High robustness discrete time quantum random walk search algorithm without marking coin

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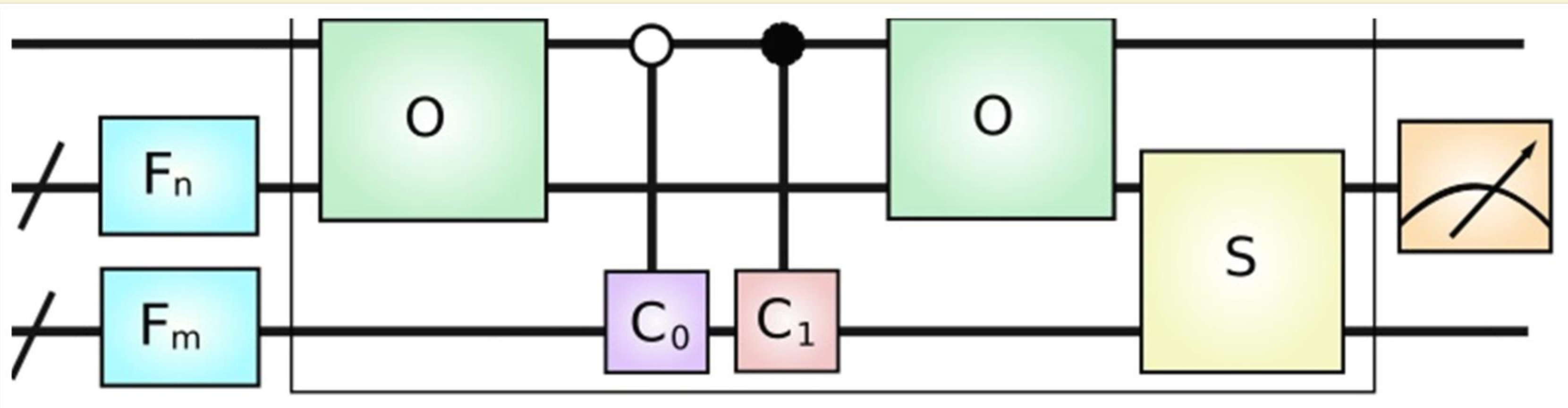
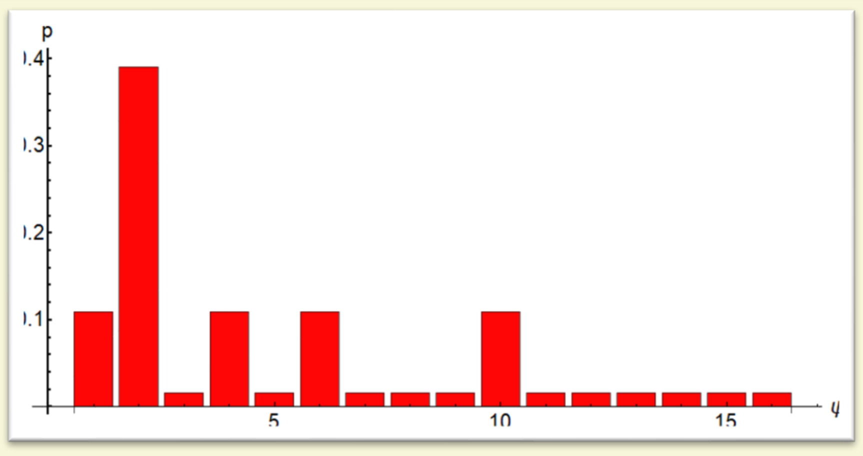
QRWS on hypercube with modified walk coin

Discrete time QRWS is probabilistic algorithm, that finds searched element in unordered database structured as graph (all elements are nodes in graph). Compared to the best classical search algorithms, it is quadratic faster. It consists of oracle function that can recognize the solution, shift operator that initiate the walk, and two coins – walk coin that changes the probability to go at each direction during the walk and mark coin that is applied on searched element. At the end of the algorithm node register is measured. Probability to find solution after measurement depends on the number of iterations and on the operators used as mark and walk coins. Quantum circuit of the algorithm is shown bellow:

