Contribution ID: **12** Contribution code: **S03-GC-105** Type: **Oral presentation**

BPU11 CONGRESS

11th International Conference of the Balkan Physical Union
28 August – 1 September 2022, Belgrade, Serbia

Analytical Quasinormal Modes of Charged Fermions in Einstein-Born-Infeld dilaton Black Hole Spacetime

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OUTLINE

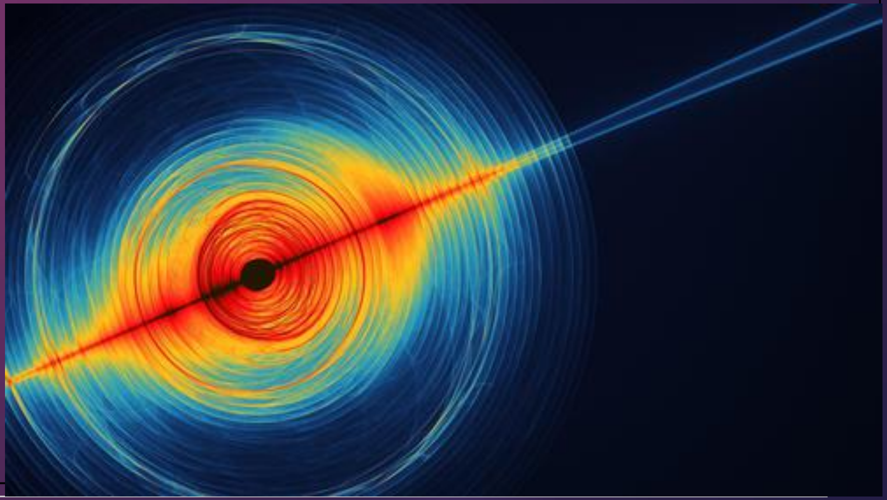
- Introduction

  BIDBH spacetime and QNMs

- Wave equations for a massless charged Dirac particle in the BIDBH geometry and their exact solutions

- Fermionic QNMs of the BIDBH

- CONCLUSION



In theoretical physics, the **Born–Infeld model** is a particular example of what is usually known as a nonlinear electrodynamics. It was historically introduced in the 1930s to remove the divergence of the electron's self-energy in classical electrodynamics by introducing an upper bound of the electric field at the origin.

Article

Foundations of the new field theory

Max Born and L. Infeld

Published: 29 March 1934 | https://doi.org/10.1098/rspa.1934.0059

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MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

PHYSICAL REVIEW D, VOLUME 64, 024009

Black holes in three-dimensional Einstein-Born-Infeld-dilaton theory

Ryo Yamazaki* and Daisuke Ida†

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

(Received 9 March 2001; published 11 June 2001)

PHYSICAL REVIEW D **72,** 044006 (2005)

Einstein-Born-Infeld-dilaton black holes in nonasymptotically flat spacetimes

Stoytcho S. Yazadjiev*

Department of Theoretical Physics, Faculty of Physics, Sofia University, 5 James Bourchier Boulevard, Sofia 1164, Bulgaria

(Received 26 April 2005; published 9 August 2005)

We derive exact magnetically charged, static, and spherically symmetric black hole solutions of the four-dimensional Einstein-Born-Infeld-dilaton gravity. These solutions are neither asymptotically flat nor (anti)-de Sitter. The properties of the solutions are discussed. It is shown that the black holes are stable against linear radial perturbations.

PHYSICAL REVIEW D **74,** 044025 (2006)

Asymptotically nonflat Einstein-Born-Infeld-dilaton black holes with Liouville-type potential

A. Sheykhi, N. Riazi,* and M. H. Mahzoon

Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran

(Received 25 February 2006; published 21 August 2006)

We construct some classes of electrically charged, static and spherically symmetric black hole solutions of the four-dimensional Einstein-Born-Infeld-dilaton gravity in the absence and presence of Liouville-type potential for the dilaton field and investigate their properties. These solutions are neither asymptotically flat nor (anti)-de Sitter. We show that in the presence of the Liouville-type potential, there exist two classes of solutions. We also compute temperature, entropy, charge and mass of the black hole solutions, and find that these quantities satisfy the first law of thermodynamics. We find that in order to fully satisfy all the field equations consistently, there must be a relation between the electric charge and other parameters of the system.

In the present work we consider stringy Einstein-Born-Infeld-dilaton (EBId) gravity described by the action [3–6]

$$S = \int d^4x\sqrt{-g}[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + L_{\mathrm{BI}}], \qquad (1)$$

where $\mathcal{R}$ is the Ricci scalar curvature with respect to the spacetime metric $g_{\mu\nu}$ and φ is the dilaton field. The Born-Infeld (BI) part of the action is given by

$$L_{\mathrm{BI}} = 4be^{2\gamma\varphi}\left[1 - \sqrt{1 + \frac{e^{-4\gamma\varphi}}{2b}F^2 - \frac{e^{-8\gamma\varphi}}{16b^2}(F\star F)^2}\right]. \qquad (2)$$

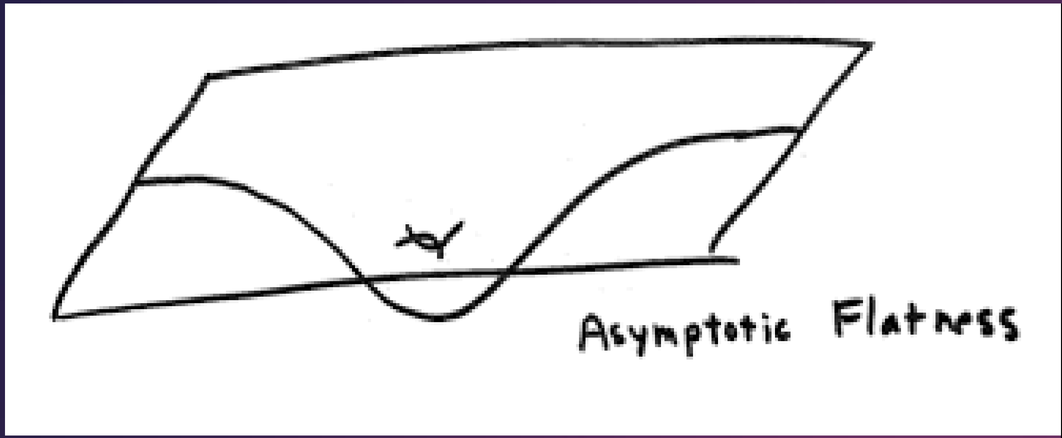
We consider the four-dimensional action in which gravity is coupled to dilaton and Born Infeld fields with an action

