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Preliminary results on the multifractal nature of the main geomagnetic field

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Introduction

The geomagnetic field of the Earth has been increasingly studied in the recent decades [1]. It is well-known that this system exhibits a very complicated dynamics [2]. The geomagnetic field that we observe on the Earth's surface is the superposition of several magnetic fields of different origins. By far the overwhelming contribution is from the dipolar magnetic field that originates in the Outer Core, OC). This ingredient of the geomagnetic field exhibits very complex dynamics and offers many possibilities to perform various statistical studies [2, 3].

The dipolar field and its magnitude in the recent geological epochs can be determined by constructing the virtual axial dipolar moment (VADM), which is virtually the palaeo-dipolar field [4]. Generally, the samples can be recovered from ocean's floor or archeological artifacts. The time series obtained is then calibrated using data recovered from the data are obtained from the samples taken in volcanic formations or lake sediments. In the actual paper we focus on three VADM time series that span the recent million years of Earth's geological history. The first series is constructed from ocean's floor samples spanning the last 4 Myr (read million years) [4].

Many geophysical series, he same be9ing observed in many complex systems, are known to posses some degree of persistence or memory. Many of them show multifractal behavior as well. In simpler words, the parts of the series exhibit different properties compared to the series itself [5]. Such behavior is observed thoroughly regarding the external geomagnetic field [6, 7]. The question addressed here is: **Does the internally generated magnetic field exhibit such behavior as well?**

Method

The generalized multifractal Detrended Fluctuation Analysis (MF – DFA) procedure consists of five steps [5]:

- Step 1: Determine the "profile" :

$$Y(i) \equiv \sum_{k=1}^{i} [x_k - \langle x \rangle], \quad i = 1, \ldots, N .$$

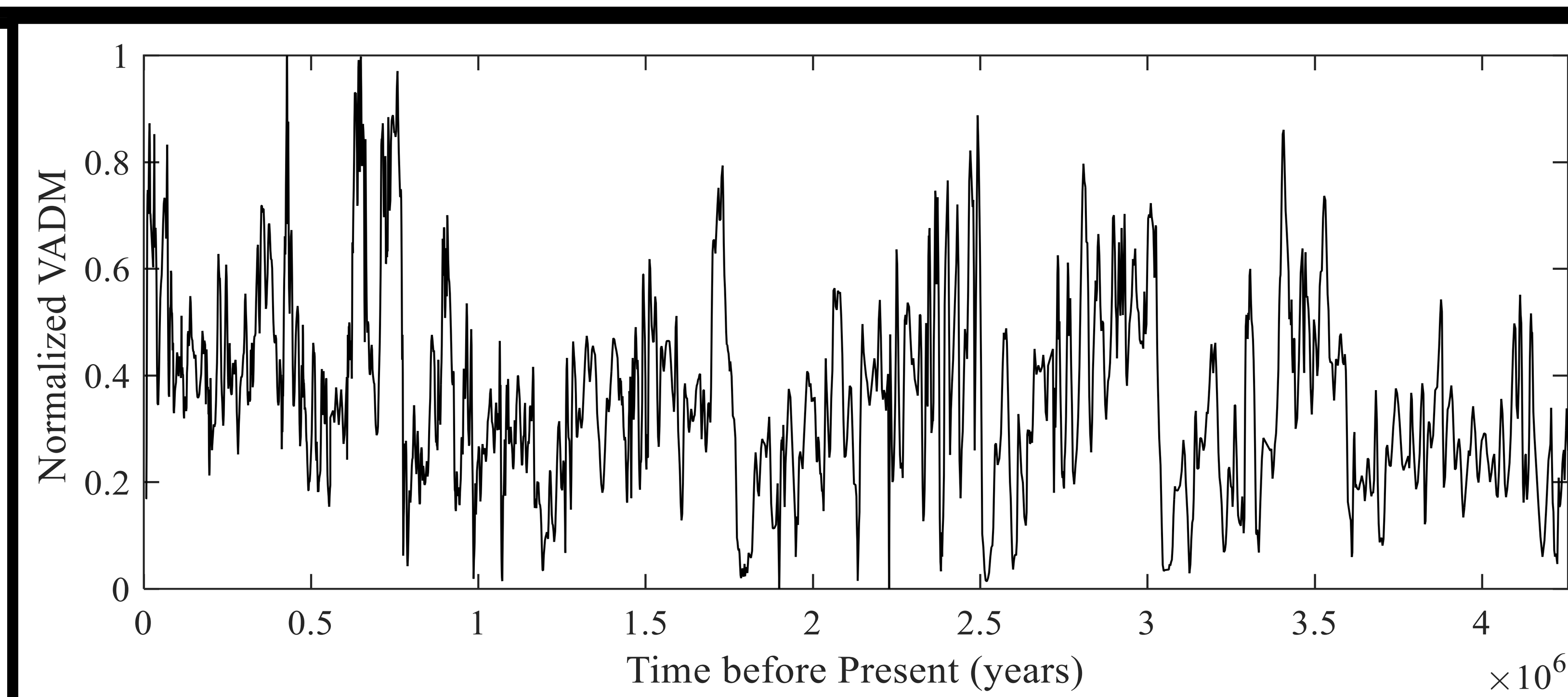
  Here N is the length of the original series. This step is not compulsory immediately because the average can be **eliminated in step 3**.

