An Alternative Approach within the CDFM for Studies of Nuclear Symmetry Energy Components and their Ratio

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Physical Motivation

Symmetry energy → crucial quantity in nuclear physics and its astrophysical applications

Study of isotopic distributions produced in $^{40,48}Ca+^{40,48}Ca$ collisions at 35 MeV/nucleon at GANIL (VAMOS spectrometer coupled with 4π INDRA detector) → $S^S/S^V \approx 1.7$

There is a little information on the volume and surface symmetry energies. This concerns mostly the surface contribution to the symmetry energy.

The main aim: to avoid the preliminary assumptions and mathematical ambiguities in our previous scheme I. In the proposed scheme II we start from the building units ("fluctons") of the CDFM and construct from them the ratio between the symmetry energy components for finite nuclei following the standard CDFM procedure.

Ni (A=74-84), Sn (A=124-156), Pb (A=202-214) isotopic chains

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Main relationships for EOS parameters in NM and finite nuclei

The Bethe-Weizsäcker semi-empirical mass formula:

$$\begin{aligned} E(A,Z) &= -B + E_S A^{-1/3} + S(A)\frac{(N-Z)^2}{A^2} + E_C\frac{Z^2}{A^{4/3}} \\ &\quad + E_{dif}\frac{Z^2}{A^2} + E_{ex}\frac{Z^{4/3}}{A^{4/3}} + a\Delta A^{-3/2} \end{aligned} \tag{1}$$

The symmetry energy $S(A)$ is expressed by the volume $S^V(A)$ and modified surface component $S^S(A)$ in the droplet model:

$$S(A) = \frac{S^V(A)}{1 + \dfrac{S^S(A)}{S^V(A)}A^{-1/3}} = \frac{S^V(A)}{1 + q(A)A^{-1/3}}, \tag{2}$$

where

$$q(A) \equiv \frac{S^S(A)}{S^V(A)} \tag{3}$$

At large A:

$$S(A) \simeq S^V(A) - \frac{S^S(A)}{A^{1/3}} \tag{1}$$

The relations of $S^V(A)$ and $S^S(A)$ with $S(A)$ in terms of $q(A)$:

$$S^V(A) = S(A)\left[1 + \frac{q(A)}{A^{1/3}}\right] \tag{2}$$

$$S^S(A) = q(A)S(A)\left[1 + \frac{q(A)}{A^{1/3}}\right] \tag{3}$$

The expression for the ratio of $S^V(A)$ to $S^S(A)$ given by Danielewicz:

$$\kappa(A) = \frac{S^V(A)}{S^S(A)} = \frac{3}{r_0}\int dr \frac{\rho(r)}{\rho_0}\left\{\frac{S^{NM}(\rho_0)}{S^{NM}[\rho(r)]} - 1\right\}, \tag{4}$$

where

$$\frac{4\pi r_0^3}{3} = \frac{1}{\rho_0} \tag{5}$$

CDFM (Sofia group 1979 - till now)

$$\rho(\mathbf{r},\mathbf{r}') = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_x(\mathbf{r},\mathbf{r}') \tag{12}$$

$$\rho_x(\mathbf{r},\mathbf{r}') = 3\rho_0(x)\frac{j_1(k_F(x)|\mathbf{r}-\mathbf{r}'|)}{(k_F(x)|\mathbf{r}-\mathbf{r}'|)}\Theta\left(x-\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right) \tag{13}$$

$$\rho_0(x) = \frac{3A}{4\pi x^3} \tag{14}$$

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\beta}{x} \qquad \beta = \left(\frac{9\pi A}{8}\right)^{1/3} \simeq 1.52A^{1/3} \tag{15}$$

$$\rho(\mathbf{r}) = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x)\Theta(x-|\mathbf{r}|) \tag{16}$$

$$\text{At } \frac{d\rho}{dr} \le 0: \qquad |\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)}\left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{17}$$

