



Fitting Potential Energy Curves for Diatomic Molecules. Extrapolation properties of Morse/Long-Range potential

Alketa Sinanaj^{1,2}, Asen Pashov¹

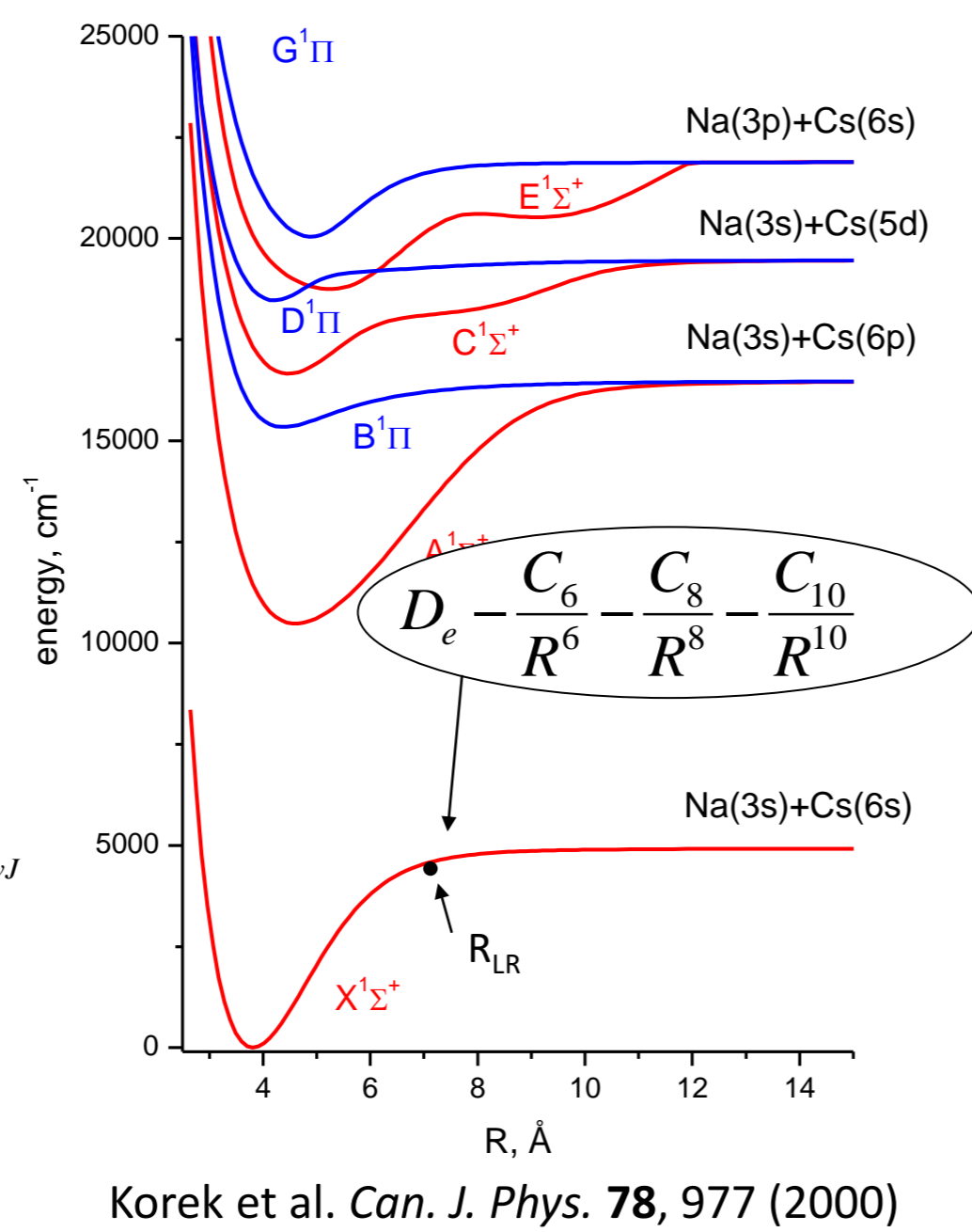
¹Faculty of Physics, Sofia University, Bul. J. Bourchier 5, 1146 Sofia, Bulgaria
²on leave from Department of Physics, University "Aleksander Xhuvani" Elbasan, Albania