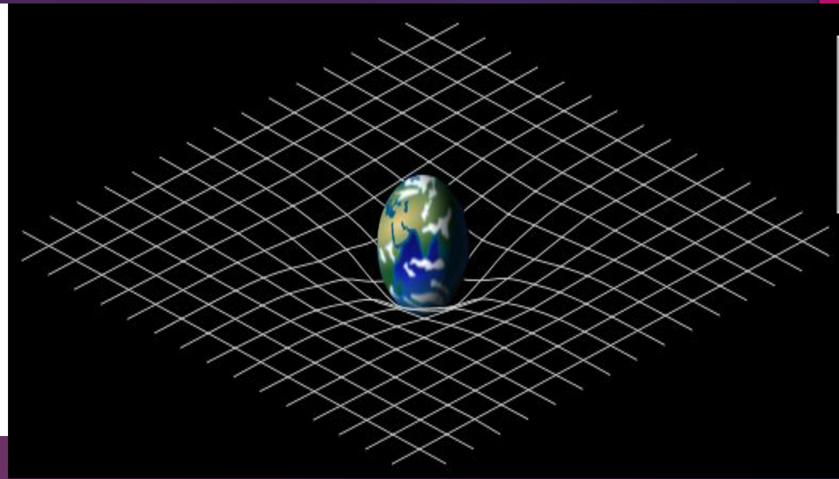
$$S = \int d^4x\sqrt{-g}(\mathcal{R} - 2(\nabla\phi)^2 - V(\phi) + L(F,\phi)) \qquad (1)$$

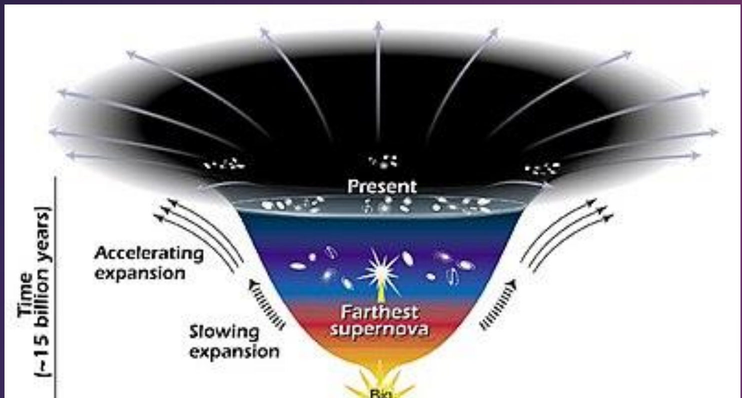
where $\mathcal{R}$ is the Ricci scalar curvature, ϕ is the dilaton field and $V(\phi)$ is a potential for ϕ. The Born-Infeld $L(F,\phi)$ part of the action is given by

$$L(F,\phi) = 4\gamma e^{-2\alpha\phi}\left(1 - \sqrt{1 + \frac{F^{\mu\nu}F_{\mu\nu}}{2\gamma}}\right). \qquad (2)$$

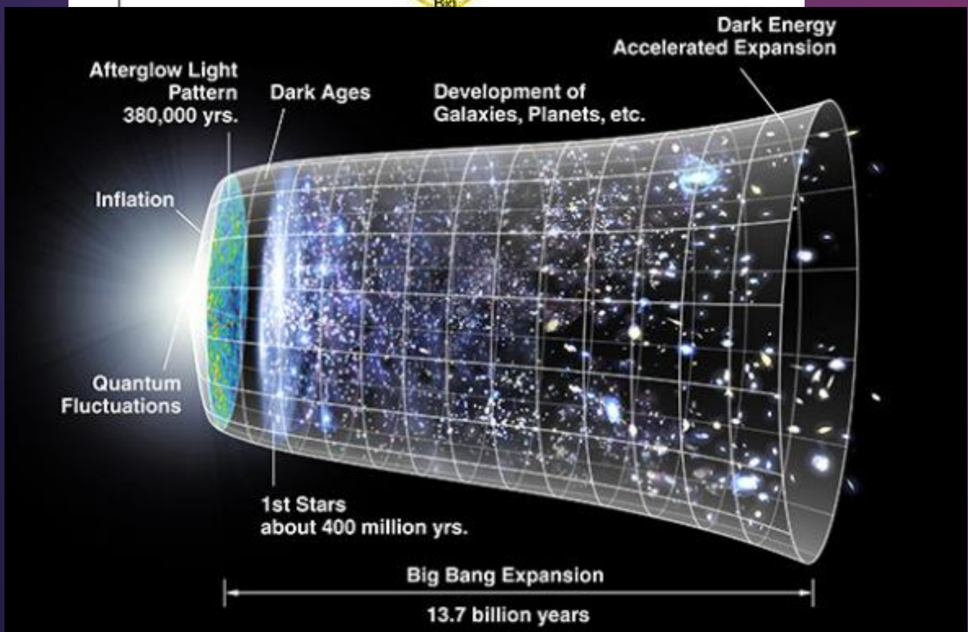
4







nonasymptotically flat



Maximally symmetric space-time

- Friedmann-Lemaître-Robertson-Walker metric

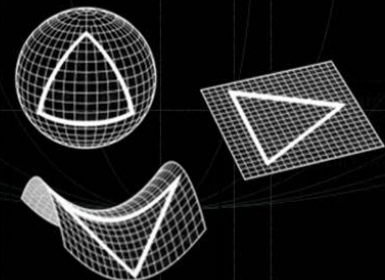
$$ds^2 = dt^2 - a^2(t)\left\{\frac{dx^2}{1-kx^2} + x^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right\}$$

equivalent to

$$ds^2 = dt^2 - a^2(t)\left\{d\chi^2 + f_k^2(\chi)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right\}$$

where

$$f_k(\chi) = \begin{cases} \sin\chi & \Leftrightarrow \quad k=+1 \quad \text{spherical} \\ \chi & \Leftrightarrow \quad k=0 \quad \text{flat} \\ \sinh\chi & \Leftrightarrow \quad k=-1 \quad \text{hyperbolic} \end{cases}$$



Special case:

In this study, we shall consider the case of magnetic charged dilatonic black hole. There is a single event horizon $r_H = Lr_0$, and therefore the line element takes the form

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + rd\Omega^2$$

nonasymptotically flat

where $h(r) = (r - r_H)/L$. Therefore the metric is characterized by two length scales L, r_H which are given functions of the three free parameters of the model, namely the black hole mass M, the Born-Infeld parameter γ, and the mass scale Λ in dilaton's potential. Note that r_H depends on all three free parameters, while L does not depend on the mass of the black hole.

The quasilocal mass

$$M = \frac{r_0}{4}.$$

$$Q^2 = \frac{1 + \sqrt{1 + 16\gamma^2}}{8\gamma}$$

$$S = \pi r_H$$

$$L^{-1} = 2(1 - \Lambda - 2H),$$

where the constant H is given by [13]

