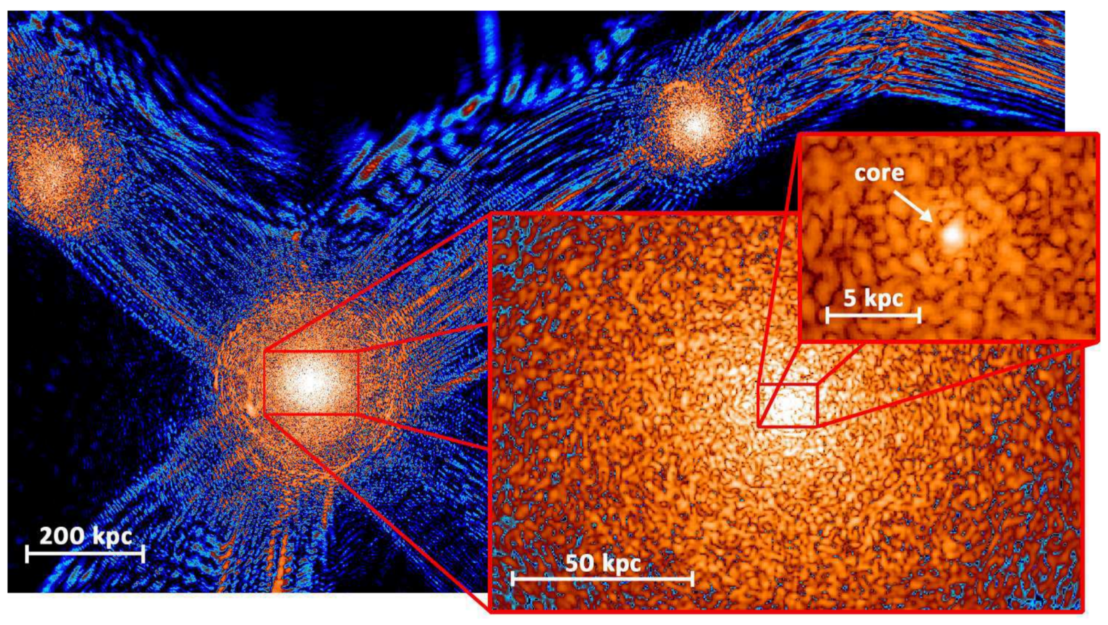
An analysis of oscillations in fuzzy dark matter cores

M. Indjin, I.-K. Liu (Gary), N.P. Proukakis, G. Rigopoulos
School of Mathematics, Statistics & Physics, Newcastle University, Newcastle-upon-Tyne, UK NE1 7RU

When considering a dark matter constituent with mass in the range $m = 10^{-22} - 10^{-20}$ eV, one obtains a large cosmological system which is well defined by a single coherent wavefunction with de Broglie wavelength of order kpc – fuzzy dark matter (FDM). On large scales, FDM replicates the cosmic web structure of cold dark matter (CDM), whilst maintaining significant benefits on galactic scales. Increased attention is motivated by the inherent properties which remedy some issues with CDM - most notably the cusp-core problem; the balance between the quantum pressure and gravitation within a FDM system naturally forms a core at the heart of the halos. Scalar field oscillations are observed in numerical simulations and have been found to send out density waves into the surrounding halo. Attempts have been made to derive an analytical expression for the frequency [1]. We repeat such an analysis with an alternate ansatz, and find agreement, whilst also obtaining a more general expression which allows for the presence of boson self-interactions.



