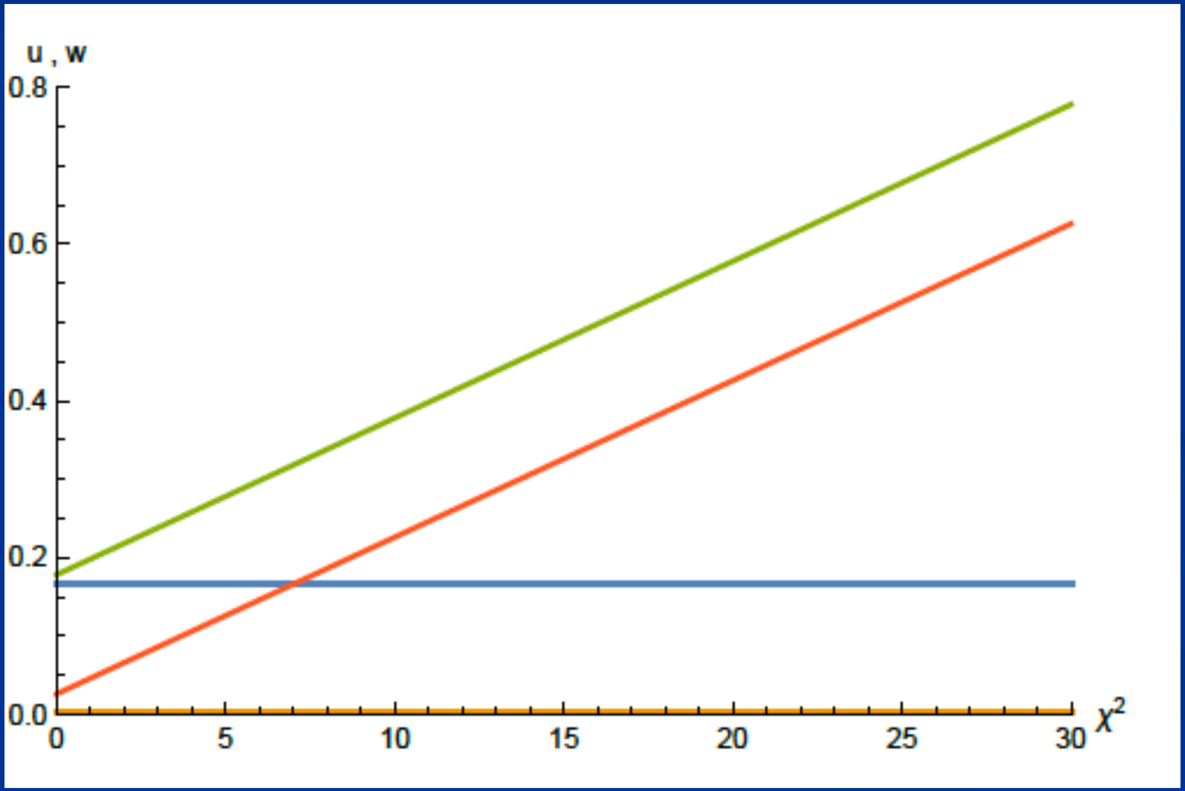
FIG. 21. Dimensionless squared Planck mass w as function of $x = \ln \tilde{\rho}$ for $\xi_\infty = 2 \cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from top to bottom on the left. The horizontal line denotes the scaling solution which is approached for $\xi_\infty \to 0$. All curves meet in a common point at $x \approx -5.05$.