- Step 2: Divide the profile $Y(i)$ into N_s non – overlapping segments of equal length s. Since N is often not a multiple of the considered time scale s, the procedure twice from both ends. Thus, the whole series is considered and there are created $2N_s$ segments are obtained altogether.
- Step 3: Calculate the local trend for each of the $2N_s$ segments by a simple least – squares method. The polynomials involved in such fit can have Then determine the variances:
  - Forward direction
    $$F^2(\nu, s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(\nu-1)s+i] - y_\nu(i)\}^2$$
  - Backward direction
    $$F^2(\nu, s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[N-(\nu-N_s)s+i] - y_\nu(i)\}^2$$
- Step 4: Average over all segments to obtain the qth order fluctuation function:

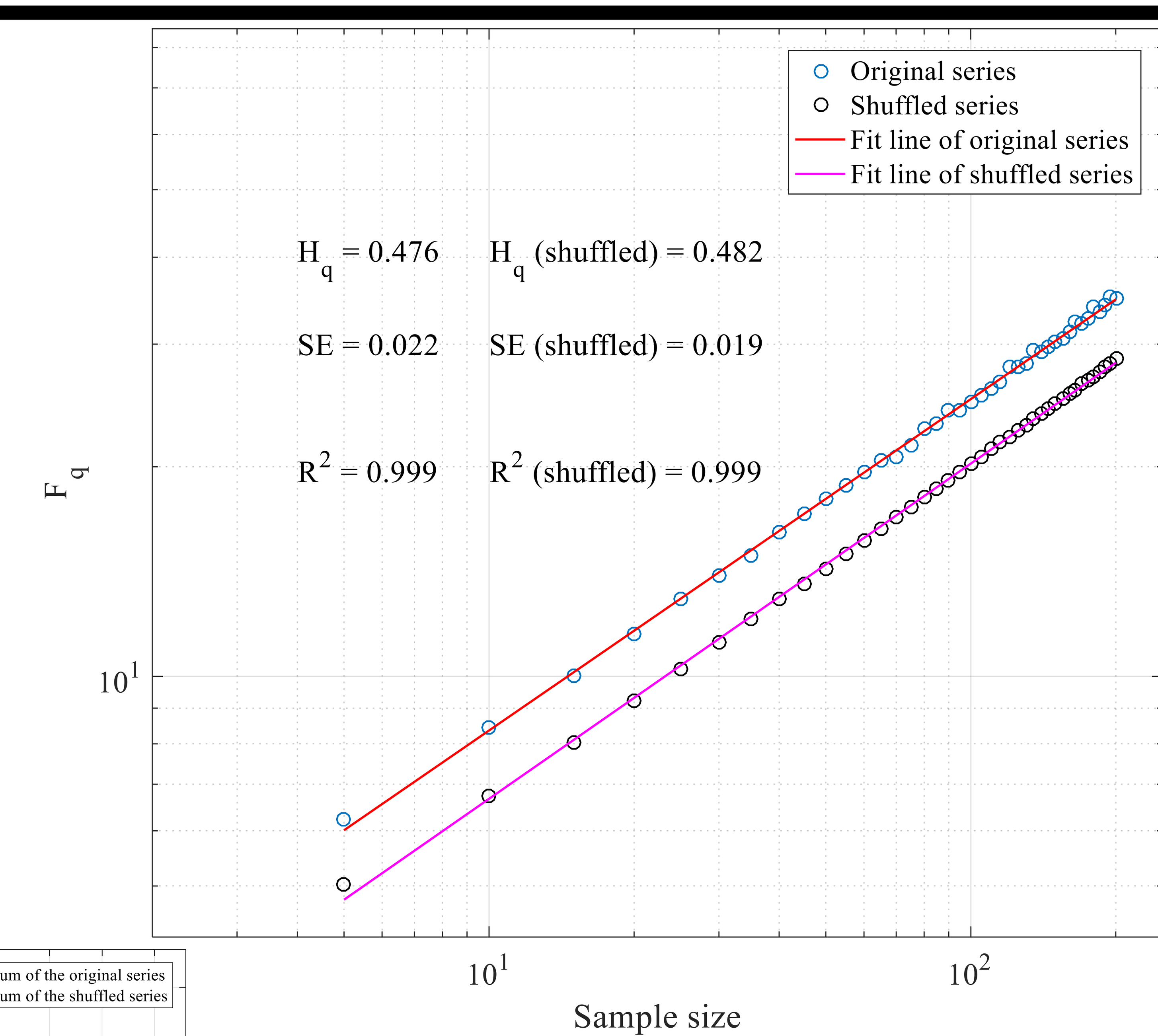
$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(\nu, s)]^{q/2} \right\}^{1/q}$$