$$H = -\gamma + \sqrt{\gamma(Q^2 + \gamma)}$$

$$dM = TdS.$$

Hawking temperature is $T_H = 1/(4\pi L)$

following Maxwell equation

$$F = d\mathcal{A},$$

with the following pure magnetic potential

$$\mathcal{A} = -Q\cos\theta d\varphi,$$

which leads to

$$F = Q\sin\theta d\theta \wedge d\varphi.$$

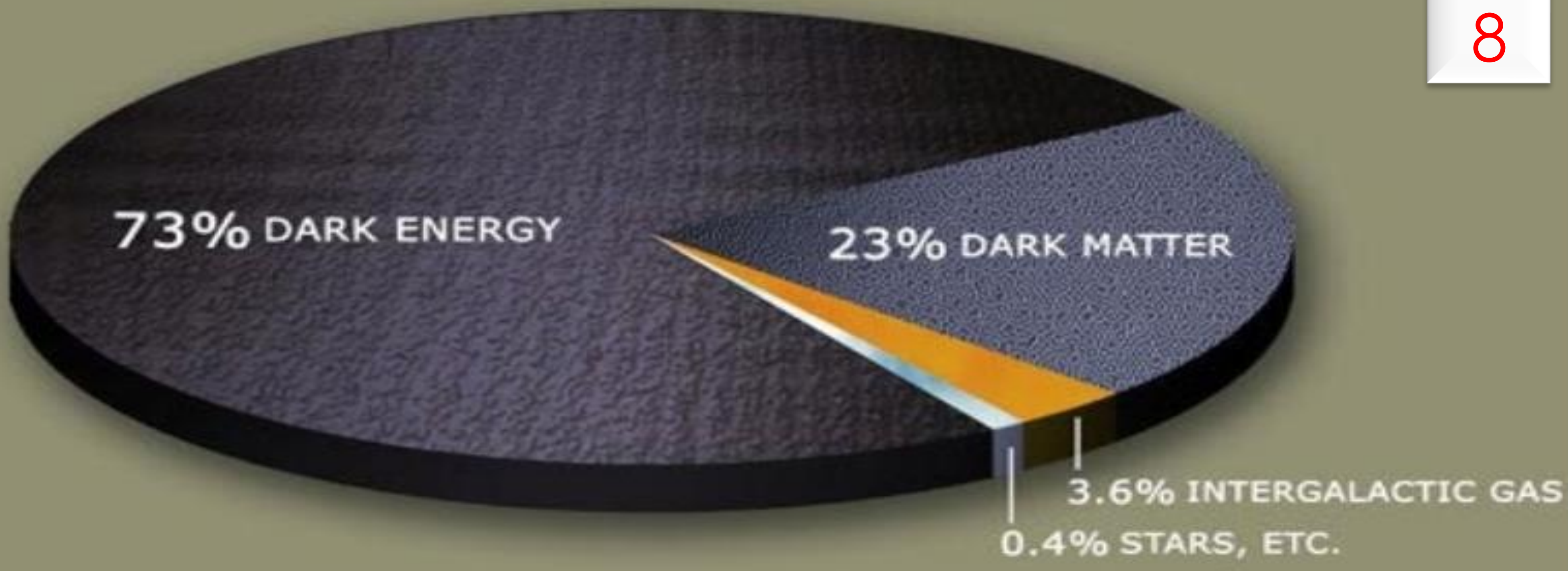
In the NP formalism , massless Dirac equations with charge coupling are given as follows

$$\left[D + iql^j\mathcal{A}_j + \varepsilon - \rho\right]\mathcal{F}_1 + \left[\overline{\delta} + iq\overline{m}^j\mathcal{A}_j + \pi - \alpha\right]\mathcal{F}_2 = 0,$$

$$\left[\delta + iqm^j\mathcal{A}_j \ \ + \beta - \tau\right]\mathcal{F}_1 + \left[\Delta + iqn^j\mathcal{A}_j \ \ + \mu - \gamma\right]\mathcal{F}_2 = 0,$$

$$\left[D + iql^j\mathcal{A}_j + \overline{\varepsilon} - \overline{\rho}\right]\widetilde{\mathcal{G}}_2 - \left[\delta + iqm^j\mathcal{A}_j + \overline{\pi} - \overline{\alpha}\right]\widetilde{G}_1 = 0,$$

$$\left[\Delta + iqn^j\mathcal{A}_j \ \ + \overline{\mu} - \overline{\gamma}\right]\widetilde{\mathcal{G}}_1 - \left[\overline{\delta} + iq\overline{m}^j\mathcal{A}_j + \overline{\beta} - \overline{\tau}\right]\widetilde{\mathcal{G}}_2 = 0,$$



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SPACE

Dark Side of Black Holes: Dark Matter Could Explain the Early Universe's Giant Black Holes

Massive black holes should not have existed in a universe less than one billion years old, yet they did

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Dilatonic dark matter and its experimental detection

Y. M. Cho and J. H. Kim
Phys. Rev. D **79**, 023504 – Published 7 January 2009

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Asimina Arvanitaki, Junwu Huang, and Ken Van Tilburg
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T. Damour, G. W. Gibbons, and C. Gundlach
Phys. Rev. Lett. **64**, 123 – Published 8 January 1990

Theoretical and Mathematical Physics
March 2018, Volume 194, Issue 3, pp 415–438 | Cite as

Anisotropic Cosmology with a Dilaton Field Coupled to Ghost Dark Energy

Authors Authors and affiliations

H. Hossienkhani, V. Fayaz, S. A. A. Terohid, N. Azimi, Z. Zarei, M. Ganji

Article
First Online: 04 April 2018

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Y. M. CHO and Y. Y. KEUM

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Dilaton-Assisted Dark Matter

Yang Bai, Marcela Carena, and Joseph Lykken
Phys. Rev. Lett. **103**, 261803 – Published 30 December 2009

Journal of Cosmology and Astroparticle Physics

Can dark matter be a scalar field?

J.F. Jesus[a], S.H. Pereira[b], J.L.G. Malatrasi[a] and F. Andrade-Oliveira[c]
Published 22 August 2016 • © 2016 IOP Publishing Ltd and Sissa Medialab srl
Journal of Cosmology and Astroparticle Physics, Volume 2016, August 2016

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Santiago Casas, Martin Pauly, and Javier Rubio
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Benjamin Fuks, Mark D. Goodsell, Dong Woo Kang, Pyungwon Ko, Seung J. Lee & Manuel Utsch

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Primordial black hole dark matter in dilaton-extended two-field Starobinsky inflation

Anirudh Gundhi, Sergei V. Ketov, and Christian F. Steinwachs
Phys. Rev. D **103**, 083518 – Published 21 April 2021