Born–Oppenheimer approximation Adiabatic approximation

Radial Schrödinger equation

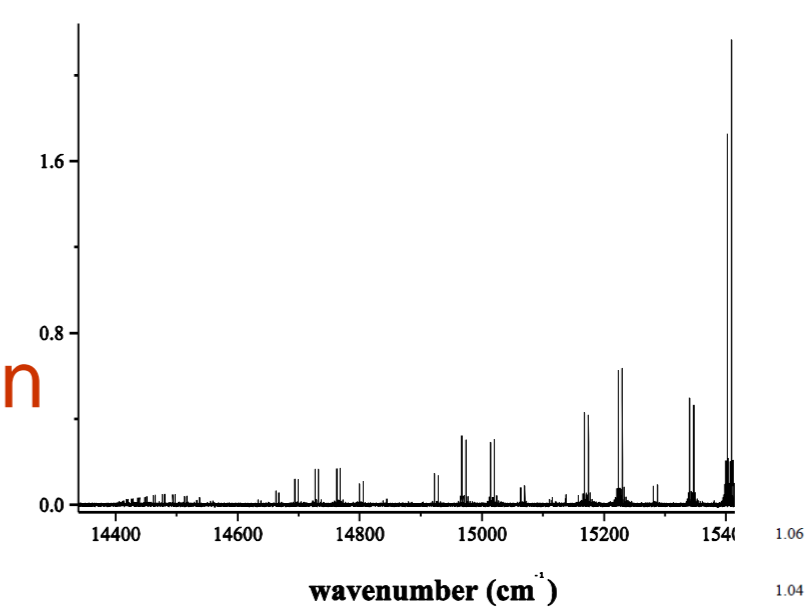
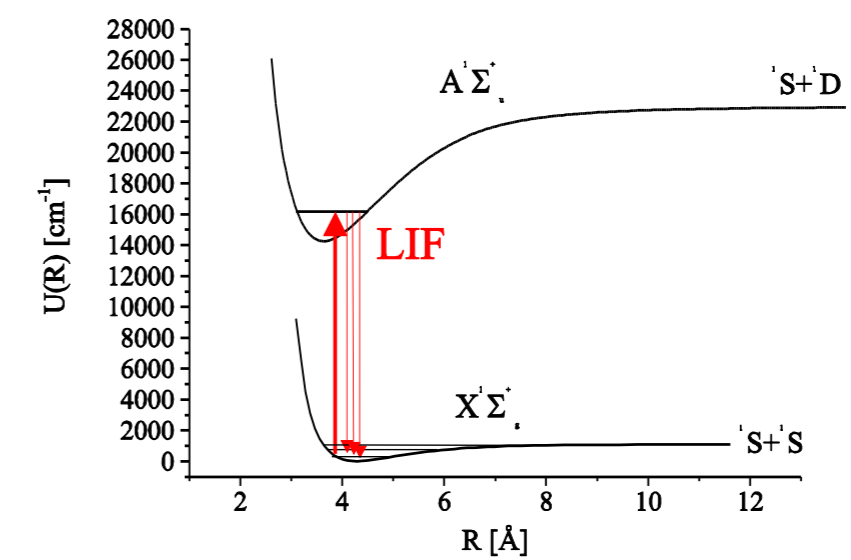
$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dR^2} + \frac{\hbar^2}{2mR^2} J(J+1) + U(R) \right] \Psi_{vJ} = E_{vJ} \Psi_{vJ}$$

No general model available!



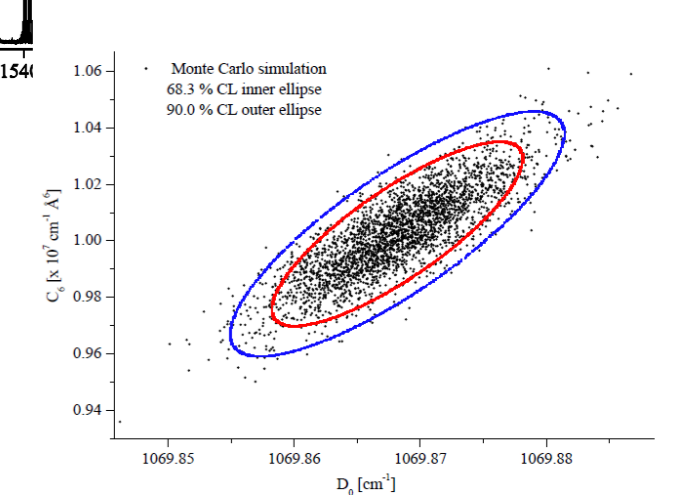
Inverse Problem in spectroscopy

Check:
Schrödinger equation



Experiment

Example: Ca₂. Allard et al. *Eur. Phys. J. D* **26**, 155–164 (2003)
About 3600 transition frequencies.



Morse/Long-Range (MLR) potential

$$V_{\text{MLR}}(r) = \mathcal{D}_e \left(1 - \frac{u_{\text{LR}}(r)}{u_{\text{LR}}(r_e)} e^{-\beta(r) \cdot y_p^{\text{eq}}(r)} \right)^2 \quad \beta(r) = \beta_{\text{MLR}}(r) = y_p^{\text{ref}}(r) \beta_\infty + [1 - y_p^{\text{ref}}(r)] \sum_{i=0} \beta_i [y_q^{\text{ref}}(r)]^i$$

$$V_{\text{MLR}}(r) \simeq \mathcal{D}_e - u_{\text{LR}}(r) + \mathcal{O}\left(\frac{u_{\text{LR}}^2}{4\mathcal{D}_e}\right) \quad u_{\text{LR}} = \frac{C_{m_1}}{r^{m_1}} + \frac{C_{m_2}}{r^{m_2}} + \dots \quad \frac{r^p - (r_e)^p}{r^p + (r_e)^p} \equiv y_p^{\text{eq}}(r)$$

R. J. Le Roy Chapter 6, pp. 159–203, of *Equilibrium Structures of Molecules*, J. Demaison and A. G. Csaszar editors, Taylor & Francis, London (2011).

Simulations

We study the extrapolation properties of MLR

- MLR potentials fitted with various combination of R_{ref} , p and q .
- β_i , C_6 , C_8 , C_{10} , D_e – fitted.
- Experimental data limited to
 - $v_{\text{max}} \leq 25$,
 - $v_{\text{max}} \leq 30$,
 - $v_{\text{max}} \leq 35$, $1069.868(10)$
 - $v_{\text{max}} \leq 38$.
- A total of 123 PEC fitted.
Dimensionless standard deviation 0.62–0.67
- Present results show very good extrapolation properties of the MLR form.
- Original study (Allard. et al.)
 $C_6 = (1.034 \pm 0.033) \times 10^7 \text{ cm}^{-1} \text{ \AA}^6$,
 $D_0 = (1069.868 \pm 0.010) \text{ cm}^{-1}$

Results