I. Chavanis' Frequency Analysis

By looking at *Chavanis, P. H. (2011)*, we can follow the necessary steps to arrive at an equation for the oscillation frequency of a fuzzy dark matter soliton

1. Take a Gaussian profile ansatz with radius r_c and mass M
2. Calculate the kinetic energy (Θ_Q), gravitational energy (W) and moment of inertia (I) integrals

$$\Theta_Q = \sigma \frac{\hbar^2 M}{m^2 r_c^2} r_c (1) \qquad \sigma = 0.75$$

$$W = -\nu \frac{GM^2}{r_c} (2) \qquad \nu = 0.3989$$

$$I = \alpha M r_c^2 \quad (3) \qquad \alpha = 1.5$$

$$M = \frac{4\pi^2 \gamma \rho_0 r_c^3}{\lambda^{3/2}} (4) \qquad \gamma = 0.0081$$

3. Enforce the virial condition to reshape the Gaussian and define a new r_c which obeys the virial condition
4. Perturb the radius and assign the time dependence to the perturbation – $r_c(t) \rightarrow r_c + \varepsilon(t)$, and keep linear terms
5. Evaluate the oscillatory equation that emerges for ω from the total energy, arriving at

$$\omega^2 = \frac{\nu^4 G^4 M^4 m^6}{8\,\sigma^3 \alpha \hbar^6} (5)$$

II. Numerical Simulation

The underlying equations for a FDM system are the Gross-Pitaevskii equations (GPPE). Naturally, in the absence of self interactions, this reduces to the Schrodinger-Poisson system of equations.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi + g|\psi|^2 \psi$$

$$\nabla^2 V = 4\pi G m |\psi|^2$$

We solve these equations with two separate methods. We use a 3D split-step Fourier method to generate a 3D simulation of the system. This is, however, computationally expensive. We also produce a 1D radial simulation of the code, where we assume spherical symmetry.

The system is propagated in imaginary time to generate the ground state solution to the GPPE, which is indeed the soliton solution. These are the cores found within the centre of FDM dark matter halos. We then apply a small perturbation and propagate the system in real time to assess the oscillation properties and frequency.

III. The Soliton Density Profile

Schive H Y, et al. [2] finds an empirically fit formula for the density profile of a FDM soliton of mass M and radius r_c, defined as the radius at which the density drops to half the peak value:

$$\rho = \left(1 + \lambda \left(\frac{r}{r_c}\right)^2\right)^{-8} (6).$$

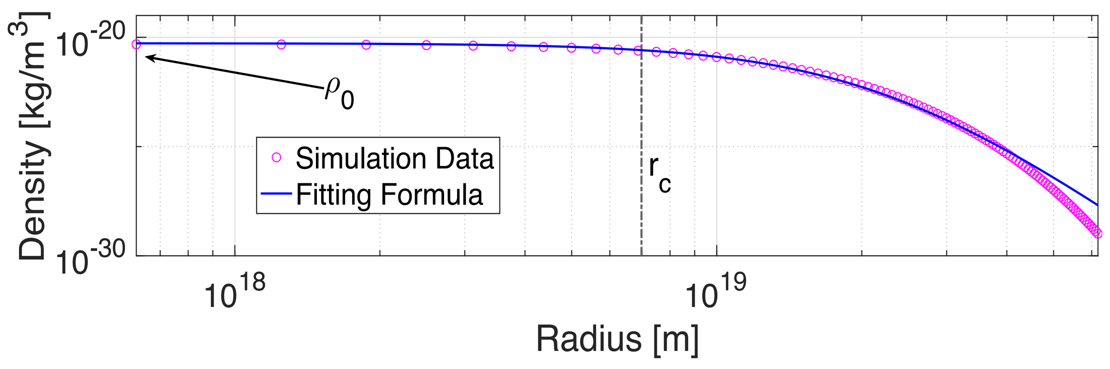


FIG. 3: A soliton density profile from simulation, compared to the fitting formula.