QUASINORMAL MODES

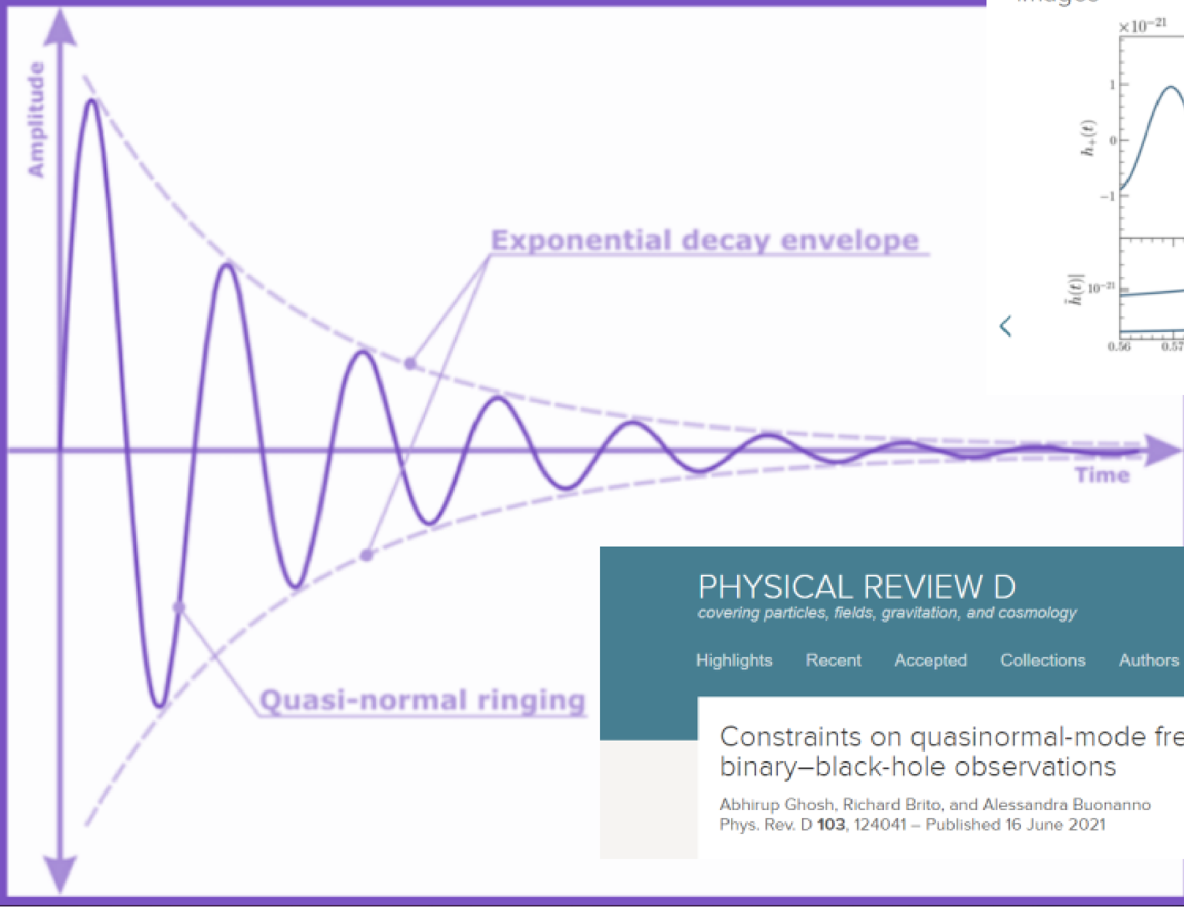
√ QNMs are the modes of "energy dissipation" of a perturbed field or object.

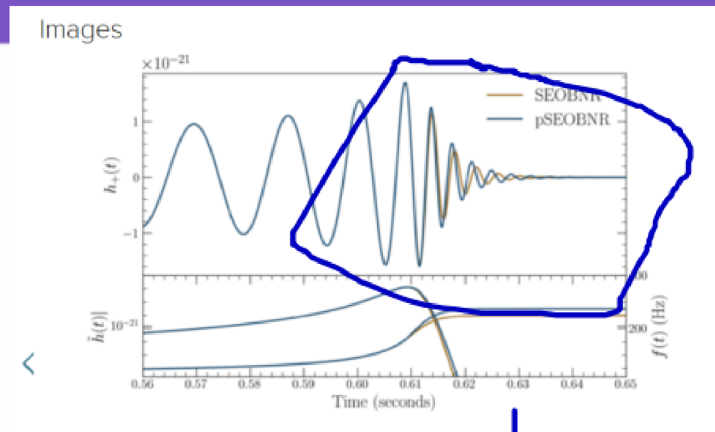
An example: Let us perturb a wine glass with a knife.

The glass begins to ring, it rings with a superposition of "its natural frequencies".

Here, we mean by the natural frequency is the mode of its sonic energy dissipation. These modes are called "Normal Modes".

If the amplitude of oscillation decay in time, we call these modes Quasinormal.





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Constraints on quasinormal-mode frequencies with LIGO-Virgo binary–black-hole observations

Abhirup Ghosh, Richard Brito, and Alessandra Buonanno

Phys. Rev. D **103**, 124041 – Published 16 June 2021

√ The amplitude of the oscillation can be approximated by

$$\begin{aligned}\Psi &\approx e^{i\omega t} \text{ and setting } \omega = \omega_R + i\omega_I \\ &\approx e^{-\omega_I t}\cos(\omega_R t)\end{aligned}$$

Here ω is referred to as the QNM frequency.

a) ω_R is the frequency of the oscillatory mode.

b) ω_I is the frequency of the exponential mode.

√ It is believed that QNM of a BH is a characteristic wave of it and would lead to the direct identification of the BH existence through gravitational wave observation. (If it will be realized in the future.)

√ Today, there are many well known studies which show that the surrounding geometry of a BH experiences QNMs under perturbations.

S. Chandrasekhar and S.L. Detweiler, "The Quasinormal Modes of the Schwarzschild Black Hole", Proc. R. Soc. London, (1975).

K.D. Kokkotas and B. Schmidt, "Quasinormal Modes of Stars and Black Holes", Living Rev. Relativity, (1999).

Main Boundary Conditions: The quasinormal modes are defined as the modes with the purely ingoing wave at the horizon and the purely outgoing wave at the spatial infinity.

Charged Dirac equation in BIDBH geometry

In the NP formalism , massless Dirac equations with charge coupling are given as follows

$$\left[D + iql^j\mathcal{A}_j + \varepsilon - \rho\right]\mathcal{F}_1 + \left[\overline{\delta} + iq\overline{m}^j\mathcal{A}_j + \pi - \alpha\right]\mathcal{F}_2 = 0,$$

$$\left[\delta + iqm^j\mathcal{A}_j \;\; + \beta - \tau\right]\mathcal{F}_1 + \left[\Delta + iqn^j\mathcal{A}_j \;\; + \mu - \gamma\right]\mathcal{F}_2 = 0,$$

$$\left[D + iql^j\mathcal{A}_j + \overline{\varepsilon} - \overline{\rho}\right]\widetilde{\mathcal{G}}_2 - \left[\delta + iqm^j\mathcal{A}_j + \overline{\pi} - \overline{\alpha}\right]\widetilde{G}_1 = 0,$$

$$\left[\Delta + iqn^j\mathcal{A}_j \;\; + \overline{\mu} - \overline{\gamma}\right]\widetilde{\mathcal{G}}_1 - \left[\overline{\delta} + iq\overline{m}^j\mathcal{A}_j + \overline{\beta} - \overline{\tau}\right]\widetilde{\mathcal{G}}_2 = 0,$$

where q is the charge of the fermion and A_j represents the j^{th} component of the vector potential

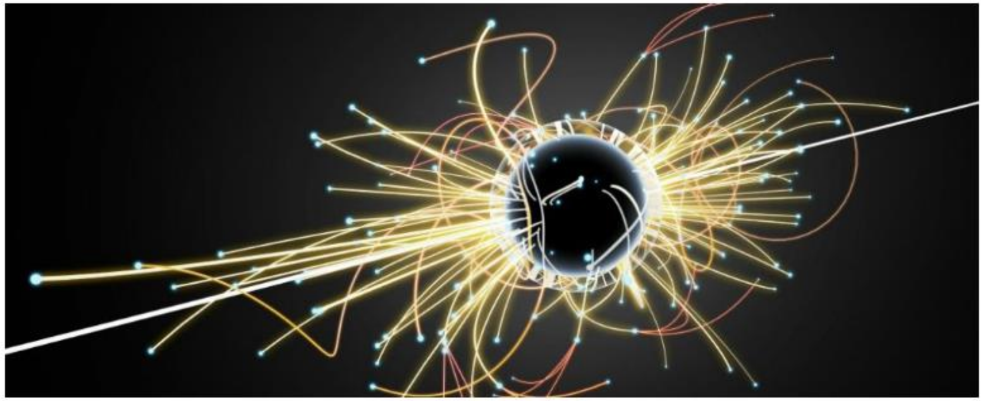
The wave functions $F_1, F_2, \widetilde{G}_1, \widetilde{G}_2$ represent the Dirac spinors

