ON SOME ANALYTICAL SOLUTIONS OF INFLATIONARY MODELS WITH DBI TACHYON FIELD

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Introduction

- The inflation theory proposes a period of extremely rapid (exponential) expansion of the universe during the an early stage of evolution of the universe
- The inflation theory predict that during inflation (it takes about 10^{-34} s) radius of the universe increased, at least e^{60} times
- Although inflationary cosmology has successfully complemented the Standard Model, the process of inflation, in particular its origin, is still largely unknown
- Over the past 40 years numerious models of inflationary expansion of the universe have been proposed
- The simplest model of inflation is based on the existence of a scalar field, which is called inflation, which drive inflation
- The most important way to test inflationary cosmological model to compare the computed and measured values of *the observation parameters*
- Is it possible to find analytical solution of dynamical equation without slow-roll approximation

Introduction

- Spatially flat FLRW universe:

$$ds^2 = g^{\mu\nu}dx_\mu dx_\nu = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \quad (1)$$

  $a(t)$ - scale factor, $k = -1, 0, +1$ - the spatial curvature parameter
- Hubble expansion rate

$$H(t) = \frac{\dot{a}}{a}, \quad \dot{a} = \frac{da}{dt} \quad (2)$$

  Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}, \quad M_{\rm Pl} = (8\pi G)^{-1/2} \quad (3)$$

- The energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + p g_{\mu\nu} \quad (4)$$

  u_μ - four-velocity of the fluid
- Friedman equations:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\rho \quad (5)$$

$$\dot{H} = -\frac{1}{2M_{\rm Pl}^2}(\rho + p) \quad (6)$$

  p - pressure
  ρ - energy density
- The energy conservation equation $\dot{\rho} + 3H(p+\rho) = 0$

Dynamics of tachyon field in cosmology

- Traditionally, the word tachyon was used to describe a hypothetical particle which propagates faster than light
- In modern physics this meaning is changed: The effective tachyonic field theory was proposed by A. Sen
- String theory - states of quantum fields with imaginary mass
- Ii was realised that the imaginary mass creates instability and tachyons spontaneously decay - *tachyon condensation*
- Tachyon matter in the holographic braneworld is described by the DBI (Dirac-Born-Infeld) Lagrangian:

$$\mathcal{L} = -V(\theta)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}, \quad \theta_{,\mu} = \frac{\partial\theta}{\partial x^{\mu}} \tag{7}$$

- Potential $V(\theta)$:

$$V(0) < \infty, \quad dV/d\theta(\theta > 0) < 0, \quad V(|\theta| \to \infty) \to 0 \tag{8}$$

Examples:

- the exponential tachyon potential

$$V(\theta) = V_0 e^{-\omega\theta}, \quad V_0 = \text{const}, \ \omega = \text{const} \tag{9}$$

- the inverse cosine hyperbolic tachyon potential

$$V(\theta) = \frac{V_0}{\cosh(\omega\theta)}, \quad V_0 = \text{const}, \ \omega = \text{const} \tag{10}$$

Dynamics of tachyon field in cosmology

- The energy-momentum tensor for tachyon matter

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} \tag{11}$$

- The pressure and energy density ($\theta \equiv \theta(t)$):

$$p \equiv \mathcal{L} = -V\sqrt{1-\dot{\theta}^2} \tag{12}$$

$$\rho \equiv \mathcal{H} = \frac{V}{\sqrt{1-\dot{\theta}^2}} \tag{13}$$

- The energy-momentum conservation gives a second order equation for θ

$$\frac{\ddot{\theta}}{1-\dot{\theta}^2} + 3H\dot{\theta} + (\ln V)' = 0, \quad (\ln V)' = \frac{V_{,\theta}}{V} \tag{14}$$

- The dynamics of tachyon inflation is the almost same as in SSFI

Dynamics of tachyon field in cosmology

- Inflation:

$$\frac{d}{dt}(aH)^{-1} < 0 \quad \Leftrightarrow \quad \ddot{a} > 0 \quad \Leftrightarrow \quad p < -\frac{1}{3}\rho \tag{15}$$

$$\frac{\ddot{a}}{a} = \frac{1}{3M_{\rm Pl}^2} \frac{V}{\sqrt{1-\dot{\theta}^2}} \left(1 - \frac{3}{2}\dot{\theta}^2\right) \tag{16}$$

The condition for accelerated expansion

$$\dot{\theta}^2 < \frac{2}{3} \tag{17}$$

- e-fold number ($N > 60$)

$$N = \int_{a_{\rm i}}^{a_{\rm f}} H dt \tag{18}$$

- Potential problems in numerical calculation (initial values, accuracy, ...)