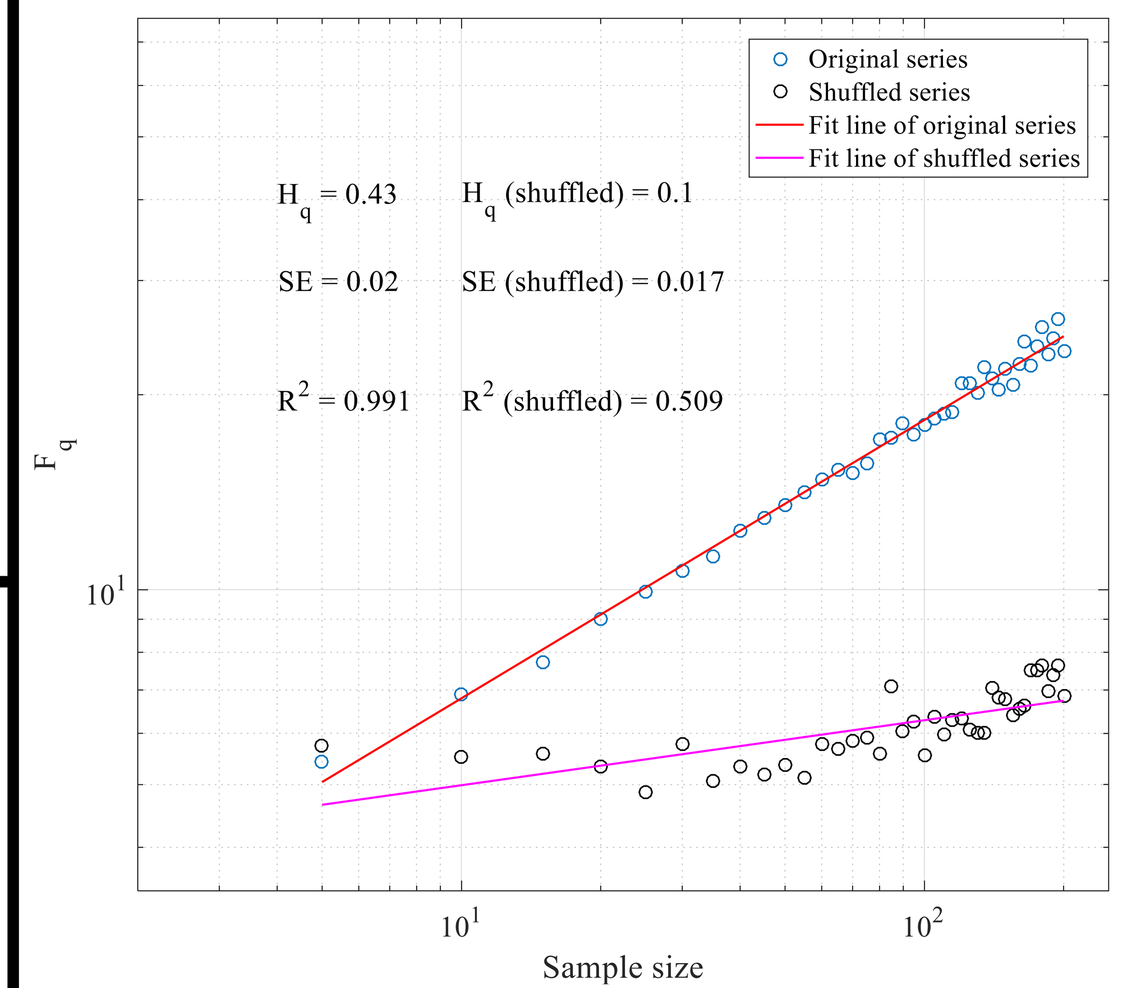
- Step 5: The number q can take any real value, but the natural values provide a measure of persistence and visualize the multifractal structure of the series. In order to Determine the scaling behavior of the fluctuation functions, there are analyzed the log–log plots of $Fq(s)$ versus s for each value of α. If the series is long – range, for an increasing timescale s, $Fq(s)$ increases as a power – law. It proven that it should follow: $F_q(s) \sim s^{h(q)}$. Here H is known as the Hurst exponent.

It is possible to extend the method by analyzing the temporal evolution of the Hurst exponent for a given series. It provides crucial information in order to gather valuable insights on the fractal structure of the series and its temporal dynamics.