The nuclear symmetry energy in finite nuclei $S(A)$ within the CDFM:

$$S(A) = \int_0^\infty dx |\mathcal{F}(x)|^2 S^{NM}[\rho(x)] \tag{1}$$

Approach I

$$\kappa(A) = \frac{3}{r_0 \rho_0} \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x) \int_0^x dr \left\{ \frac{S^{NM}(\rho_0)}{S^{NM}[\rho_0(x)]} - 1 \right\} \tag{2}$$

$$\kappa(A) = \frac{3}{r_0 \rho_0} \int_0^\infty dx |\mathcal{F}(x)|^2 x \rho_0(x) \left\{ \frac{S^{NM}(\rho_0)}{S^{NM}[\rho_0(x)]} - 1 \right\} \tag{3}$$

An alternative approach II

First, to construct $q(x) = S^S(x)/S^V(x)$ we recall that the flucton is a sphere of nuclear matter of radius x with density $\rho_0(x)$. Thus, inside each flucton we can read

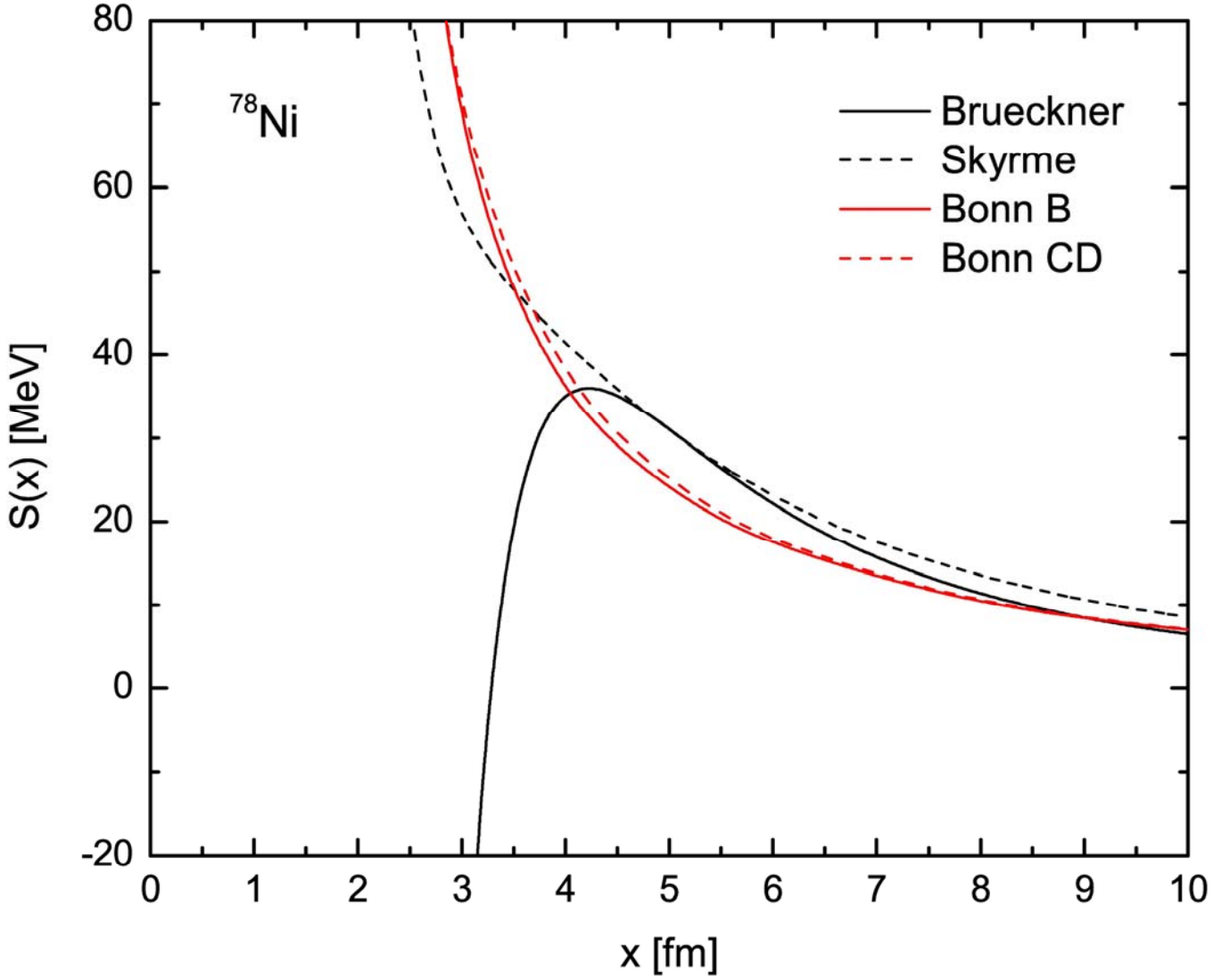
$$q(x) = \frac{S^S(x)}{S^V(x)} = A^{1/3}\left[1 - \frac{S(x)}{S^V(x)}\right] \tag{1}$$