$$dN = H dt \tag{19}$$

Dynamics of tachyon field in cosmology

- The slow-roll conditions:

$$\dot{\theta}^2 \ll 1 \quad \text{and} \quad |\ddot{\theta}| \ll 3H\dot{\theta} \tag{20}$$

- The equations in *slow-roll regime*

$$H^2 \sim \frac{V}{3M_{\rm Pl}^2} \quad \dot{\theta} \sim -\frac{(\ln V)'}{3H} \tag{21}$$

- For some type of potential analytical solution for dynamical equation can be find (in slow-roll regime)
- The slow-roll inflation parameters

$$\dot{\epsilon}_j = H\epsilon_j\epsilon_{j+1} \tag{22}$$

$$\epsilon_0 \equiv \frac{H_*}{H}, \quad \epsilon_{j+1} \equiv \frac{d\ln|\epsilon_j|}{dN}, \quad j \geq 0 \tag{23}$$

  H_* is the Hubble rate at some chosen time
- During inflation both parameters are less then 1 and inflation ends when ϵ_1 exceeds unity

$$\epsilon_{1_{\rm f}}(\theta_{\rm f}) = 1 \tag{24}$$

Dynamics of tachyon field in cosmology

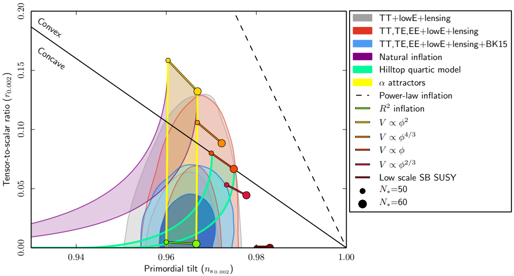
The inflation parameters:

n_s - the scalar spectral index

r - the tensor-to-scalar ratio

$$n_s = 1 - 2\epsilon_1 - \epsilon_2 - (2\epsilon_1^2 + (2C + 3 - \frac{1}{3})\epsilon_1\epsilon_2 + C\epsilon_2\epsilon_3), \tag{25}$$

$$r = 16\epsilon_1(1 + C\epsilon_2 - \frac{1}{3}\epsilon_1). \tag{26}$$



Constraints from Planck 2018:

$$n_s = 0.9668 \pm 0.0037 \tag{27}$$

$$r < 0.058 \tag{28}$$

Analytical solutions

- Friedman equations

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\frac{V}{\sqrt{1-\dot{\theta}^2}} \tag{29}$$

$$\dot{H} = -\frac{1}{2M_{\rm Pl}^2}\frac{V\dot{\theta}^2}{\sqrt{1-\dot{\theta}^2}} \tag{30}$$

- Using the expression $H = H(\theta)$, the system of Friedman equations can be written in the form of of the Hamilton-Jacobi type equation

$$\dot{H} = -\frac{3}{2}H^2\dot{\theta}^2, \quad \dot{H} < 0 \tag{31}$$

$$\dot{H} = H_{,\theta}\dot{\theta} \tag{32}$$

$$\boxed{-\frac{2}{3}dt = \frac{H^2}{H_{,\theta}}d\theta} \tag{33}$$

Assumption: $H \sim 1/f(\theta)$

Expected form of solution: $H \sim \frac{1}{\theta^n}, \quad H \sim \frac{1}{\sin^n\theta}, \quad H \sim \frac{1}{\cos^n\theta}, \ldots$

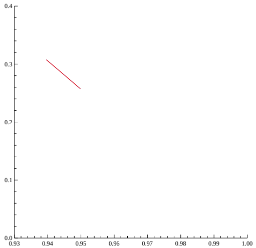
Case 1: $H = A\frac{1}{\theta^2}$

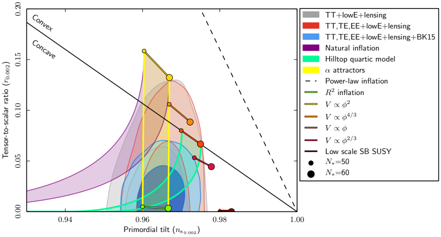
$$\theta = e^{\frac{4t}{3A}}, \quad H(t) = Ae^{-\frac{8t}{3A}} \tag{34}$$

$$\epsilon_1 = \frac{8}{3A^2}\theta^2, \quad \epsilon_2 = \epsilon_1 \tag{35}$$

$$\epsilon_2 = -\frac{1}{\epsilon_1}\frac{d\epsilon_1}{dN}, \quad \epsilon_1(N=0) = 1 \tag{36}$$