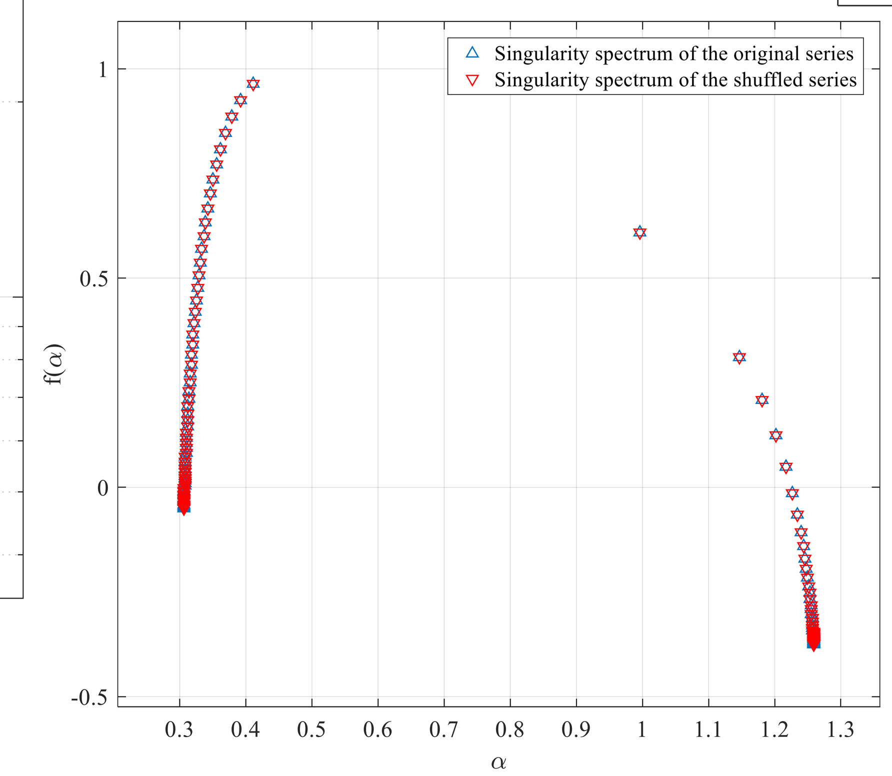


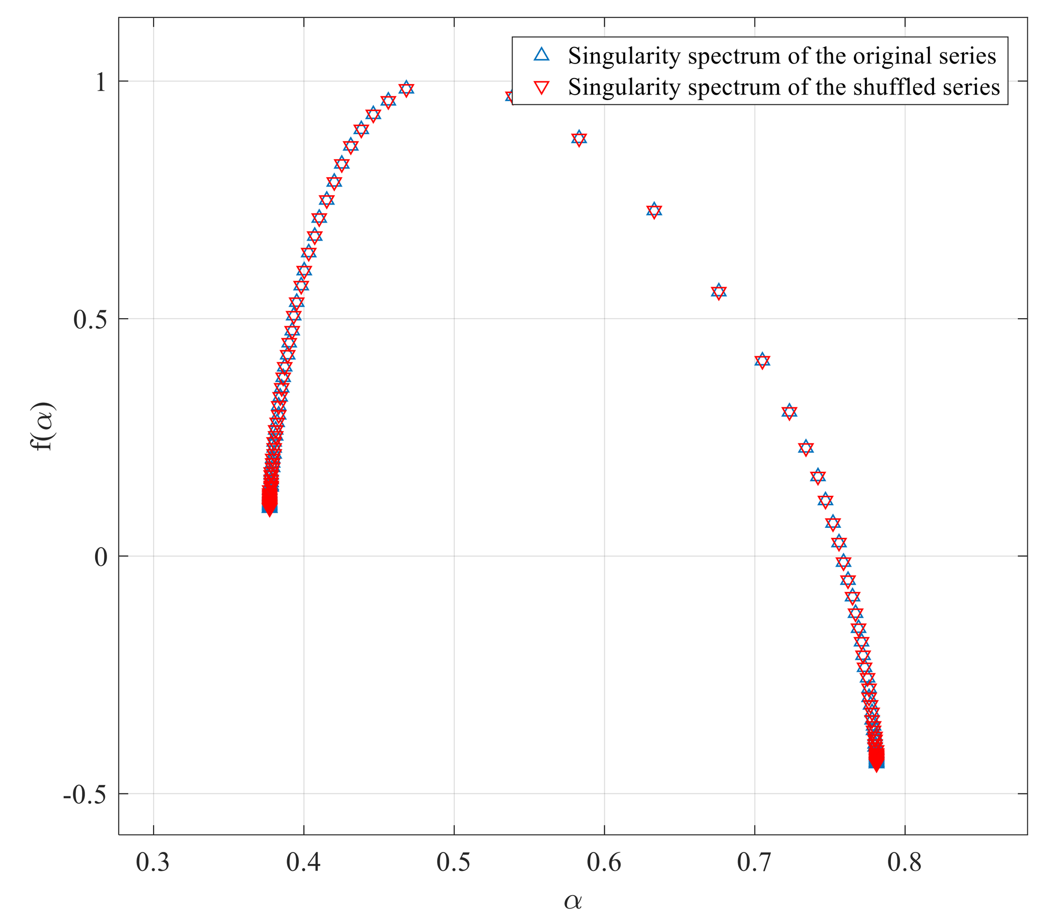
The series contains 2160 data, covering a time span of more than 4 Million years. They provide information about the axial component of the geomagnetic dipolar moment (DM). Due to the impossibility to faithfully reconstructing the DM and its temporal evolution without having knowledge about the non-dipolar field, we obtain a virtual axial dipolar moment (VADM). Thus, we analyze only the dipolar ingredient of the geomagnetic field. The series magnitude of the VADM has been normalized. The times when the magnitude drops to nearly zero constitute either a reversal or an excursion.