$\alpha, \beta, \gamma, \varepsilon, \mu, \pi, \rho, \tau$ are the spin (Ricci rotation) coefficients. The directional derivatives for NP tetrads are defined as

$$D = l^j\nabla_j,\; \Delta = n^j\nabla_j,\; \delta = m^j\nabla_j,\; \bar{\delta} = \bar{m}^j\nabla_j,$$

Scientists Have Finally Discovered Massless Particles, and They Could Revolutionise Electronics

23 July 2015 By FIONA MACDONALD



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After 85 years of searching, researchers have confirmed the existence of a massless particle called the Weyl fermion for the first time ever. With the unique ability to behave as both matter and anti-matter inside a crystal, this strange particle can create electrons that have no mass.

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RESEARCH ARTICLE

Discovery of a Weyl fermion semimetal and topological Fermi arcs

SU-YANG XU, ILYA BELOPOLSKI, NASSER ALIDOUST, MADHAB NEUPANE, GUANG BIAN, CHENGLONG ZHANG, RAMAN SANKAR, GUOQING CHANG, ZHUJUN YUAN, [...] M. ZAHID HASAN +12 authors Authors Info & Affiliations

SCIENCE • 16 Jul 2015 • Vol 349, Issue 6248 • pp. 613-617 • DOI: 10.1126/science.aaa9297

Weyl physics emerges in the laboratory

$$l_j = \frac{1}{\sqrt{2}}\left[\sqrt{h(r)}, -\frac{1}{\sqrt{h(r)}}, 0, 0\right],$$

$$n_j = \frac{1}{\sqrt{2}}\left[\sqrt{h(r)}, \frac{1}{\sqrt{h(r)}}, 0, 0\right],$$

$$m_j = \sqrt{\frac{r}{2}}\,[0, 0, 1, i\sin\theta],$$

$$\overline{m}_j = \sqrt{\frac{r}{2}}\,[0, 0, 1, -i\sin\theta],$$

orthogonality conditions: $l.n = -m.\overline{m} = 1$

spinors can be chosen as follows

$$\mathcal{F}_1 = f_1(r)\,A_1(\theta)\,e^{i(kt+m\varphi)},$$

$$\widetilde{\mathcal{G}}_1 = g_1(r)\,A_2(\theta)\,e^{i(kt+m\varphi)},$$

$$\mathcal{F}_2 = f_2(r)\,A_3(\theta)\,e^{i(kt+m\varphi)},$$

$$\widetilde{\mathcal{G}}_2 = g_2(r)\,A_4(\theta)\,e^{i(kt+m\varphi)},$$

The non-zero spin coefficients

$$\rho = \mu = -\frac{-1}{2\sqrt{2}}\frac{\sqrt{h(r)}}{r},$$

$$\epsilon = \gamma = \frac{b}{4\sqrt{2h(r)}}\left(1 - \frac{r_2 r_1}{r^2}\right),$$

$$\alpha = -\beta = \frac{\cot\theta}{2\sqrt{2}r}.$$

Dirac equations

$$\frac{1}{f_2}\widetilde{Z} f_1 - \frac{(LA_3)}{A_1} = 0,$$

$$\frac{1}{f_1}\overline{\widetilde{Z}} f_2 + \frac{(L^\dagger A_1)}{A_3} = 0,$$

$$\frac{1}{g_1}\widetilde{Z} g_2 - \frac{(L^\dagger A_2)}{A_4} = 0,$$

$$\frac{1}{g_2}\overline{\widetilde{Z}} g_1 + \frac{(LA_4)}{A_2} = 0.$$

The radial operators

$$\widetilde{Z} = \sqrt{h(r)}\partial_r + H + \frac{ikr}{\sqrt{h(r)}},$$

$$\overline{\widetilde{Z}} = \sqrt{h(r)}\partial_r + H - \frac{ikr}{\sqrt{h(r)}},$$

$$H = \frac{1}{4r}\left[\frac{r^2}{L\sqrt{h(r)}} + 2\sqrt{h(r)}\right].$$

$$L = \partial_\theta + \frac{m}{\sin\theta} + \left(\frac{1}{2} - p\right)\cot\theta,$$

$$L^\dagger = \partial_\theta - \frac{m}{\sin\theta} + \left(\frac{1}{2} + p\right)\cot\theta,$$

where $p = qQ$.

Further, choosing

$$f_1 = g_2,\ f_2 = g_1,\ A_1 = A_2,\ A_3 = A_4,$$

The radial master equations

$$\overline{\widetilde{Z}} g_1 = -\lambda g_2,$$

$$\widetilde{Z} g_2 = -\lambda g_1,$$

the angular master equations

$$L^\dagger A_2 = \lambda A_4,$$

$$L A_4 = -\lambda A_2.$$

a real eigenvalue λ which is the separation constant of the complete Dirac equations

The laddering operators L and $L^\dagger$ govern the spin-weighted spheroidal harmonics

$$\lambda = -\sqrt{\left(l+\frac{1}{2}\right)^2 - p^2}, \quad \left(\text{Real: } \left(l+\frac{1}{2}\right) \geq p\right),$$

$$\therefore\ \lambda^2 = \left(l+\frac{1}{2}\right)^2 - p^2.$$

$$\left(\partial_\theta - \frac{m}{\sin\theta} - s\cot\theta\right)\left({}_sY_l^m(\theta)\right) = -\sqrt{(l-s)(l+s+1)}\,{}_{s+1}Y_l^m(\theta),$$

$$\left(\partial_\theta + \frac{m}{\sin\theta} + s\cot\theta\right)\left({}_sY_l^m(\theta)\right) = \sqrt{(l+s)(l-s+1)}\,{}_{s-1}Y_l^m(\theta).$$

$$2qQ = 2p = n, \qquad n = 0, \pm 1, \pm 2...,$$

$l = |s|, |s|+1, |s|+2, ...$ and $-l < m < +l.$

Since l and s both must be integers or half-integers, we impose the "Dirac quantization condition"