$$\boxed{\epsilon_1 = \frac{1}{1+N}} \tag{37}$$





$\Rightarrow$ *The parameters is inconsistent with the observations*

Case 2: $H = A\frac{1}{\sin^2\theta}$

$$\theta = \cot^{-1}(e^{-\frac{4t}{3A}}), \quad H(t) = Ae^{-\frac{8t}{3A}}\left(1 + e^{\frac{8t}{3A}}\right) \tag{38}$$

$$\epsilon_1 = \frac{2}{3A^2}\sin^2(2\theta) \tag{39}$$

$$\epsilon_2 = \frac{8}{3A^2}\sin^2(\theta)\cos(2\theta) \tag{40}$$

The approach from the previous example can not be applied!

$$\theta_{\rm f} = \frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}A\right) \tag{41}$$

$$N = \int H dt = \int \frac{H}{\dot{\theta}} d\theta \tag{42}$$

$$N = \frac{3A^2}{4}\left(-\frac{1}{2\sin^2(\theta)} + \log\frac{\sin(\theta)}{\cos(\theta)}\right)\Big|_{\theta_{\rm cmb}}^{\theta_{\rm f}} \tag{43}$$

Case 2: $H = A\frac{1}{\sin^2\theta}$

- For given A the value of $\theta_{\rm f}$ is calculated

$$\theta_{\rm f} = \frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}A\right) \tag{44}$$

- The field value at the end of inflation - $\theta_{\rm cmb}$

$$N = \frac{3A^2}{4}\left(-\frac{1}{2\sin^2(\theta)} + \log\frac{\sin(\theta)}{\cos(\theta)}\right)\Big|_{\theta_{\rm cmb}}^{\theta_{\rm f}} \tag{45}$$

$$\epsilon_1 = \epsilon_1(\theta_{\rm cmb}), \quad \epsilon_2 = \epsilon_2(\theta_{\rm cmb}) \tag{46}$$

- Parameters $n_{\rm s}$ and r

$$n_{\rm s} = n_{\rm s}\left(\epsilon_1(\theta_{\rm cmb}), \epsilon_2(\epsilon_1(\theta_{\rm cmb})\right) \tag{47}$$

$$r = r(\epsilon_1\left(\theta_{\rm cmb}), \epsilon_2(\epsilon_1(\theta_{\rm cmb})\right) \tag{48}$$

$\Rightarrow$ *The agreement with observations is better for larger values of the numbers of e-folds* N

Final remarks

- We have investigated a model of tachyon inflation based on a dynamic of tachyon field
- Concrete functions of $H(\theta)$ for which there are analytical solutions were analyzed

$$H = A\frac{1}{\theta^2} \quad H = A\frac{1}{\sin^2\theta}$$

- It is an interesting task to examine the another possibility for function $H(\theta)$ and calculate observational parameter for that cases

$$H = A\frac{1}{\sinh^2\theta}, \cdots$$

Task for future research:

- Reconstruction of the corresponding inflationary potentials
- Prove that inflation solution is an attractor, as expected
- ...

Some references

- D.D. Dimitrijevic, N. Bilić, G.S. Djordjevic, M. Milosevic, M. Stojanovic, *Tachyon scalar field in a braneworld cosmology*, Int. J. Mod. Phys. A (2018), Vol 33, No. 34, 1845017.
- D.A.Steer, F.Vernizzi, *Tachyon inflation: tests and comparison with single scalar field inflation*, Phys.Rev.D70 (2004),043527.
- N. Bilić, D.D. Dimitrijević, G.S. Djordjevic, M. Milošević, M. Stojanović, *Tachyon inflation in the holographic braneworld*, J. Cosmol. Astropart. Phys. 2019 (2019) 034–034.
- M. Milosevic, M. Stojanović, D.D. Dimitrijevic, G.S. Djordjevic, *On an inflation in holographic cosmology with inverse cosh potential*, Annals of the University of Craiova, Physics 30 (part II) (2020).
- D.D. Dimitrijević, G. Djordjević, M. Milošević and M. Stojanović, *Attractor Behaviour of Holographic Inflation Model for Inverse Cosine Hyperbolic Potential*, Facta Universitatis, Series: Physics, Chemistry and Technology, Vol. 18, No 1, 65-73 (2020), ISSN 0354-4656
- M. Stojanović, M. Milošević, G.S. Djordjević and D.D. Dimitrijević, *Holographic Inflation with Tachyon Field as an Attractor Solution*, SFIN year XXXIII Series A: Conferences, No. A1, (2020) 311-318.

THANK YOU FOR YOUR ATTENTION!

HVALA VAM NA PAŽNJI!