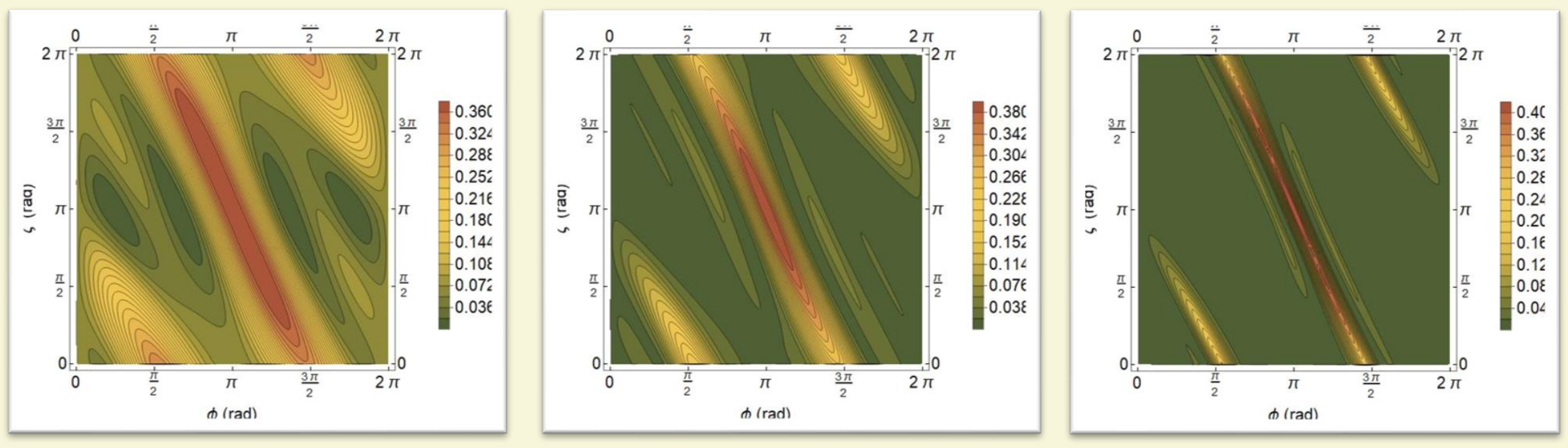




Example for probability to obtain each state after the algorithm finishes. Here, the solution is state $|\mathbf{1}\rangle$ and Grover coin is used for traversing. The coin size is $m = 4$ (in this case number of nodes is $2^m = 16$).

Monte-Carlo simulations

MC simulations of the probability $\boldsymbol{p(\phi, \zeta)}$ to find solution in the plane spawned by $\boldsymbol{\varphi}$ and $\boldsymbol{\zeta}$ for coin sizes 4, 6 and 8 are presented below.



Higher probability is shown with brown colour. On the figures we can see that there is connected region where:

$$p\left(\phi \in (\phi_{max} - \varepsilon, \phi_{max} + \varepsilon)\right) \cong \mathbf{p}(\phi_{max}) \equiv p_{max}$$

Here, $\boldsymbol{p_{max}}$ is maximum probability to find a solution and $\boldsymbol{\varepsilon}$ is region of stability (there the probability to find solution is high)

Different correlations

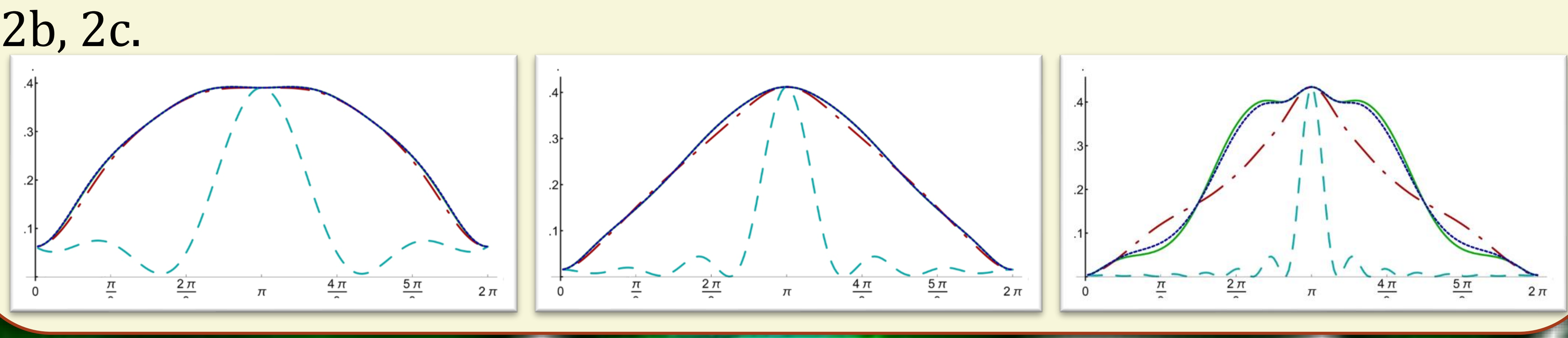
The correlation between phases that gives highest robustness can be approximated as a function $\boldsymbol{\zeta(\phi)}$, so the probability to find solution will be:

$$p(\phi, \zeta, \mathrm{m}) \to p(\phi, \zeta(\phi), \mathrm{m} = const) \equiv p(\phi)$$