with A, the number of nucleons in the flucton, given by $(x/r_0)^3[\rho_0(x)/\rho_0]$, and S the nuclear symmetry energy in the flucton, $S^{NM}[\rho_0(x)]$, with volume component $S^V = S^{NM}(\rho_0)$.

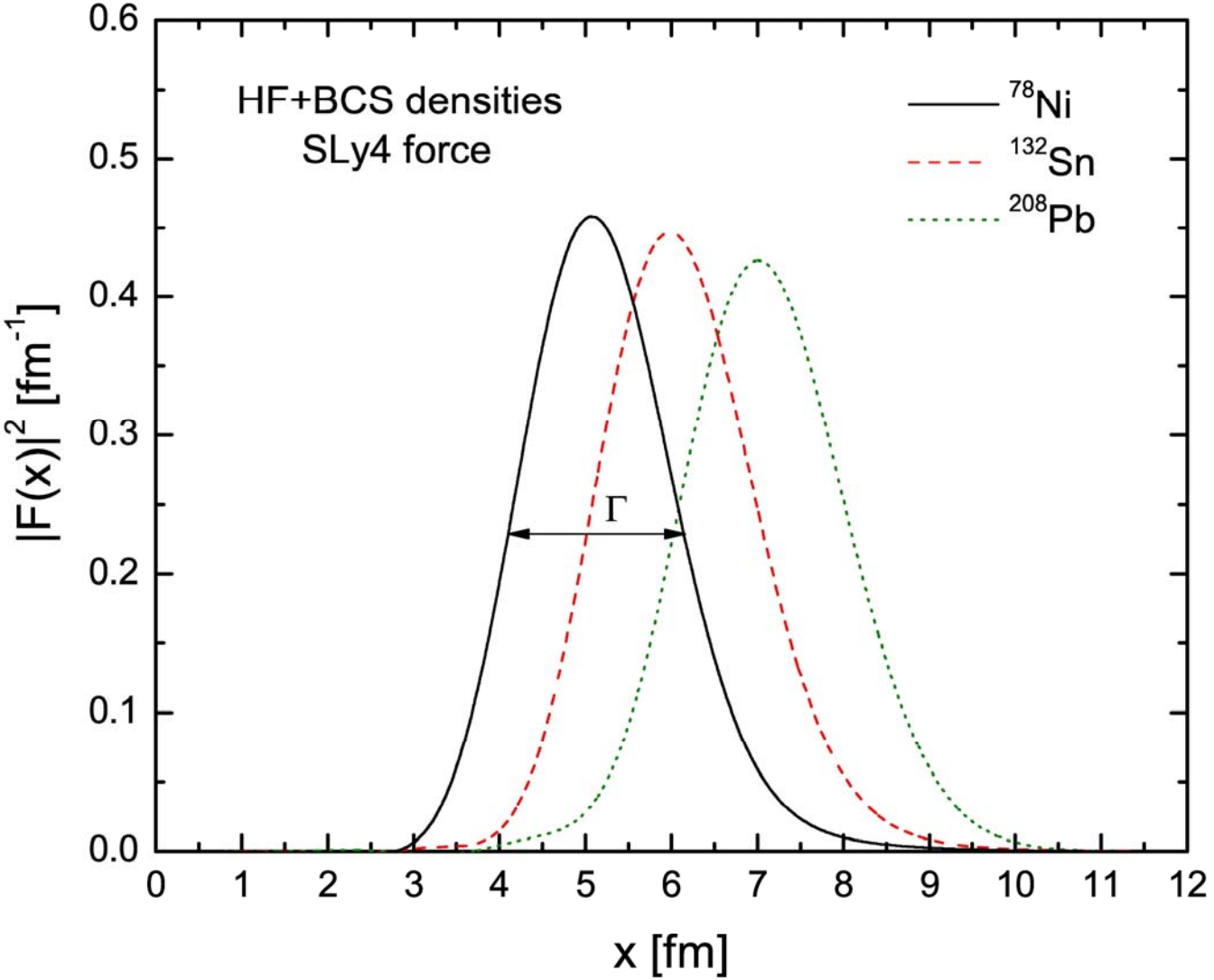
$$q(x) = \frac{x}{r_0}\left[\frac{\rho_0(x)}{\rho_0}\right]^{1/3}\left\{1 - \frac{S^{NM}[\rho_0(x)]}{S^{NM}(\rho_0)}\right\} \tag{2}$$