Here, $\lambda = 0.091$. We now calculate the relevant energy integrals without enforcing virialisation, and find solutions in the same form as the Gaussian approach. However, we do find that the constants have changed:

$$\sigma = 0.3943, \qquad \nu = 0.2924, \qquad \alpha = 2.9970$$

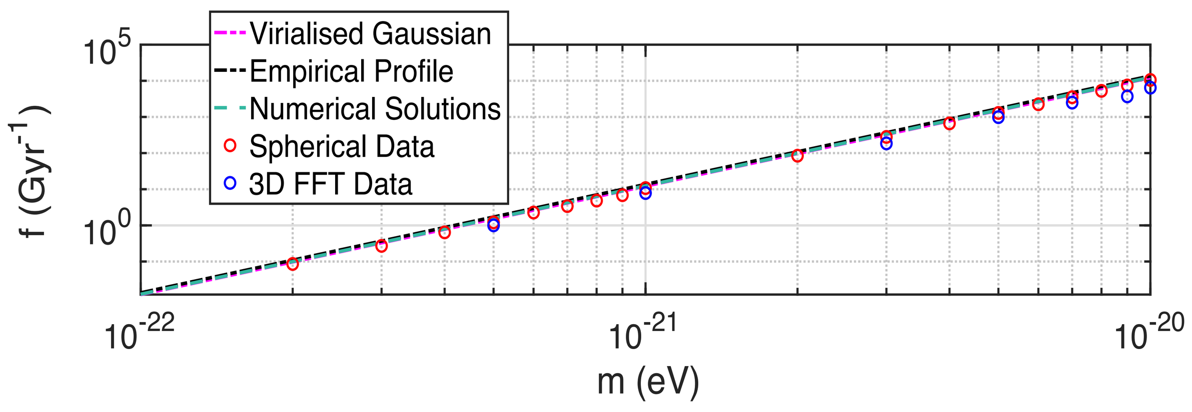


FIG. 1: The soliton core oscillation frequency for varied boson mass. Virialised Gausisan – Eq. (4) using Chavanis' constants. Empirical profile – Eq. (4) using the analytically calculated constants using Eq. (5) as the ansatz. Numerical solutions – Eq. (4) using the constant values for a numerically generated soliton.

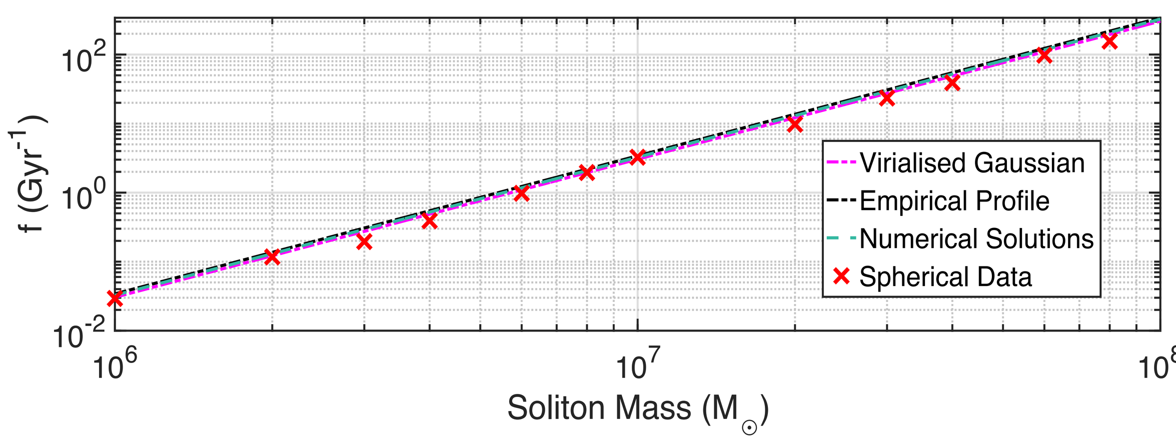


FIG. 2: As adjacent, but for varied soliton core mass, and constant boson mass of 10^{-21} eV.

