ON VARIOUS ASPECTS OF DBI LAGRANGIAN DYNAMICS AND ITS MECHANICAL AND COSMOLOGICAL REALIZATION

Goran S. Djordjević[1], Danilo Delibašić[2], Milan Popović[3]

[1] *Faculty of Science, University of Nis, Serbia*
[2] *Faculty of Sciences and Mathematics, University of Nis, Serbia*
[3] *Faculty of Natural Sciences and Mathematics, University of Banja Luka, Republic of Srpska, BiH*

Abstract. The DBI Langrangian (or a family of DBI Lagrangians) has been known and widely considered in many different fields of physics.

For instance, classical and quantum dynamics of tachyon systems have been examined, describing spatially homogeneous scalar fields, in the limit of classical and quantum mechanics.

Understanding and modelling of these systems are of particular importance in the development of field and string theory. We review its application in modern cosmology and in the theory of inflation. The focus is on the study of the dynamics of a scalar tachyonic field with a non-standard Lagrangian of a DBI type, or a Lagrangian used in the so-called effective field theories, with various potentials.

It is suitable to use lower-dimensional models, including a zero-dimensional classical-mechanical analogue.

The original calculations for several specific and important (tachyon) potentials are presented.

Those potentials are also exactly solvable in the framework of Friedmann cosmology, and they have physical motivation in inflationary cosmology.

In addition, the so-called locally-equivalent Lagrangians of the standard type are considered. The classical and quantum formalism for a harmonic oscillator with time-dependent frequency is given as an example.

Introduction:

The DBI-type Lagrangian is non-standard type Lagrangian. It contains potential as a multiplicative factor and a term with derivatives ("kinetic" term) inside the square root:

$$\mathcal{L}_{tach} = \mathcal{L}(T, \partial_\mu T) = -V(T)\sqrt{1+(\partial T)^2}$$

where T is tachyonic scalar field, $V(T)$ - tachyonic potential, $(\partial T)^2 = g_{\mu\nu}\partial^\mu T \partial^\nu T$ and $g_{\mu\nu}$ - components of the metric tensor. The tachyon potential $V(T)$ has a positive maximum at $T=0$ and a minimum at T_0 with $V(T_0)=0$. General expression of the energy-momentum tensor:

$$T_{\mu\nu} = \mathcal{L}_{tach} g_{\mu\nu} + V(T)\frac{\partial_\mu T \partial_\nu T}{\sqrt{1+(\partial T)^2}}$$

written in the form of the energy-momentum tensor for the ideal fluid defines pressure and energy density of a fluid described by the tachyonic scalar fields

$$\rho = \frac{V(T)}{\sqrt{1+(\partial T)^2}}$$

$$p = \mathcal{L}_{tach}$$

The parameter w is then

$$w = \frac{p}{\rho} = -1 - (\partial T)^2$$

One can define the so called effective sound speed c_s

$$c_s^2 = \frac{\partial \mathcal{L}_{tach}}{\partial(\partial T)^2}\left(\frac{\partial \rho}{\partial(\partial T)^2}\right)^{-1}$$

which takes into account "friction" effects and it is a generic feature of theories with nonstandard Lagrangians of this type.

Equation of motion in curved spacetime is

$$D_\mu \partial^\mu T - \frac{D_\mu \partial^\nu T}{1-(\partial T)^2}\partial_\mu T \partial_\nu T - \frac{1}{V(T)}\frac{dV}{dT} = 0.$$

where D_μ is the covariant derivative with respect to $g_{\mu\nu}$. For the spatially homogenous tachyon field in flat spacetime background the last equation is reduced to

$$\ddot{T}(t) - \frac{1}{V(T)}\frac{dV}{dT}\dot{T}^2(t) = -\frac{1}{V(T)}\frac{dV}{dT}$$

Results:

By applying some appropriate transformations, the tachyonic equation of motion can be rewritten in the following form:

$$\ddot{T} + 3H(t)\left(1 + \frac{2}{3}\frac{\dot{H}(t)}{H^2(t)}\right)\dot{T} + \frac{V'(T)}{V(T)}(1-\dot{T}^2) = 0$$

Next, after the use of the method of Darboux [5], we obtain the following localy-equivalent Lagrangian:

$$L = a(t)\dot{a}^2(t)\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2V^2(T(\phi))}\right]$$

where ø is the "new" field.

Recasting the problem in the framework of classical mechanics, while also taking the concrete case of an exponential tachyonic potential:

$$V(T) = V_0 e^{-\omega T}, \quad V_0 = const, \ \omega = const$$

with $\phi \equiv x$ and $m(t) = a(t)\dot{a}^2(t)$, the tachyonic Lagrangian takes the following form:

$$L = \frac{1}{2}m(t)\dot{x}^2 + \frac{1}{2}m(t)\omega^2 x^2$$

which corresponds to an inverted harmonic oscillator with a time-dependent mass. For an even more concrete case, it we take the scale factor to be exponentially expanding (which approximately holds for inflation), we find that:

$$m(t) = me^{rt}$$

where r = *const*. The inverted harmonic oscillator with constant frequency and exponentially increasing mass is well-known in the literature [6] as the inverted Caldirola-Kanai oscillator. We can now finally obtain the Feynman propagator:

$$K(x'', t''; x', t') = \left(\frac{m\Omega}{2\pi i\hbar \sinh \Omega t}\right)^{\frac{1}{2}} \times e^{\frac{1}{4}rt} \times \exp\left(\frac{im}{4\hbar}(x'^2 - e^{rt}x''^2)\right) \times \exp\left(\frac{im\Omega}{2\hbar \sinh\Omega t}\left[(e^{rt}x''^2 + x'^2)\cosh\Omega t - 2x'x''e^{\frac{1}{2}rt}\right]\right)$$

Acknowledgements:

This work was supported by ICTP-SEENET-MTP project NT-03, CEEPUS project "Gravitation and Cosmology" CIII-RS-1514-02-2122 and by Serbian Ministry for Education, Science and Technologilal Development under Contract No. 451-03-68/2022-14/200124.

References

1. Dragoljub D. Dimitrijević, *Dinamika tahionskih polja u klasičnoj i kvantnoj kosmologiji*, Prirodno-matematički fakultet Niš, doktorska disertacija, (2015).
2. Marko D. Stojanović, *Izračunavanje parametara inflacije u modelima sa tahionskim poljem*, Prirodno-matematički fakultet Niš, master rad, (2015).
3. Neven Bilić, Dragoljub D. Dimitrijević, Goran S. Djordjevic, Milan Milošević and Marko Stojanović, *Tachyon inflation in the holographic braneworld*, Journal of Cosmology and Astroparticle Physics, Volume 2019, August 2019.
4. Dimitrijevic, D.D.; Djordjevic, G.S.; Milosevic, M.; *Classicalization and quantization of tachyon-like matter on (non)archimedean spaces*, Romanian Reports in Physics, 2016, Journal article.
5. G. Darboux, *Leçons sur la Théorie Générale des Surfaces*, Gauthier- Villars, Paris (1891)
6. S. Baskoutas, A. Jannussis, *Quantum Mechanics of the Inverted Caldirola-Kanai Oscillator*, Il Nuovo Cim. 107(3), 255-267 (1992)