Radial equation and Zerilli potential

$$g_1 = \frac{G_1}{[rh(r)]^{\frac{1}{4}}},$$
$$g_2 = \frac{G_2}{[rh(r)]^{\frac{1}{4}}},$$

$$G_{1,r_*} - ik = -\frac{\lambda\sqrt{h(r)}G_2}{r},$$
$$G_{2,r*} + ik = -\frac{\lambda\sqrt{h(r)}G_1}{r}.$$

$$dr_* = \frac{dr}{h(r)} \quad \longrightarrow \quad r_* = \frac{L}{r_H}\ln\left[\frac{(r-r_H)^{r_H}}{r}\right],$$

$$G_1 = P_1 + P_2,$$
$$G_2 = P_1 - P_2,$$

Two decoupled radial equations, which correspond to 1-dimensional Schrödinger equation or the so-called Zerilli equation

$$P_{j,r*r*} + k^2 P_j = V_j P_j, \qquad j = 1, 2$$

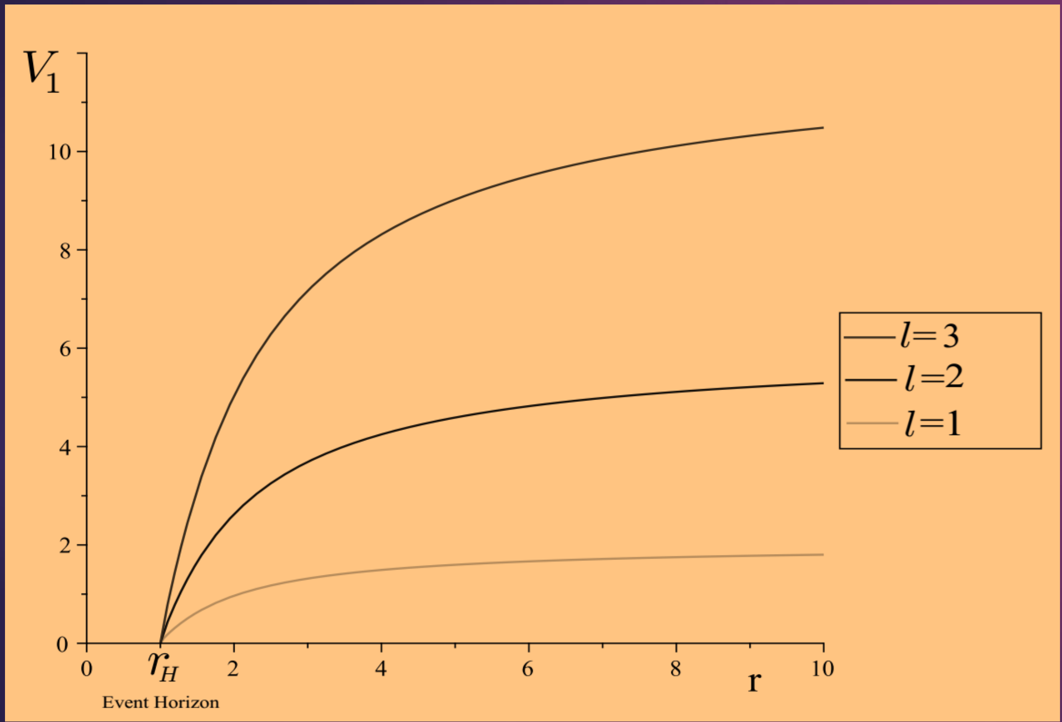
the effective potentials are given by

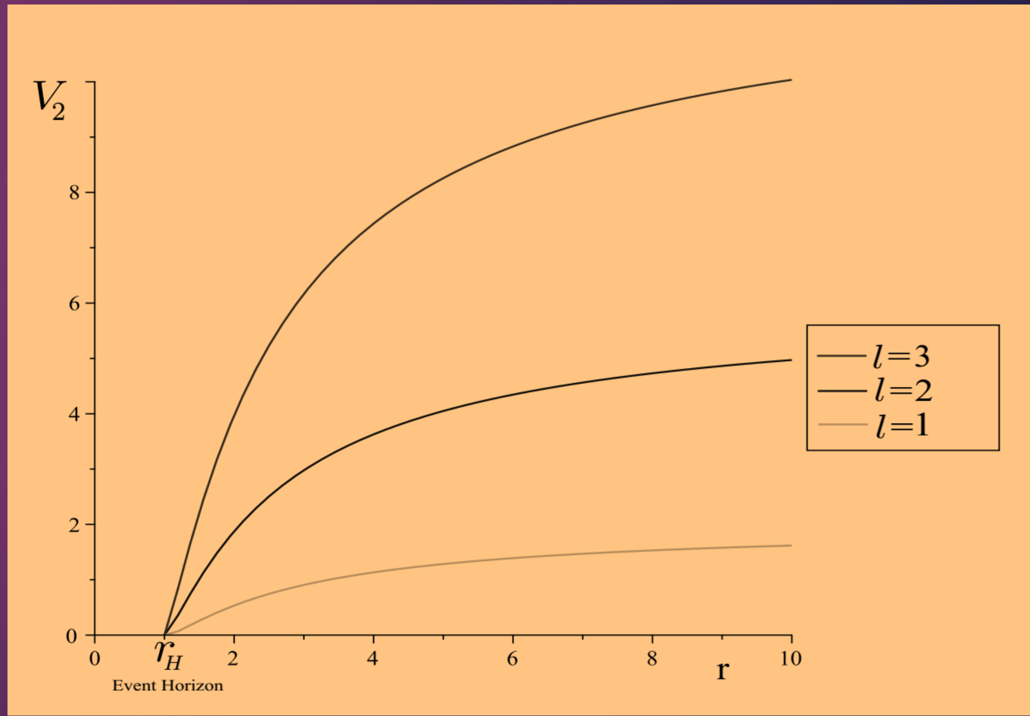
$$V_j = \lambda^2\frac{h(r)}{r} + (-1)^j\frac{\lambda r_H}{2Lr}\sqrt{\frac{h(r)}{r}}.$$

The near-horizon and asymptotic limits of the potentials are as follows

$$\lim_{r\to r_H} V_j \equiv \lim_{r^*\to -\infty} V_j = 0,$$

$$\lim_{r\to\infty} V_j \equiv \lim_{r^*\to\infty} V_j = \frac{\lambda^2}{L}.$$





decoupling of G_j

$$h(r)G_j'' - \frac{b(r_H - 2r)}{2}G_j' + \left\{(-1)^j ik\left[\frac{-r^2}{h(r)L} - \frac{(r_H - 2r)}{2h(r)}\right] + \frac{k^2r^2}{h(r)} - \lambda^2\right\}G_j = 0.$$

$$G_j = H_j e^{(-1)^{j+1}ikr_*},$$

$$z = -x = -\frac{r - r_H}{r_H}.$$

Euler's hypergeometric differential equation

$$z(1-z)H_j'' + \left[\boldsymbol{c} - (1 + \boldsymbol{a} + \boldsymbol{b})z\right]H_j' - \boldsymbol{a}\boldsymbol{b}H_j = 0,$$

general solution

$$\boldsymbol{a} = i\left[(-1)^{j+1}kL + \widetilde{\alpha}\right],$$

$$\boldsymbol{b} = i\left[(-1)^{j+1}kL - \widetilde{\alpha}\right],$$

$$\boldsymbol{c} = \frac{1}{2} + 2ik(-1)^{j+1}L.$$

$$H_j = C_2 F\boldsymbol{a}, \boldsymbol{b}; \boldsymbol{c}; z) + C_1 z^{1-\boldsymbol{c}} F(\widetilde{\boldsymbol{a}}, \widetilde{\boldsymbol{b}}; \widetilde{\boldsymbol{c}}; z),$$

$$\widetilde{\boldsymbol{a}} = \boldsymbol{a} - \boldsymbol{c} + 1 = \frac{1}{2} - i\left[k(-1)^{j+1}L + \widetilde{\alpha}\right],$$

$$\widetilde{\boldsymbol{b}} = \boldsymbol{b} - \boldsymbol{c} + 1 = \frac{1}{2} - i\left[k(-1)^{j+1}L - \widetilde{\alpha}\right],$$

$$\widetilde{\boldsymbol{c}} = 2 - \boldsymbol{c} = \frac{3}{2} - 2ik(-1)^{j+1}L.$$