w : dimensionless field dependent squared Planck mass

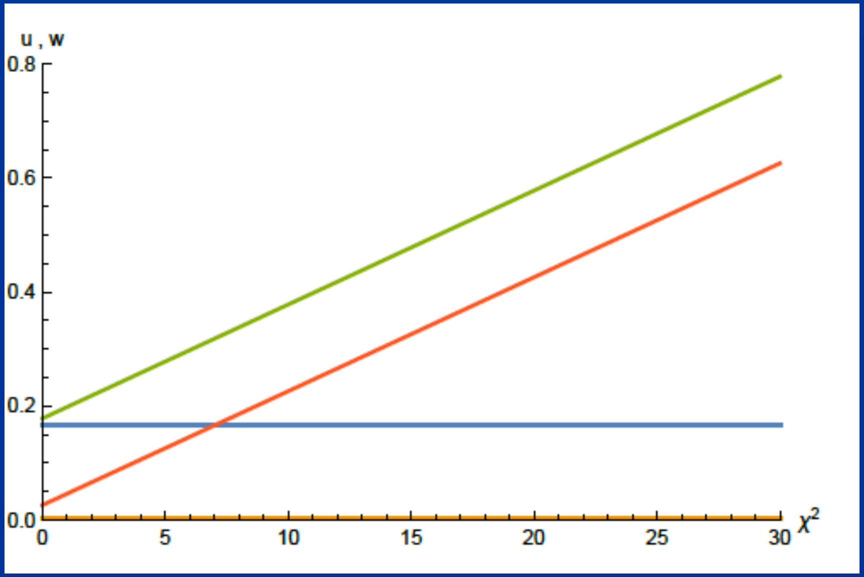
$w = 2 F / k^2$

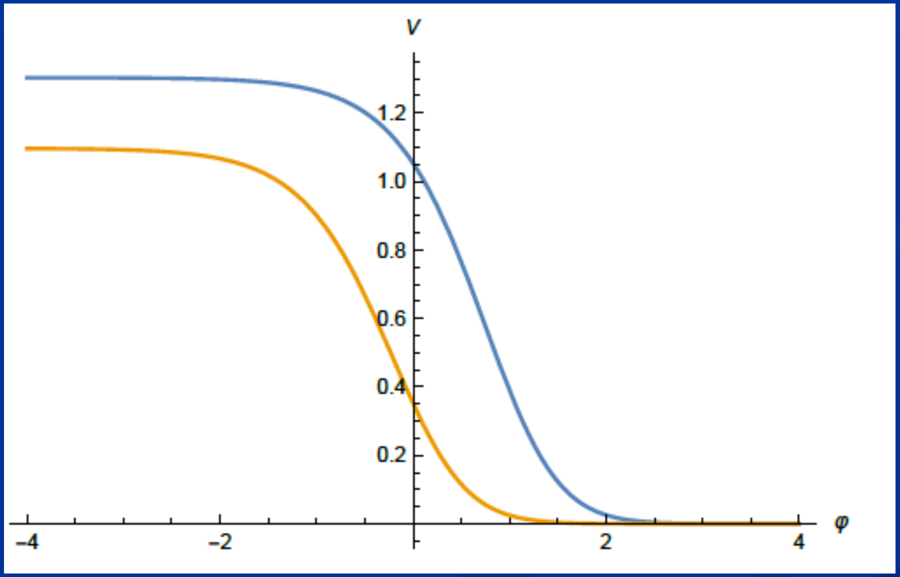
non-minimal coupling of scalar field to gravity: $\xi\chi^2 R$