Weighting $q(x)$ by the function $|F(x)|^2$ leads to the following CDFM relationship for the ratio:

$$q(A) = \int_0^\infty dx|F(x)|^2 q(x) = \int_0^\infty dx|F(x)|^2 \frac{x}{r_0}\left[\frac{\rho_0(x)}{\rho_0}\right]^{1/3}\left\{1 - \frac{S^{NM}[\rho_0(x)]}{S^{NM}(\rho_0)}\right\}$$



The symmetry energy ***S(x)*** of the double-magic nucleus ^{78}**Ni** as a function of the flucton radius ***x*** calculated with Brueckner EDF, Skyrme EDF, and BHF method with Bonn B and Bonn CD potentials



The weight functions $|F(x)|^2$ of double-magic ^{78}**Ni**, ^{132}**Sn**, and ^{208}**Pb** nuclei calculated in the Skyrme HF+BCS method with SLy4 force.

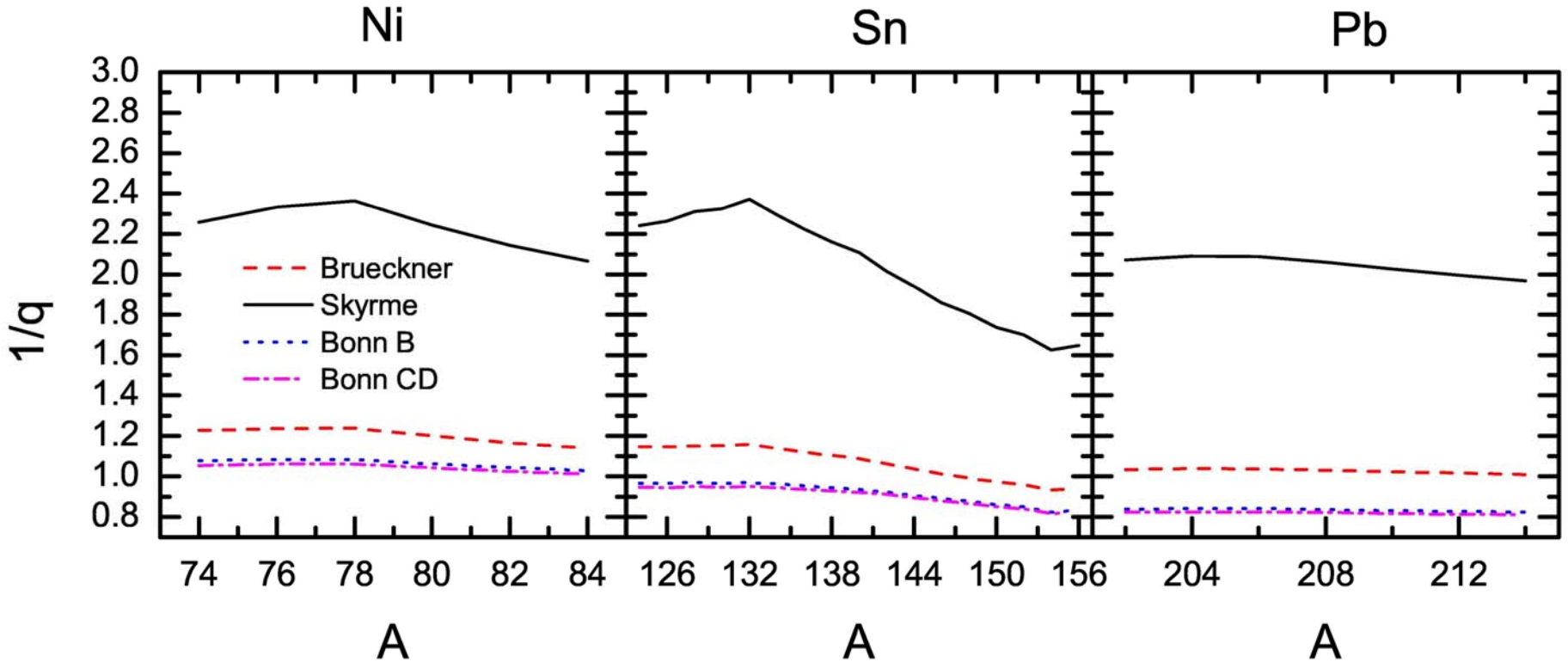
The function $|F(x)|^2$ has the form of a bell with a maximum around $x = R_{1/2}$ at which the value of the density $\rho(x = R_{1/2})$ is around half of the value of the central density equal to ρ_c $[\rho(R_{1/2})/\rho_c = 0.5]$.