In the near-horizon region: $r \to r_2,\ r_* \to -\infty \quad \Rightarrow \quad z \approx e^{r_*} \to 0$. By recalling that $F(\boldsymbol{a}, \boldsymbol{b}; \boldsymbol{c}; 0) = 1$

Only ingoing waves can propagate near the horizon

$$G_2 = H_2 e^{-ikr_*} = C_2 e^{-ikr_*} F(\boldsymbol{a}, \boldsymbol{b}; \boldsymbol{c}; z),$$

$$G_1 = H_1 e^{ikr_*} = C_1 z^{1-\boldsymbol{c}} e^{+ikr_*} F(\widetilde{\boldsymbol{a}}, \widetilde{\boldsymbol{b}}; \widetilde{\boldsymbol{c}}; z).$$

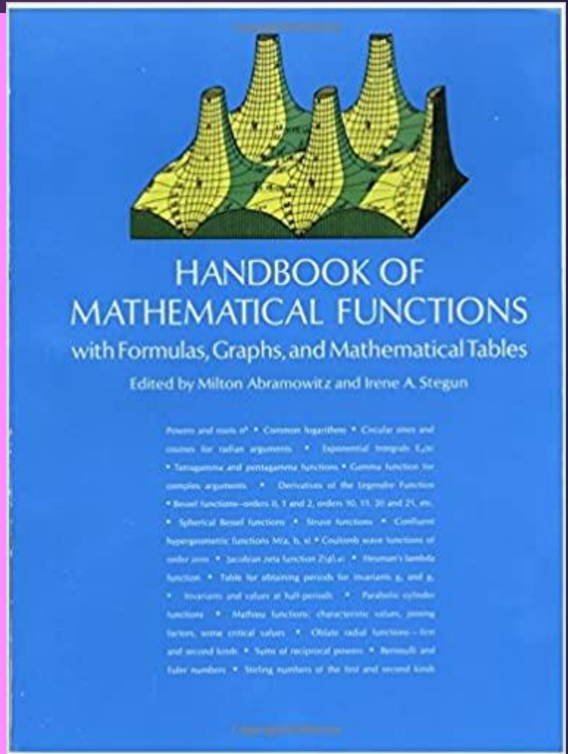
spatial infinity $(r \to \infty,\ r_* \to \infty \ \Rightarrow\ \frac{1}{z} \to 0)$

$$G_1 = C_1 z^{1-\boldsymbol{c}} e^{ikr_*} \left[\frac{\Gamma(\widetilde{\boldsymbol{c}})\Gamma(\widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{a}})}{\Gamma(\widetilde{\boldsymbol{b}})\Gamma(\boldsymbol{c} - \widetilde{\boldsymbol{a}})} (x)^{-\widetilde{\boldsymbol{a}}} F(\widetilde{\boldsymbol{a}}, \widetilde{\boldsymbol{a}} + 1 - \widetilde{\boldsymbol{c}}; \widetilde{\boldsymbol{a}} + 1 - \widetilde{\boldsymbol{b}}; 1/z) \right.$$

$$\left. + \frac{\Gamma(\widetilde{\boldsymbol{c}})\Gamma(\widetilde{\boldsymbol{a}} - \widetilde{\boldsymbol{b}})}{\Gamma(\widetilde{\boldsymbol{a}})\Gamma(\widetilde{\boldsymbol{c}} - \widetilde{\boldsymbol{b}})} (x)^{-\widetilde{\boldsymbol{b}}} F(\widetilde{\boldsymbol{b}}, \widetilde{\boldsymbol{b}} + 1 - \widetilde{\boldsymbol{c}}; \widetilde{\boldsymbol{b}} + 1 - \widetilde{\boldsymbol{a}}; 1/z) \right],$$

$$G_2 = C_2 e^{-ikr_*} \left[\frac{\Gamma(\boldsymbol{c})\Gamma(\boldsymbol{b} - \boldsymbol{a})}{\Gamma(\boldsymbol{b})\Gamma(\boldsymbol{c} - \boldsymbol{a})} (x)^{-\boldsymbol{a}} F(\boldsymbol{a}, \boldsymbol{a} + 1 - \boldsymbol{c}; \boldsymbol{a} + 1 - \boldsymbol{b}; 1/z) \right.$$

$$\left. + \frac{\Gamma(\boldsymbol{c})\Gamma(\boldsymbol{a} - \boldsymbol{b})}{\Gamma \boldsymbol{a})\Gamma(\boldsymbol{c} - \boldsymbol{b})} (x)^{-\boldsymbol{b}} F(\boldsymbol{b}, \boldsymbol{b} + 1 - \boldsymbol{c}; \boldsymbol{b} + 1 - \boldsymbol{a}; 1/z) \right].$$



Since $x = \frac{r - r_H}{r_H}$, and thus $x \simeq r \simeq e^{\frac{r_*}{L}}$ at the infinity, we have

$$G_1 \approx C_1 \frac{\Gamma(\widetilde{\boldsymbol{c}})\Gamma(\widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{a}})}{\Gamma(\widetilde{\boldsymbol{b}})\Gamma(\boldsymbol{c} - \widetilde{\boldsymbol{a}})} r^{i\widetilde{\alpha}} + C_1 \frac{\Gamma(\widetilde{\boldsymbol{c}})\Gamma(\widetilde{\boldsymbol{a}} - \widetilde{\boldsymbol{b}})}{\Gamma(\widetilde{\boldsymbol{a}})\Gamma(\widetilde{\boldsymbol{c}} - \widetilde{\boldsymbol{b}})} r^{-i\widetilde{\alpha}}.$$

$$G_2 \approx C_2 \frac{\Gamma(\boldsymbol{c})\Gamma(\boldsymbol{b} - \boldsymbol{a})}{\Gamma(\boldsymbol{b})\Gamma(\boldsymbol{c} - \boldsymbol{a})} r^{i\widetilde{\alpha}} + C_2 \frac{\Gamma(\boldsymbol{c})\Gamma(\boldsymbol{a} - \boldsymbol{b})}{\Gamma(\boldsymbol{a})\Gamma(\boldsymbol{c} - \boldsymbol{b})} r^{-i\widetilde{\alpha}}.$$

$$h(r)G_j'' - \frac{b(r_H - 2r)}{2} G_j' + \left\{(-1)^j ik\left[\frac{-r^2}{h(r)L} - \frac{(r_H - 2r)}{2h(r)}\right] + \frac{k^2 r^2}{h(r)} - \lambda^2\right\} G_j = 0.$$