Scaling solution : flat potential, non-minimal scalar- gravity coupling

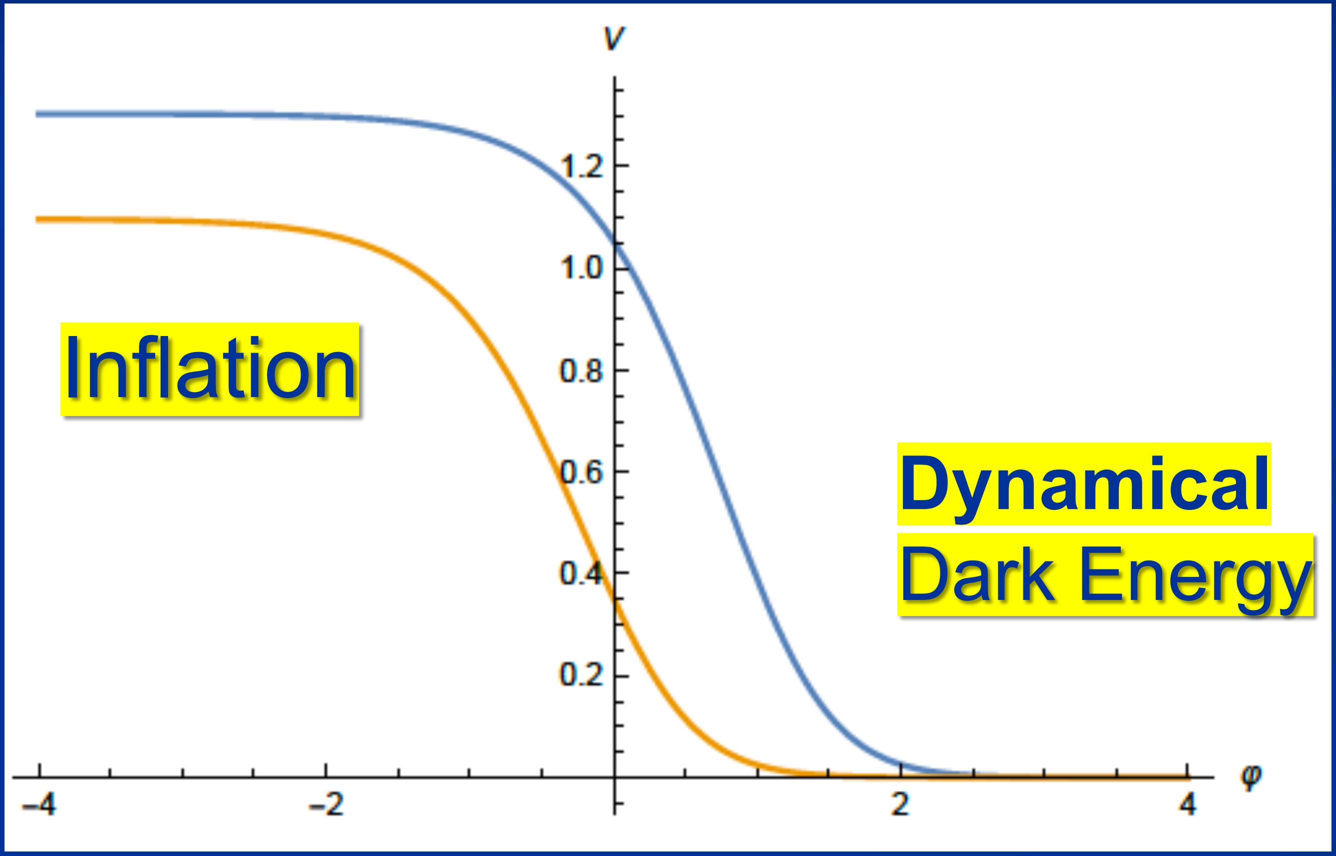


Scaling solution in Einstein frame





Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini, Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio…

Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F) g'_{\mu\nu} \qquad \varphi = 4M \ln(\chi/k)$$

$$\Gamma = \int_\chi \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

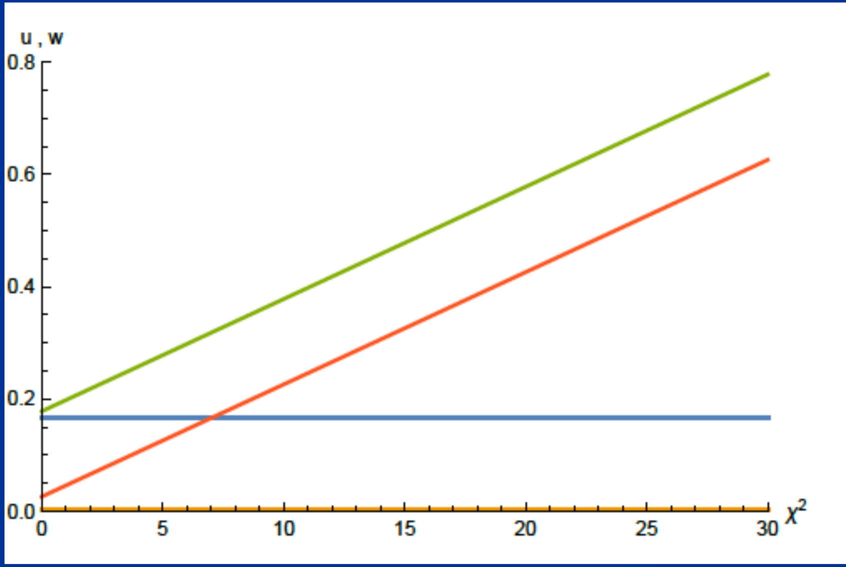
$$\Gamma = \int_\chi \sqrt{g}\Big\{ -\frac{M^2}{2}R' + \frac{1}{2}Z(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\Big\}$$

