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LatticeQCD simulations using Boriçi – Creutz fermions

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Abstract

Minimally doubled fermions are a class of lattice fermions, which preserve exact chiral symmetry and are strictly local, details that make them very suitable for lattice simulations. Borici - Creutz fermions are one kind of minimally doubled fermions that have certain properties adaptable for lattice studies and simulations. In this work we present some of the lattice simulations made by using this kind of fermions, in light hadrons spectrum and spontaneous chiral symmetry breaking. We study and analyze the properties which make them desirable and the problems they show in lattice simulations.

Introduction

Minimally doubled fermions have been proposed as a strictly local discretization of the QCD fermionic action, which preserves chiral symmetry at finite cut-off, while break the hyper-cubic symmetry. Being strictly local and chiral, they present the possibility of fast simulations. Anyway the broken hyper-cubic symmetry has to be restore, possibly with a minimal numerical cost. There are several types of MDF fermions, and each one of them breaks the hyper - cubic symmetry in a specific direction [8, 16]. One kind of minimally doubled fermions are Boriçi - Creutz fermions, proposed by Creutz and Boriçi, in 2008 [1, 5]. The Dirac operator for these fermions in the momentum space:

$$D(p)=\sum_\mu i\gamma_\mu \sin p_\mu + \sum_\mu i\gamma'_\mu \cos p_\mu - 2i\Gamma$$

where $\gamma'_\mu = \Gamma\gamma_\mu\Gamma$ $\Gamma=\sum_\mu \gamma_\mu/2$ has two poles: $p_1 = (0,0,0,0)$ and $p_2 = (\pi/2, \pi/2, \pi/2, \pi/2)$.

These poles, that correspond to the two physical flavours, have a preferred direction in the euclidean space - time, defined by the line that connect them, which correspond to the major hyper - cube diagonal. Eventually, the hyper - cubic symmetry is broken [8,16]. In our previous publications we have shown the effects of this broken symmetry in the light hadrons spectrum [9, 10, 13, 14], have proposed a method of how to restore it partially for BC fermions using the chiral condensate [12, 13, 15] and also have presented some preliminary results of the calculations of the charged pion mass, in two different lattice directions, using the same conditions in which we have restored the hyper - cubic symmetry [14]. We have found that there a reduction of the difference of the masses calculated in different directions, which means that partially the broken hyper cubic symmetry is corrected. Regardless of these results, we have to see if this effect is represented even in the other light mesons and baryons masses. Also, the total computational cost (for defining the value of the counter-term in each case and for the calculation of the hadrons spectrum) has to be defined, in order to understand if this kind of fermions is suitable for the hadrons spectrum calculation.

Materials and methods

What we have done in this work, is to calculate the masses of the rho meson, nucleon and delta baryon in two different directions: in the hypercube diagonal direction and in one edge of the hypercube x_1 with the original Boriçi – Creutz action and then with the corrected Boriçi – Creutz action, where is added one of the counter-terms, dimension-3 counter-term. Let's remember that because of the broken hyper - cubic symmetry, the counter - terms are necessary for a renormalized theory. The allowed counter - terms for the BC action are dimension-4 counter-term $c_4(g_0)\bar{\psi}\Gamma_\mu D_\mu\psi$ and dimension-3 counter-term $ic_3(g_0)\bar{\psi}\Gamma\psi$ [16]. The coefficients of the dimension 3 and 4 counter-terms, c_3 and c_4, respectively, have been evaluated in one-loop lattice perturbation theory [16].