IV. New Findings & Conclusion

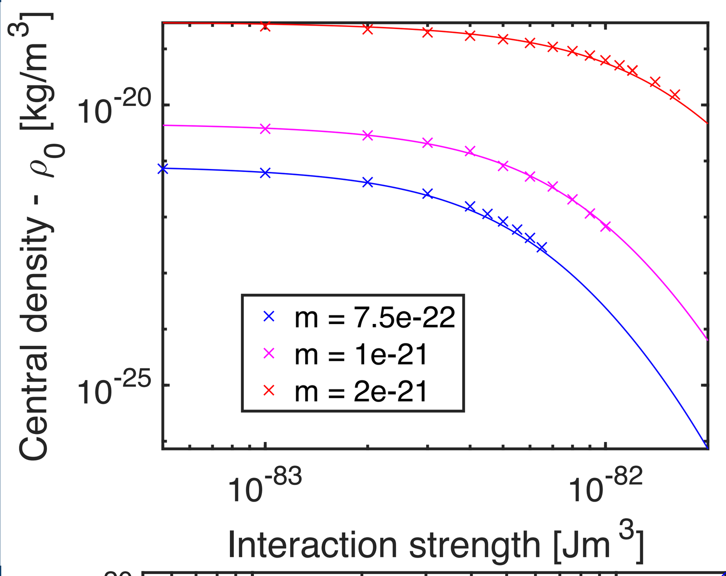


FIG. 4: The soliton core central density obtained from numerical simulation, with respect to the repulsive self-interaction strength, plotted alongside the fitting formula.

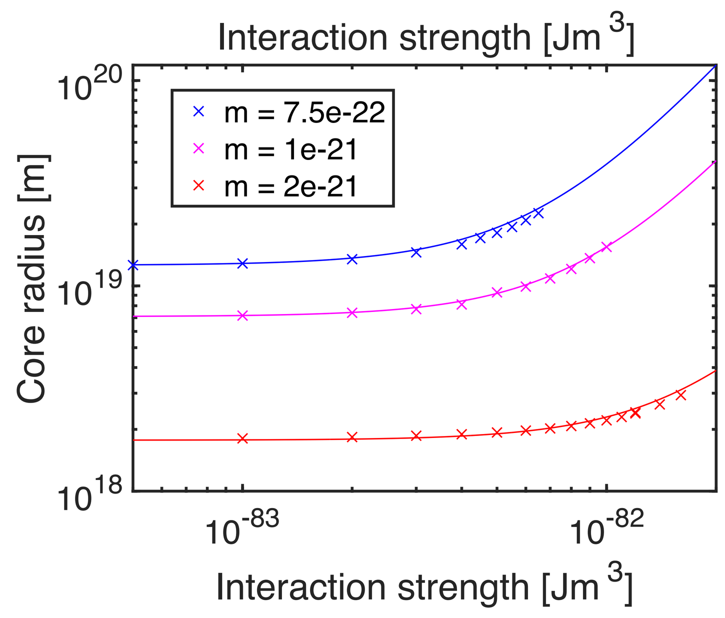


FIG. 5: The soliton core radius obtained from numerical simulation, with respect to the repulsive self-interaction strength, plotted with the fitting formula.

Equation (5) can be re-written in terms of the peak density, obtaining

$$\omega = \left(\frac{511}{291}\pi G\right)^{1/2} \rho_0^{1/2}. (7)$$

Through numerical fitting of core radius and central density in the presence of self interactions (FIG. 4, 5) we semi-analytically derive the following relationship for the oscillation frequency in the self interacting limit:

$$\omega = \left(\frac{\nu G}{\alpha}\right)^{1/2} \left(\frac{4\pi^2 \gamma}{\lambda^{3/2}}\right)^{5/4} \left(\frac{2\sigma\hbar^2}{G\nu m^2}\right)^{9/4} \frac{\rho_0^{5/4}}{M^3}. (8)$$

This expression reduces to Eq. (7) in the limit of no self-interactions ($g = 0$). Eq. (8) thus accurately depicts the oscillation frequency of soliton cores in the presence of self interactions, which can be seen in the adjacent figures.

To conclude, we have found a new, generalized formula for the oscillation frequency of FDM soliton cores, which accounts for the boson mass, soliton mass and now also the repulsive self-interaction strength. This will allow us to open up the parameter space for allowed boson masses based on oscillation frequency derived constraints [3].