$$V(\varphi) = \frac{UM^4}{F^2} \qquad Z(\varphi) = \frac{1}{16}\Big\{\frac{\chi^2 K}{F} + \frac{3}{2}\Big(\frac{\partial \ln F}{\partial \ln \chi}\Big)^2\Big\}$$

Scaling solution

$$U = u_0 k^4$$

$$F = 2 w_0 k^2 + \xi \chi^2$$



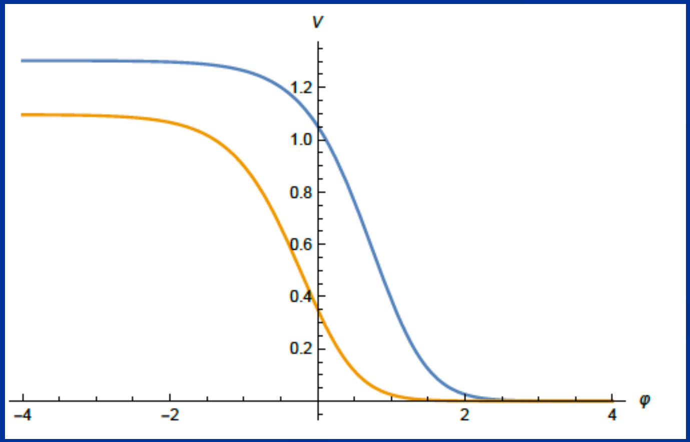
For low energy standard model :

$$u_\infty = \frac{7}{256\pi^2}$$

Scaling solution in Einstein frame

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$

$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$

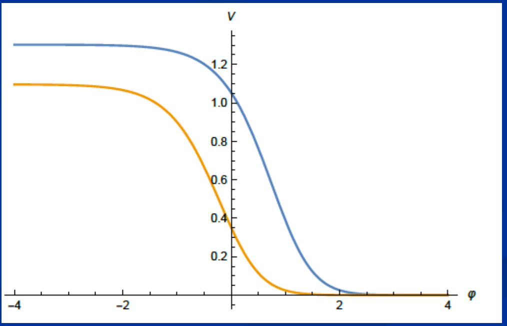


Mass scales in Einstein frame

Renormalization scale k is no longer present

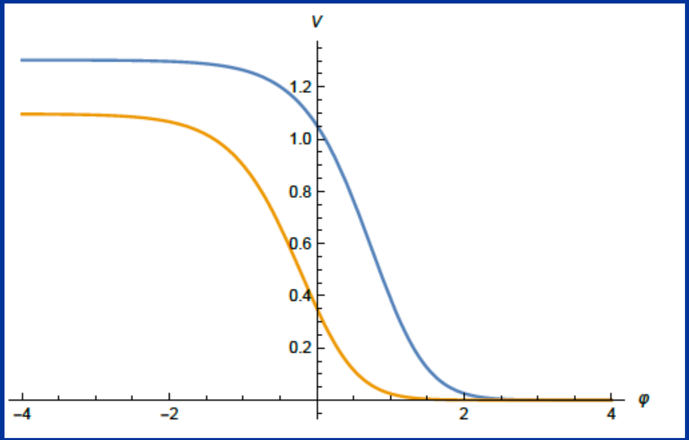
Planck mass M not intrinsic: introduced only by change of variables

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2} = \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