The first step was to write the proper hadrons operators using the point splitting method, [6,11] as $\psi(p)$ contains two degenerate flavors. Details are described in the reference [11,13]. Sure, writing the operators using two flavours isn't very simple, and for complicated operators, this step will be challenging. Then we studied the dependence of the dimension-3 counter-term from the lattice spacing. The methodology of how the counter-term value is defined for each lattice spacing, is presented in ref [12,15]. What was interesting was the fact that c_3 is almost independent form the lattice spacing. After this, we have repeated exactly the same procedure for the calculation of the light hadron masses, as described in the references [13, 14], but now even for the modified Boriçi – Creutz fermion action, where is added a counter-term $c_3 = 0.4$, that partially restore the hyper-cubic broken symmetry.
Boriçi – Creutz action in the second case (the corrected action) is:

$$D(p)=\Sigma_\mu[i\gamma_\mu \sin p_\mu + i(\Gamma-\gamma_\mu)\cos p_\mu + i\,(c_3-2)\Gamma$$

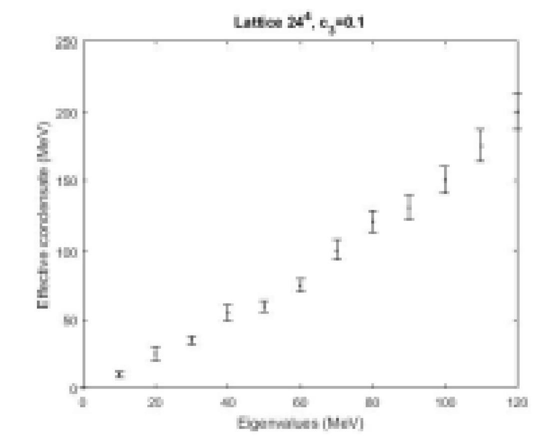
Simulations are carried out on QCDLAB (Tirana, Albania) in the quenched approximation for the Wilson gauge action, for the original BC action and the modified BC action for five different bare mass quarks, five lattice spacings, using FermiQCD C++ package [18]. The codes written for the Boriçi – Creutz action and for the hadrons propagators are implemented in this package.
The gauge configurations are generated using the Wilson gauge action:

$$S_g[U] = -\beta\sum \frac{1}{3}\Re\, Tr\, U_{\mu,i}\, U_{\vartheta,i+\mu}\, U^{-1}_{\mu,i+\vartheta}\, U^{-1}_{\vartheta,i}$$

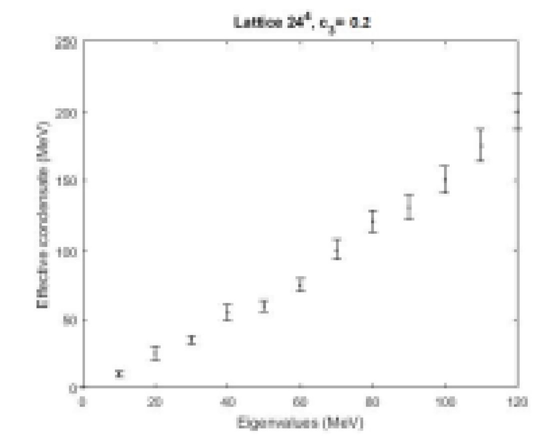
Let's remember that using Gauss - Lanczos quadrature we have calculated the modes number for minimally doubled fermions, and therefore the effective chiral condensate, in a background of 1000 gauge configurations, generated using the SU(3) theory. The results are averaged over 1000 configurations. As expected by the chiral perturbation theory, the low-modes of the operator condense and reach a "plateau". In order to find the effective condensate we have taken some "representative" data using the original data's histogram, and perform a linear fit of them, considering the fact that the datas have different weights. It's clear that for $c_3 < 0.4$ we have the phase with chiral symmetry, while for $c_3 \geq 0.4$, are in the SCHSB phase.