Different correlation functions between phases $\boldsymbol{\zeta(\varphi)}$ lead to different robustness (for fixed coin size $\mathbf{m}$) of the algorithm against changes in phases:

1	$\zeta = \pi$	NA	Teal
2a	$\zeta = -2\phi + 3\pi + \alpha \sin(2\phi)$	$\alpha = 0$	Red
2b	$\zeta = -2\phi + 3\pi + \alpha \sin(2\phi)$	$\alpha = \frac{-1}{2\pi}$	Blue
2c	$\zeta = -2\phi + 3\pi + \alpha \sin(2\phi)$	$\alpha = \alpha_{ML}$	Green

Worst performance is in the case of Eq. 1. Using Eq. 2 instead gives higher robustness for appropriate values $\boldsymbol{\alpha}$ (QRWS is also robust against inaccuracies in $\boldsymbol{\alpha}$). Examples for some $\boldsymbol{\alpha}$ are shown in Eq. 2a, 2b, 2c.



QRWS on hypercube with modified walk coin

Here we study QRWS on hypercube. Walk coin can be constructed by generalized Householder reflection and phase multiplier.

$$C_0(\phi, \chi, \zeta) = e^{i\zeta}(I - (1 - e^{i\phi})|\chi\rangle\langle\chi|)$$

The probability p to find the searched element depends on both phase multipliers in the walk coin $\varphi, \zeta \in [0, 2\pi]$ and coin size m. Coin can be constructed by using qudits so it can have arbitrary dimension. When probability to go at each direction should be the same, χ is equal weight superposition vector.

Modification of the marking coin

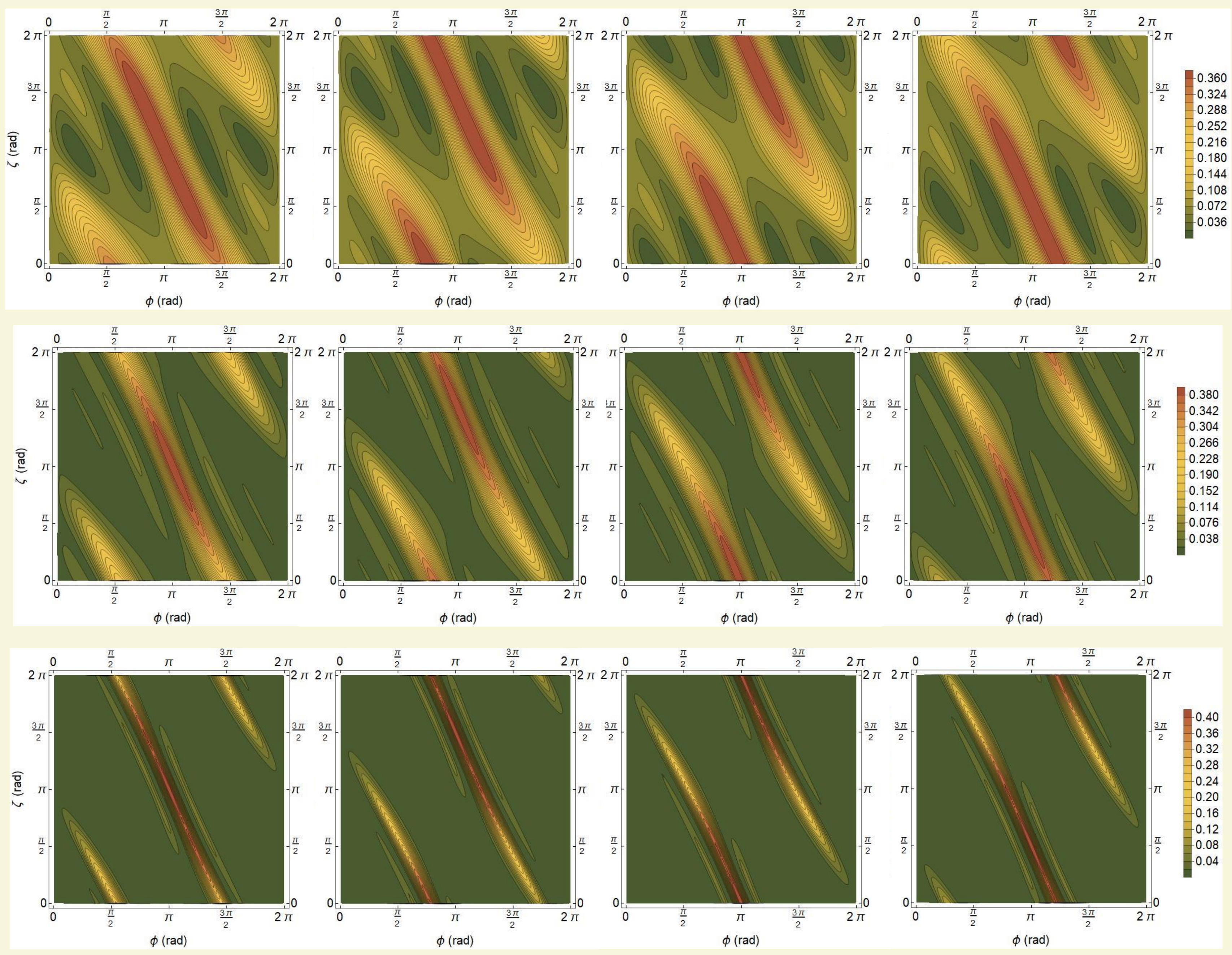
In the original QRWS algorithm the mark coin is: $C_1 = -\hat{1}$, however here we modify mark coin to be:

$$C_1 = -e^{-i\omega}\hat{1}$$

It can be shown that the additional phase of both walk and marking coin is not important by itself unlike their phase difference:

$$\Delta = \zeta - \omega$$

On Figure below shows Monte Carlo simulation for different value of $\mathbf{\Delta}$. Each row corresponds to different coin size (4, 6 and 8). Each column corresponds of $\mathbf{\Delta} = \mathbf{0}$; $\boldsymbol{\pi}/2$; $\boldsymbol{\pi}$ and $3\boldsymbol{\pi}/\mathbf{2}$ accordingly:



This modification changes the equations for correlation between parameters:

$$\zeta = -2\phi + \omega + 3\pi + \alpha \sin(2\phi)$$

Simplified quantum cirquit

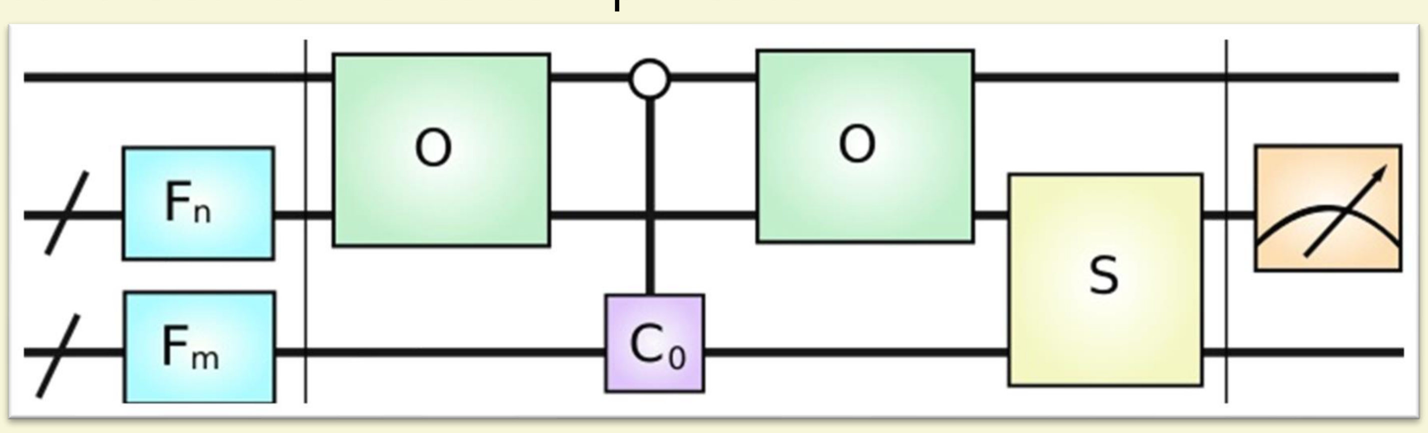
When $\boldsymbol{\omega} = \boldsymbol{\pi}$ the marking coin becomes the identity operator:

$$C_1 = -e^{-i\pi}\hat{1} = \hat{1}$$

Walk coin changes to:

$$C_0(\phi, \chi, \zeta) = e^{i(\xi - \pi)}(I - (1 - e^{i\phi})|\chi\rangle\langle\chi|)$$

In this case both marking coin and one of the control gates becomes redundant. Quantum circuit simplifies to:



Correlation that should be used in the case of those simplifications is:

$$\zeta = -2\phi + \alpha \sin(2\phi)$$

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