Asymptotic solution of cosmological constant problem

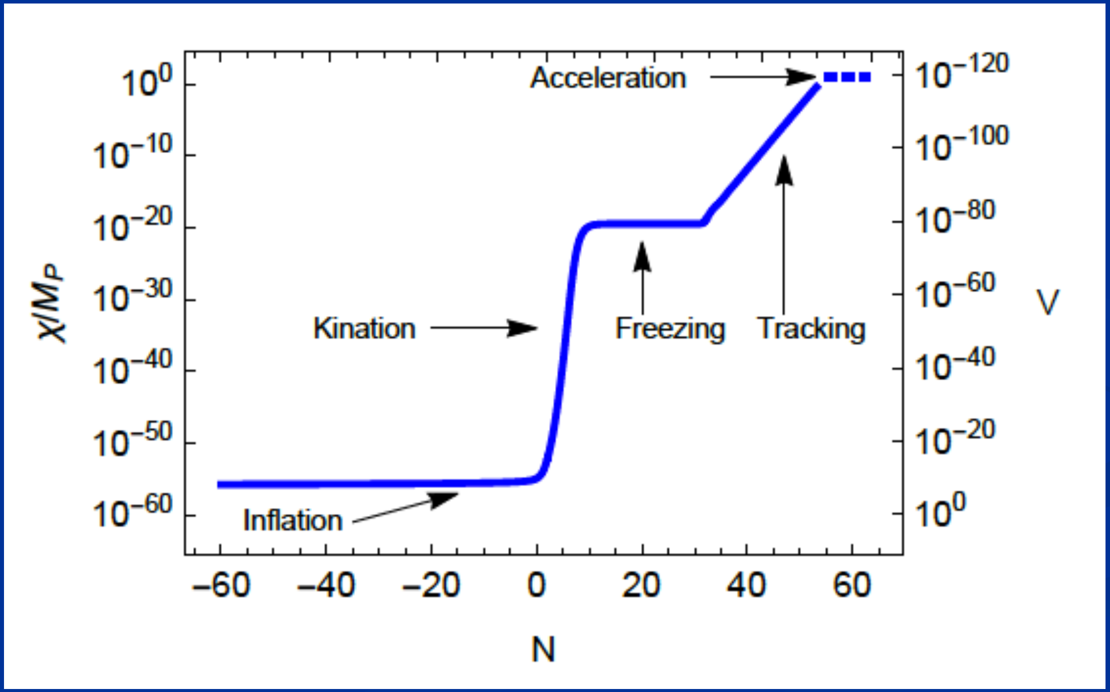
$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



no tiny parameter !

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Conclusion (2)