Results

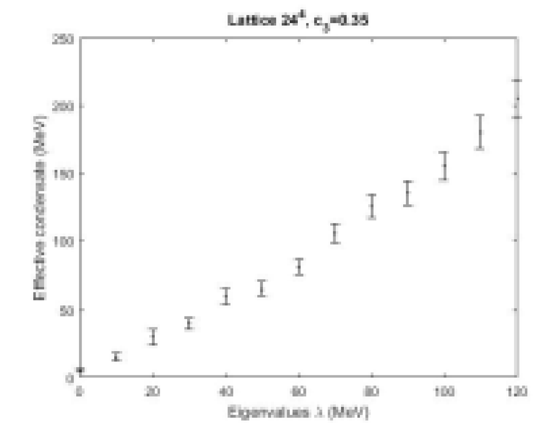
The results of how the countert-term is defined and the light spectrum using both actions are presented in the figures below.



(a) $c_3 = 0.1$
The calculations show that there is no formation of chiral condensate

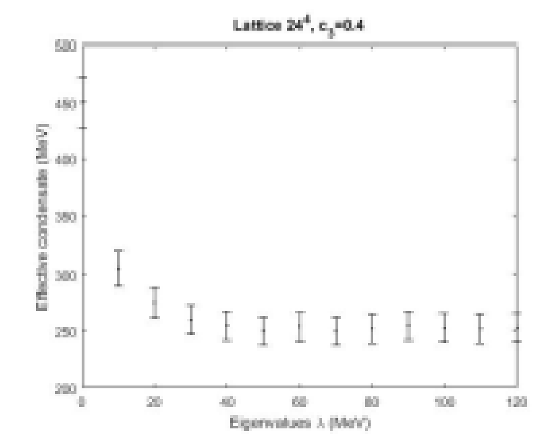


(b) $c_3 = 0.2$
The calculations show that there is no formation of chiral condensate

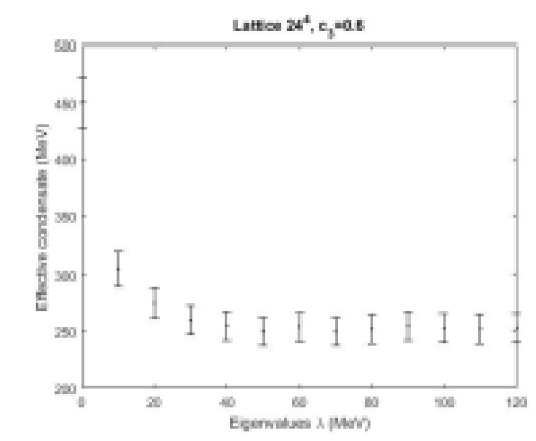


(c) $c_3 = 0.35$
The calculations show that there is a chance for the formation of chiral condensate

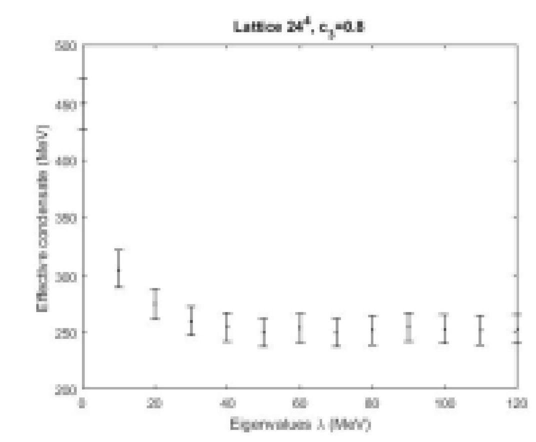
Figure 1. Effective condensate for different values of the counterterms $c_3 < 0.4$ added in BC action. It can be shown that for these values of the countertems, we don't have low-lying modes condensation



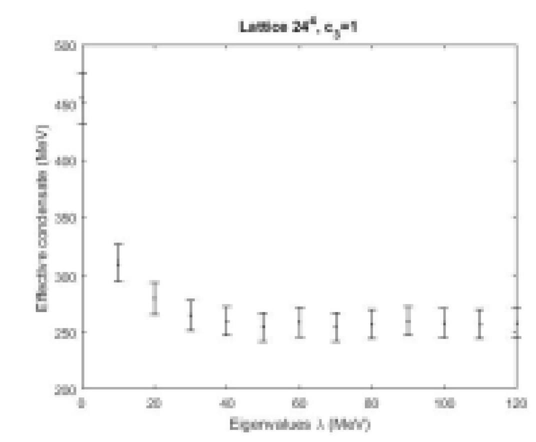
(a) $c_3 = 0.4$
Can be easily shown that for this value the low-lying modes condense, therefore there is a value of effective condensate different from zero