A physical criterion related to the weight function $|F(x)|^2$: the width Γ

Typically, the central density of the nucleus has values around $\rho_c \approx$ 0.10-0.16 fm^{-3}. Consequently, the maximum of the weight function $|F(x)|^2$ is around $\rho(R_{1/2}) \approx$ 0.05-0.08 fm^{-3}. In the case of ^{78}Ni the maximum of $|F(x)|^2$ is at $\rho = 0.05$ fm^{-3} and, within its width range, the density ρ is between 0.12 fm^{-3} and 0.01 fm^{-3}.

The lower limit of integration: the lower value of the radius x, corresponding to the left point of the half-width Γ.

Additional calculations taking $\Gamma \pm 10\%$



The quantity $1/q=S^V/S^S$ as a function of A for the isotopic chains of **Ni**, **Sn**, and **Pb** obtained using Brueckner EDF, Skyrme EDF, and BHF method with Bonn B and Bonn CD potentials. The weight function $|F(x)|^2$ used in the calculations is obtained by means of the densities derived within a self-consistent Skyrme-Hartree-Fock plus BCS method with SLy4 force.

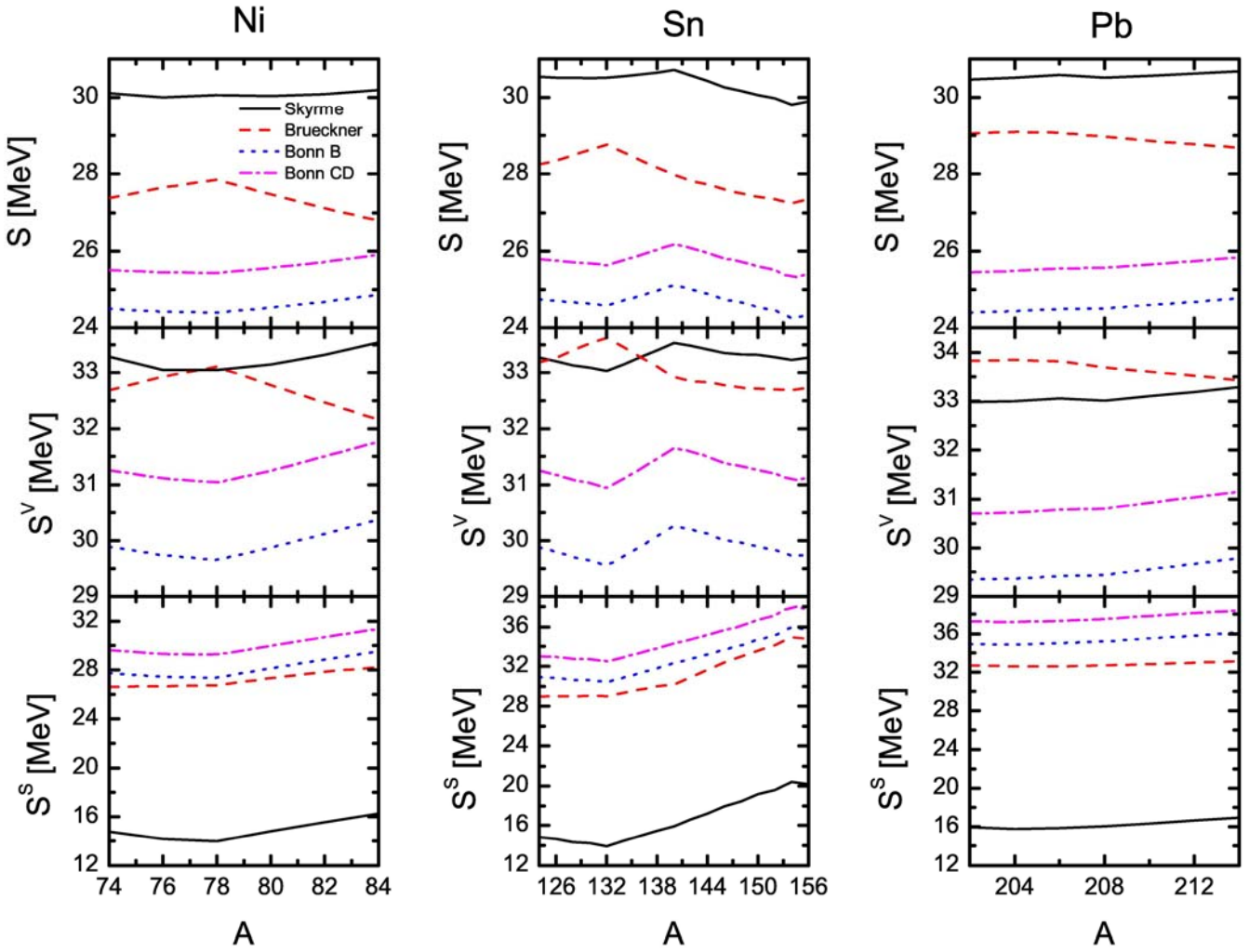
		Skyrme	Brueckner	Bonn-B	Bonn-CD
Ni	1/q	2.07 - 2.36	1.14 - 1.24	1.03 - 1.08	1.01 - 1.06
	k	1.53 - 1.70	2.22 - 2.44	1.8 - 1.9	1.8 - 2.0
Sn	1/q	1.63 - 2.37	0.94 - 1.16	0.83 - 0.97	0.82 - 0.95
	k	1.58 - 2.02	2.4 - 2.9	2.00 - 2.48	
Pb	1/q	1.97 - 2.09	1.01 - 1.04	0.84 - 0.88	0.81 - 0.83
	k	1.67 - 1.71	2.62 - 2.64	2.54 - 2.80	

In general, the values of 1/q within the new CDFM approach calculated using the Skyrme EDF for the isotopic chains of Ni, Sn, and Pb are between 1.70 and 2.40.