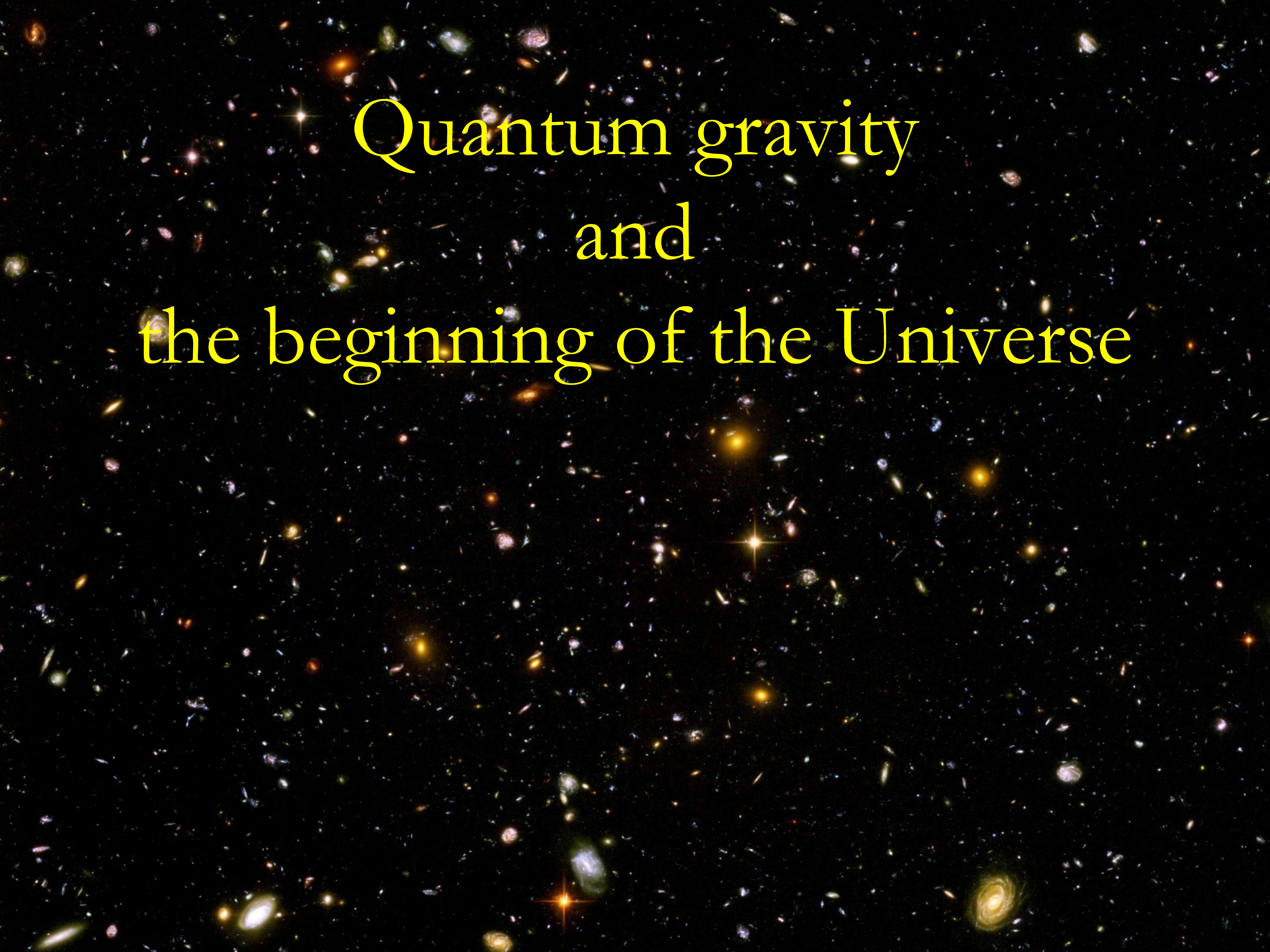
Fixed points of quantum gravity

and associated quantum scale symmetry

are crucial for understanding the

evolution of our Universe

Quantum gravity
and
the beginning of the Universe



Beginning of Universe

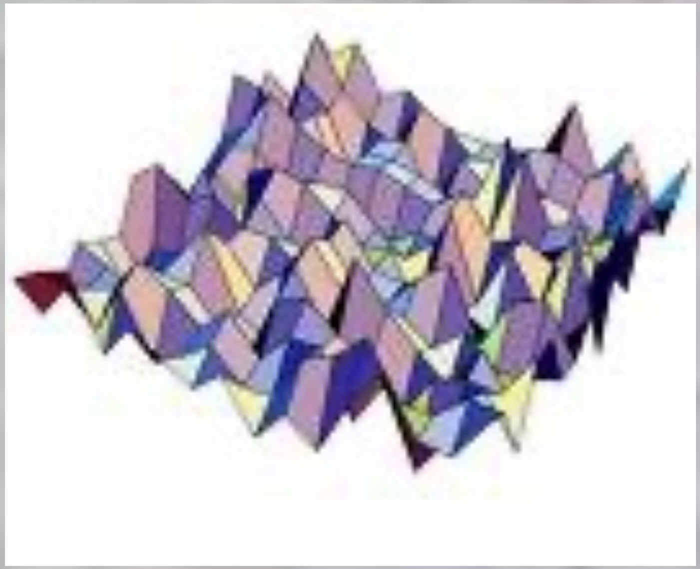
Zu Anfang war die Welt öd und leer und währte ewig.