(b) $c_3 = 0.6$
It can be seen that there is a value of effective condensate different from zero



(c) $c_3 = 0.8$
It can be seen that there is a value of effective condensate different from zero



(d) $c_3 = 1$
It can be seen that there is a value of effective condensate different from zero

Figure 2. Formation of effective condensate for different values of the counterterms $c_3 \geq 0.4$ added in BC action.

In the first figure, we have presented one of our previous results, the square mass of the neutral pion, using BC action with the added dimension 3 counter-term, as a function of the bare quark masses. As it expected, it goes to zero, in the chiral limit.

In the figures below are presented the results of the rho meson, nucleon and delta baryon masses, calculated in two different directions. In figure 2, are presented the results using the original BC action, while in figure 3 the case when the corrected BC action, with the dimension 3 counter-term is added.

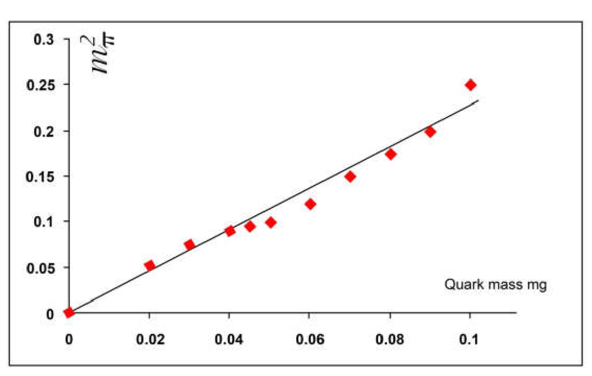


Figure 3 Pion mass as a function of the bare quark masses with the corrected BC action.

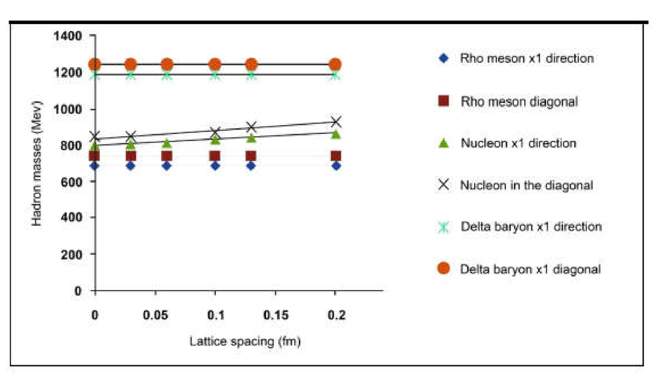


Figure 4 Hadron masses of the light hadrons, in two different directions using the original BC action