As $r \to \infty$

$$r^2 G_j'' + r G_j' + \widetilde{\alpha}^2 G_j = 0.$$

$$G_j = \widetilde{D}_j r^{i\widetilde{\alpha}} + \cancel{\widehat{C}_j r^{-i\widetilde{\alpha}}},$$

The correspondence between the asymptotic solutions

$$\widetilde{D}_1 = C_1 \frac{\Gamma(\widetilde{\boldsymbol{c}})\Gamma(\widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{a}})}{\Gamma(\widetilde{\boldsymbol{b}})\Gamma(\boldsymbol{c} - \widetilde{\boldsymbol{a}})},$$

$$\widetilde{D}_2 = C_2 \frac{\Gamma(\boldsymbol{c})\Gamma(\boldsymbol{b} - \boldsymbol{a})}{\Gamma(\boldsymbol{b})\Gamma(\boldsymbol{c} - \boldsymbol{a})},$$

$$\widetilde{D}_3 = C_1 \frac{\Gamma(\widetilde{\boldsymbol{c}})\Gamma(\widetilde{\boldsymbol{a}} - \widetilde{\boldsymbol{b}})}{\Gamma(\widetilde{\boldsymbol{a}})\Gamma(\widetilde{\boldsymbol{c}} - \widetilde{\boldsymbol{b}})},$$

$$\widetilde{D}_4 = C_2 \frac{\Gamma(\boldsymbol{c})\Gamma(\boldsymbol{a} - \boldsymbol{b})}{\Gamma(\boldsymbol{a})\Gamma(\boldsymbol{c} - \boldsymbol{b})}.$$

the wave should be purely outgoing at the spatial infinity, we set

$$\widehat{C}_j = 0$$

to have only pure outgoing waves at the spatial infinity (i.e., $\widetilde{D}_3, \widetilde{D}_4 = 0$)

we appeal to the pole structure of the Gamma functions

$$\boldsymbol{a} = i\,(-kL + \widetilde{\alpha}) = -n,$$

$$\boldsymbol{c} - \boldsymbol{b} = \frac{1}{2} - i\,(kL + \widetilde{\alpha}) = -n,$$

$$\widetilde{\boldsymbol{a}} = \frac{1}{2} - i\,(kL + \widetilde{\alpha}) = -n,$$

$$\widetilde{\boldsymbol{c}} - \widetilde{\boldsymbol{b}} = 1 + i\,(kL - \widetilde{\alpha}) = -n.$$

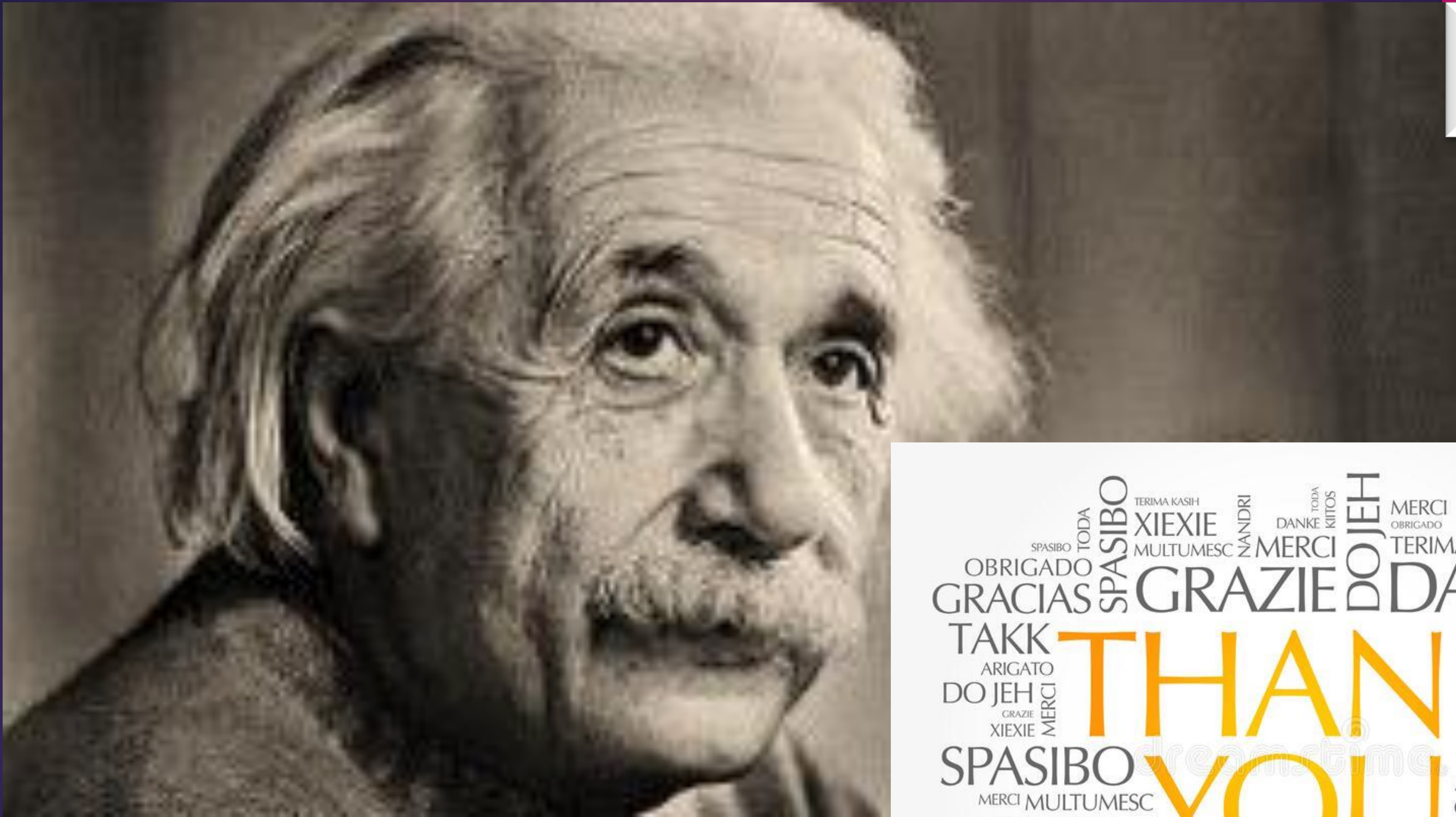
The Gamma functions $\Gamma(\widetilde{x})$ have poles at $\widetilde{x} = -n$ for $n = 0, 1, 2....$

Conclusion

QNM frequencies for $G_{1,2}$.

Frequencies	Stable Modes (QNMs)	Unstable Modes
$k = \frac{i(\lambda^2 L - n^2)}{2Ln}$	if $\lambda^2 > \frac{n^2}{L}$	if $\lambda^2 < \frac{n^2}{L}$
$k = \frac{i(4\lambda^2 L - (2n+1)^2)}{4L(2n+1)}$	if $\lambda^2 > \frac{(2n+1)^2}{4L}$	if $\lambda^2 < \frac{(2n+1)^2}{4L}$

Thus, we have shown that there are two sets of frequencies for $G_{1,2}$ of the fermionic fields in the BIDBH geometry.

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TERIMA KASIH
NANDRI
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DOJEH
KIITOS
TERIMA KASIH
SHUKRAN
KOP KHUN
DANKE
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