In the beginning the Universe was empty and lasted since ever.

Eternal light-vacuum

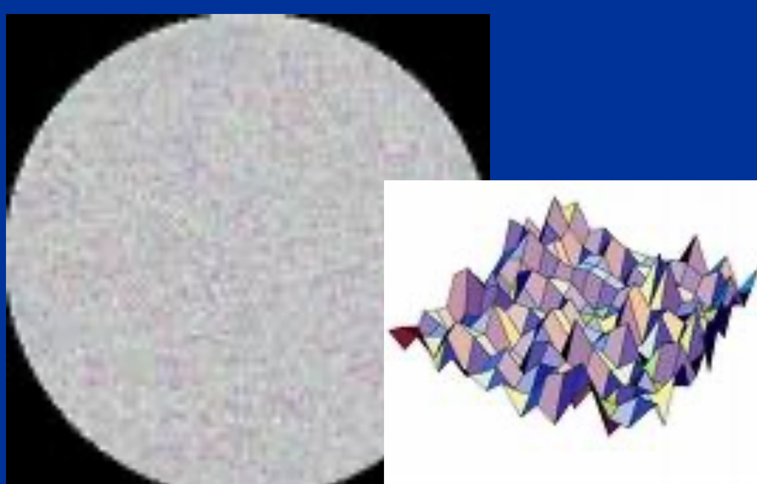
Everywhere almost nothing
only fields and their fluctuations

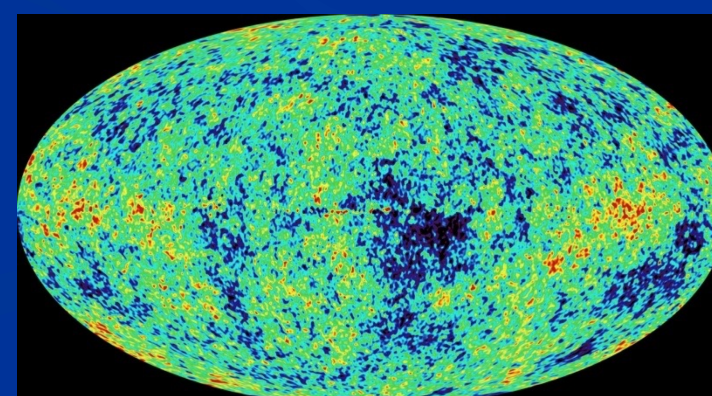
All particles move
with light velocity,
similar to photons



Eternal light-vacuum is unstable

- Slow increase of particle masses
- Only slow change of space-time geometry
- Creation of particles and entropy
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage 5000 billion years ago.



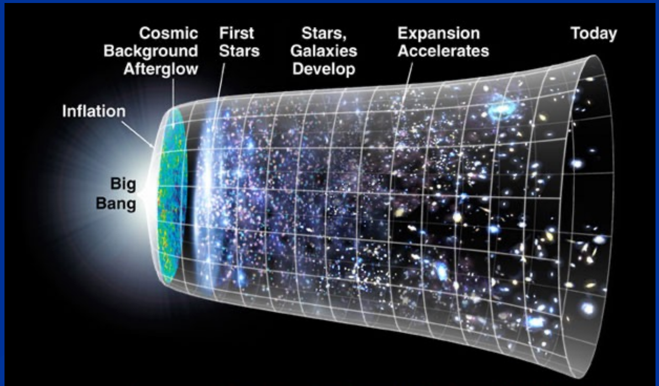


The great emptiness story

In the beginning was light-like emptiness.

The big bang story

- dramatic hot big bang
- started 13.7 billion years ago
- at the beginning extremely short period of cosmic inflation with almost exponential expansion of the Universe, duration around 10^{-40} seconds
- start with singularity : our whole observable Universe evolves from one point



Field relativity

- Both stories are equivalent
- related by field transformation of the metric

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

- different metrics related by Weyl transformation, which depends on scalar field (inflaton)

Field - singularity

- Big Bang is field - singularity
- similar (but not identical with) coordinate - singularity

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

end