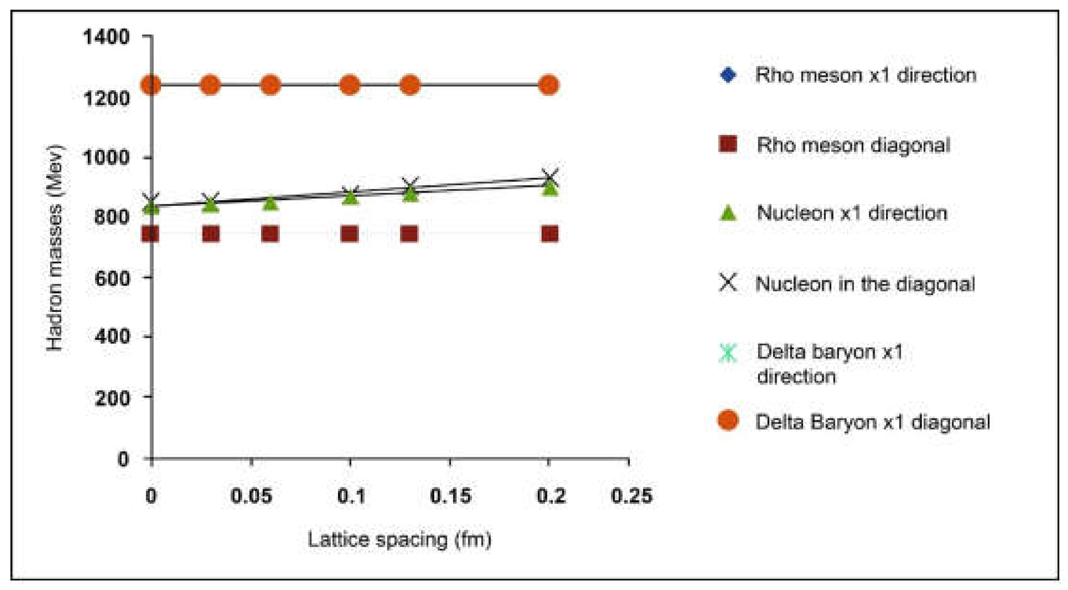


Figure 5 Hadron masses of the light hadrons, in two different directions using the corrected BC action, where c_3 is added

In every case we have extrapolated the results in the continuum limit, and the taken results are presented in Table 1 (BC original action) and Table 2 (corrected BC actions)

Hadron	Masses in the diagonal direction (MeV/c²)	Masses in the x_1 direction(MeV/c²)
Rho meson	741	690
Nucleon	849	805
Delta baryon	1240	1187

Table 1. Hadron masses calculated with the original BC action

Hadron	Masses in the diagonal direction (MeV/c²)	Masses in the x_1 direction(MeV/c²)
Rho meson	741	730
Nucleon	860	852
Delta baryon	1240	1236

Table 2. Hadron masses calculated with the corrected BC action, where c_3 is added

Discussions and conclusions

Chiral symmetry and spontaneous chiral symmetry breaking are very imporant in QCD. Using minimally doubled fermions (as chiral fermions) and Lanczos quadrature we can explore and understand the dynamical mechanism of spontaneous chiral symmetry breaking in a very simple way. The metodology we propose has a minimal numerical cost, because we utilise the optimal properties of Krylov subspaces in approximating the distribution of the eigenvalues of the Dirac operator, not in calculating every eigenvalue of the operator. One important thing of this work is the fact that the chiral condensate can be used as an order parameter for Boriçi - Creutz and help us to find the proper counterterms that restore partially the broken hypercubic symmetry. This work aims to the use of this methodology for further detailed studies of minimally doubled fermions.

Boriçi - Creutz fermions (minimally doubled fermions) are known to preserve chiral symmetry for a degenerate quark doublet and are strictly local. These fermions are ideal for studying QCD with *u* and *d* quarks, can be helpful for $N_f = 2$ lattice simulations, relatively simpler and possibly faster than Ginsparg-Wilson fermions. The chiral condensate can be used as an order parameter for BC fermions, and help us to find the proper counter-term that restore partially the broken hyper-cubic symmetry. Tuning of counter-term coefficient c_3 doesn't seem difficult and expensive, because apparently it is almost independent from the lattice spacing and coupling constant (further studies). Tuning c_4 will require more care and work so as to properly restore the desired symmetry, because it depends on the lattice and the coupling constant we use on simulations. (further studies) Using BC fermions with the added counter-term c_3, the difference between the light hadrons masses calculated in different directions decrease, so we can think that partially have restore the broken hyper - cubic symmetry. Anyway….BC fermions, seem to be not the most suitable fermions for spectroscopy in full QCD

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