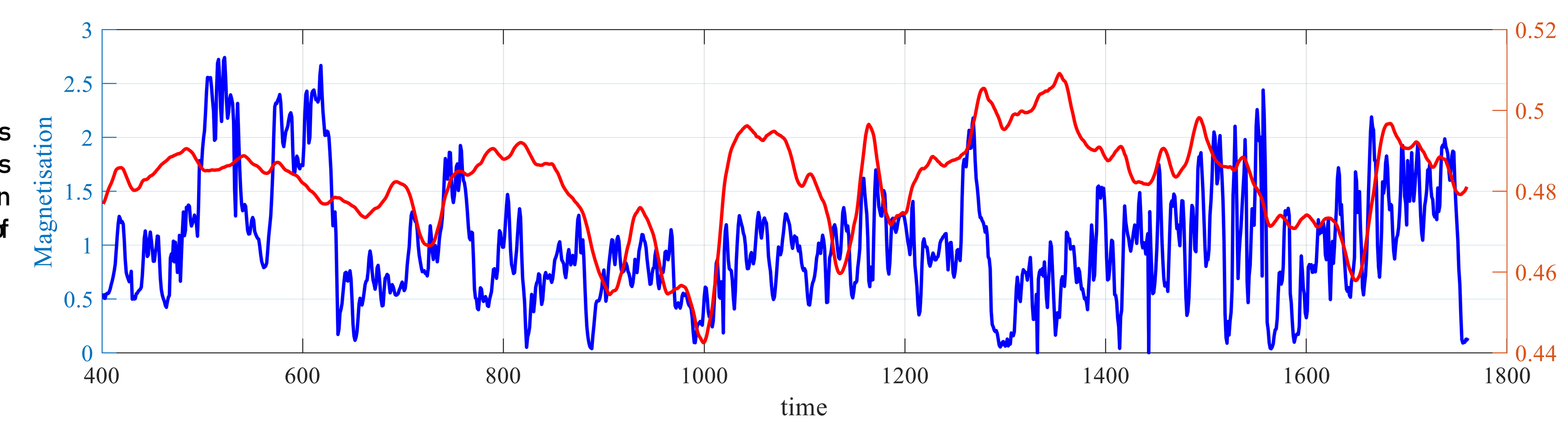
Same procedure, but with the series that considers polarity changes.





The shuffling of the series allows the disruption of all possible long scale correlations. If the series is intrinsically multifractal, it should be reflected in the singularity spectrum

The temporal dynamics of H is quite complex and deserves future attention. Here is shown the case with window width of 400 units.



Conclusions

➢ The Hurst exponent H (for $q = 2$) is very close to 0.5 for the series of magnitudes. This signifies that the VADM series has characteristics of a random walk. The shuffling of the series does not alter the nature of the series considerably.

➢ The series of VADM containing the polarity changes also exhibits random walk properties. However, the shuffling process strongly affects such properties indicating the presence of long-range correlations. The consideration of polarity makes the calculation of Hurst exponent less reliable, because the changes between adjacent entries of the series become considerable.

➢ Both original and shuffled series exhibit identical singularity spectra, either for magnitudes series or the polarity series. This fact implies that the VADM series is intrinsically multifractal.

➢The time evolution of Hurst exponent can be evaluated for each series. The time window width plays a crucial role and as a rule of thumb, the greater the width, the smoother the Hurst curve. Further attention will be dedicated future studies.

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