Our results with Skyrme EDF for 1/q agree with empirical values of κ:
- from excitation energies to IAS and skins (Danielewicz) - $2.6 \leq \kappa \leq 3.0$
- from masses and skins (Danielewicz) - $2.0 \leq \kappa \leq 2.8$

Our results with Brueckner EDF for 1/q for the Ni isotopic chain agree with κ:
- from the analyses of masses and skins (Dieperink and V. Isacker) - $1.6 \leq \kappa \leq 2.0$

Our results with Brueckner EDF and Bonn potentials for 1/q for Sn and Pb isotopes:
- are close to the theoretically estimated values of 1.14 (Bethe) and of 1.1838 (Myers and Swiatecki)



The symmetry energy ***S*** and its volume S^V and surface S^S components for the isotopic chains of **Ni**, **Sn**, and **Pb** obtained using Brueckner EDF, Skyrme EDF and BHF method with Bonn B and Bonn CD potentials. The weight function $|F(x)|^2$ used in the calculations is obtained by means of the densities derived within a self-consistent Skyrme-Hartree-Fock plus BCS method with SLy4 force.

Results for S for all chains: between 24 and 31 MeV

- predictions for the symmetry energy vary substantially (28–38 MeV); empirical value of the symmetry energy: 30 ± 4 MeV

Results for S^V: between 29 and 34 MeV

- $30.0 \leq S^V \leq 32.5$ MeV (Danielewicz)
- $31.5 \leq S^V \leq 33.5$ MeV (Danielewicz)

Results with Skyrme EDF for S^S for all chains: between 14 and 18 MeV

- are close to the limits on the surface symmetry parameter 11 MeV$\leq \beta \leq$14 MeV (Danielewicz)

Conclusions

1. We provide an alternative approach to calculate the ratio $q(A)=S^S(A)/S^V(A)$ of the surface to volume components of the NSE within the framework of the CDFM in a more direct and simple way and having stronger physical grounds than the former one that had been used in our previous works. In it we first determine the ratio $q(x)$ for a "flucton" in the CDFM from the basic droplet model mass formula and then we use the convolution of $q(x)$ with $|F(x)|^2$ to construct $q(A)$ for finite nuclei following the standard CDFM procedure.

2. The results for $q(A)$ and the components $S^S(A)$ and $S^V(A)$ are obtained from calculations based on Skyrme and Brueckner EDFs for nuclear matter and nonrelativistic Brueckner-Hartree-Fock method with realistic Bonn B and Bonn CD *NN* potentials. As in our previous scheme, by applying the CDFM the finite nuclei densities from the isotopic chains of Ni, Sn, and Pb are obtained in self-consistent Hartree-Fock+BCS calculations with SLy4 Skyrme effective interaction.

3. We would like to note the dependence of the results for the ratio of S^S to S^V on the effective nuclear potentials used in the calculations. In this respect, the results of our calculations using Skyrme EDF turn out to be close to the different estimations obtained from a fit to nuclear properties, such as the excitation energies to IAS and neutron-skin thickness, masses, and others. The values of $1/q$ obtained using the Brueckner EDF for the Ni isotopic chain are in agreement with those obtained from the analyses of masses and skins. In the case of Bonn B and Bonn CD two-body potentials the results for the ratio $1/q$ approach the estimated values of Bethe and Myers and Swiatecki. Overall, the results of the proposed scheme for $1/q$ cover reasonably the whole region of estimated values for κ (between 1.14 and 2.80) and in some cases are somewhat better than the values obtained in our previous scheme.

4. The values of the symmetry energy S for the three isotopic chains and all considered potentials are between 24 and 31 MeV, which is in accordance with the region of its empirical values 30±4 MeV. The results for $S^V(A)$ in scheme II (between 29 and 34 MeV) are in good agreement with those of Danielewicz (between 30 and 33.5 MeV). The values of $S^S(A)$ in scheme II in the case of Skyrme EDF (14-18 MeV) come closer to the region of 11-14 MeV established by Danielewicz.

5. Analyzing the isotopic sensitivity of $S^V(A)$, $S^S(A)$, and their ratio $1/q(A)$ we observe peculiarities ("kinks") of these quantities as functions of the mass number A in the cases of the double-magic ^{78}Ni and ^{132}Sn isotopes, as well as a "kink" of $S^V(A)$ for ^{140}Sn. No pronounced peak at the double-magic nucleus with A=208 in the Pb chain is found. The mentioned peculiarities in the behavior of the corresponding curves for the same quantities have been observed also in our previous CDFM approach.

Thank you!