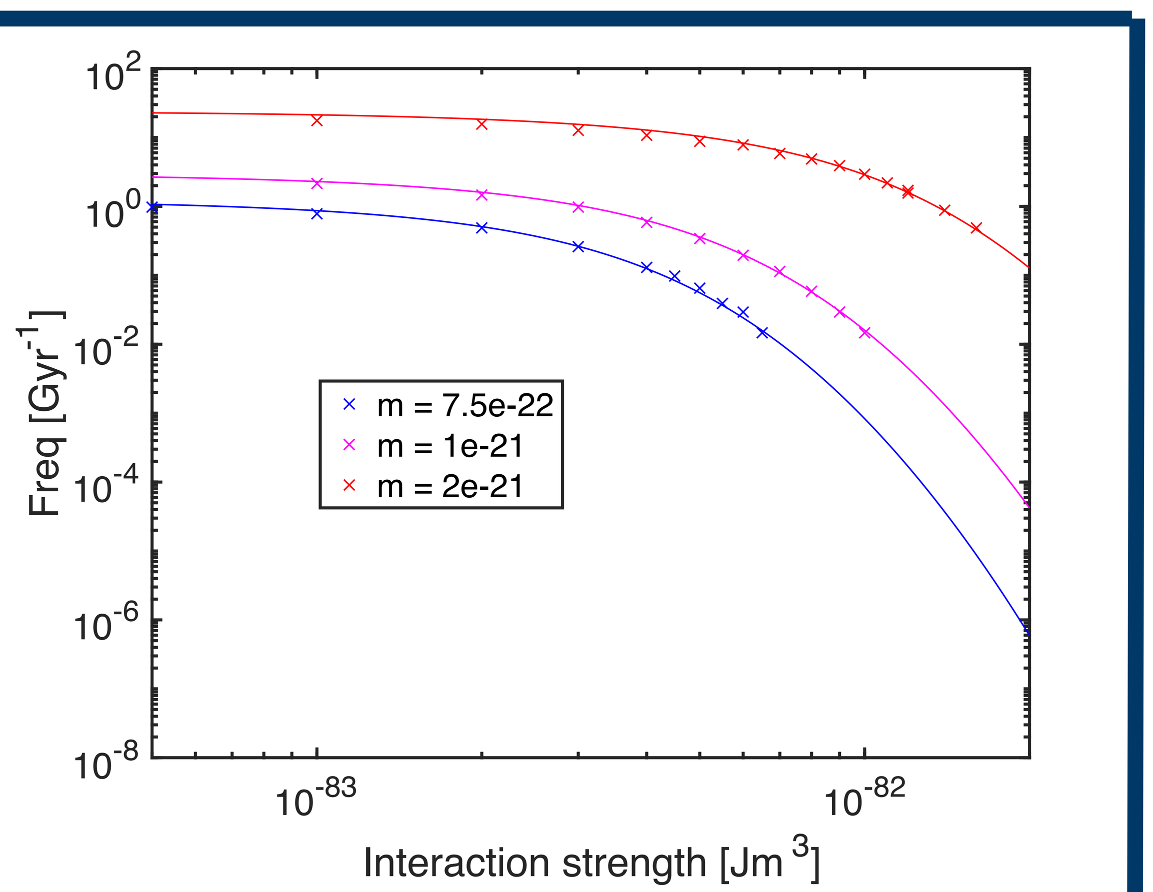


FIG. 6: The soliton core oscillation frequency with respect to the repulsive self-interaction strength, plotted alongside the semi-analytical predictive formula in Eq. (8).

[1] Chavanis P H 2011 Physical Review D - Particles, Fields, Gravitation and Cosmology **84** 1–27
[2] Schive H Y, Chiueh T and Broadhurst T 2014 Nature Physics **10** 496–9
[3] Marsh, D.J.E., & Niemeyer, J.C., Phys. Rev. Lett. **123**, 051103