On the quarks transverse momenta in SIDIS experiments

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SIDIS

In Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes, $lN \to l'hX$, one detects the scattered lepton l', and a hadron h. The contemporary approach is to go beyond collinear momenta picture of hadron and constituent partons and to use the so-called Transverse Momentum Dependent (TMD) Partonic Distribution and Fragmentation Functions (PDF and FF). The PDF $\hat f_{aN}(x_B, k_\perp)$ gives the number density of a with light-cone momentum fraction x_B and transverse momentum $k_\perp$ inside a fast moving nucleon. The FF $\hat D_{ha}(z_h, p_\perp)$ gives the number density of hadron h resulting in the fragmentation of parton a with a light-cone momentum fraction z_h and a transverse momentum $p_\perp$, relative to the original parton motion. The SIDIS cross section is factorizable, i.e. $d\sigma^{lN\to l'hX} = \sum_q \int_{int} \hat f_{qN}(x_B, k_\perp; Q^2) d\sigma^{lq\to l'q} \hat D_{hq}(z_h, p_\perp; Q^2)$ and can be written in FSE on the angles (defined on Fig.1) ϕ_h and $\phi_s - \phi_h$.

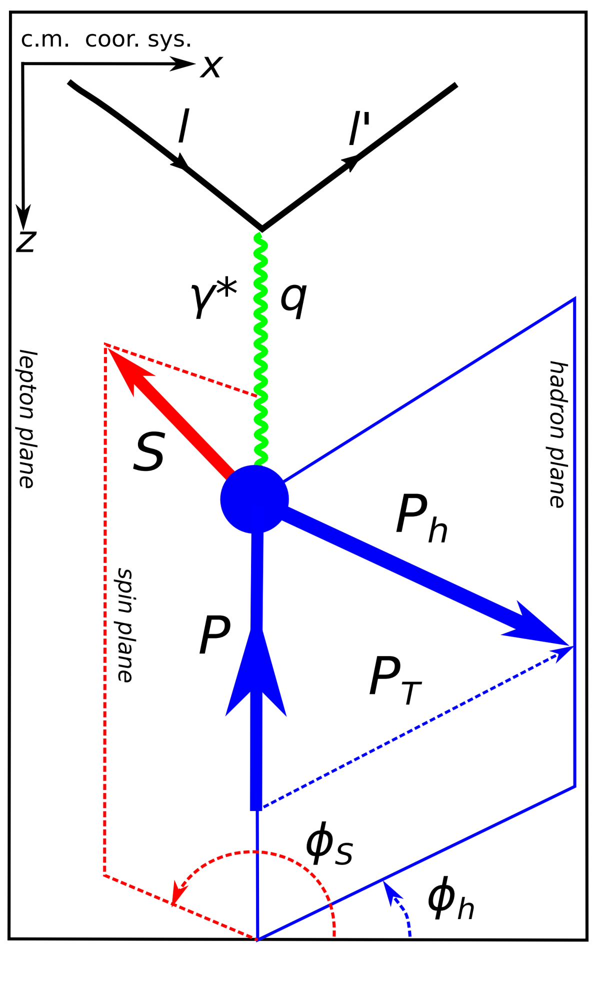


Figure 1: SIDIS kinematics

Notations (not a full list at all): l and P: initial lepton and nucleon 4-momenta; l' and P_h: final lepton and (final) hadron 4-momenta; $z_h = \frac{(P\cdot P_h)}{(P\cdot q)}$, $q = l - l'$; $y = \frac{(P\cdot q)}{(P\cdot l)}$, $Q^2 = -q^2 = 2M_d E x_B y$; M_d is the target (deuteron) mass; E is the lepton laboratory energy; S_T is the nucleon polarization; P_T is the measured transverse momentum of the final hadron which (at order $k_\perp/Q$) is $P_T = z_h k_\perp + p_\perp$. We assume some relations between collinear and non-collinear PDFs and FFs, some factorization of their variables dependence and simple Gaussian dependence on transverse momenta. Thus only 2 parameters are needed ($\langle k_\perp^2\rangle$ and $\langle p_\perp^2\rangle$). However different publications give incompatible values for them (all values are in GeV^2):

$$\langle k_\perp^2\rangle \approx 0.25,\ \langle p_\perp^2\rangle \approx 0.20 \quad \text{Ref.[1], data from [2]} \quad (1)$$

$$\langle k_\perp^2\rangle = 0.18,\ \langle p_\perp^2\rangle = 0.20 \quad \text{Ref.[3], data from []} \quad (2)$$

$$\langle k_\perp^2\rangle = 0.57 \pm 0.08,\ \langle p_\perp^2\rangle = 0.12 \pm 0.01 \quad \text{Ref.[4], data from [5]} \quad (3)$$

$$\langle k_\perp^2\rangle = 0.61 \pm 0.20,\ avp = 0.19 \pm 0.02 \quad \text{Ref.[4], data from [6]} \quad (4)$$

To clarify the problem we work with following asymmetries:

$$A^{\cos\phi_h,h-\bar h}_{UU,d}(x_B) = \sqrt{\frac{\pi}{\langle Q^2\rangle}}\Big(\underbrace{C^h_{Cahn}}_{const.} + 2\mathcal{N}^{BM}_{q_v}(x_B)\underbrace{C^h_{BM}}_{const.}\Big), \quad (5)$$

$$A^{\cos 2\phi_h,h-\bar h}_{UU,d}(x_B) = \mathcal{N}^{BM}_{q_v}(x_B)\underbrace{\hat C^h_{BM}}_{const.} + \frac{MM_d}{\langle Q\rangle^2}\underbrace{\hat C^h_{Cahn}}_{const.} \quad (6)$$

$$A^{Siv,h-\bar h}_{UT,d}(x_B) = \frac{\sqrt{e\pi}}{\sqrt{2}\,M_{Siv}}\frac{\langle k_\perp^2\rangle^2_{Siv}}{\langle k_\perp^2\rangle}\underbrace{C^h_{Siv}}_{const.}\mathcal{N}^{Siv}_{q_v}(x_B) \quad (7)$$

Here $A^{h^+-h^-} \equiv \frac{\Delta A^{h^+}-\Delta A^{h^-}}{A^{h^+}-A^{h^-}}$, where Δ indicates polarized target, and A is the corresponding Fourier coefficient of σ integrated over redundant variables and normalized to the 0-th coefficient.). These asymmetries (5, 6, 7) are ploted on Fig.2 below.

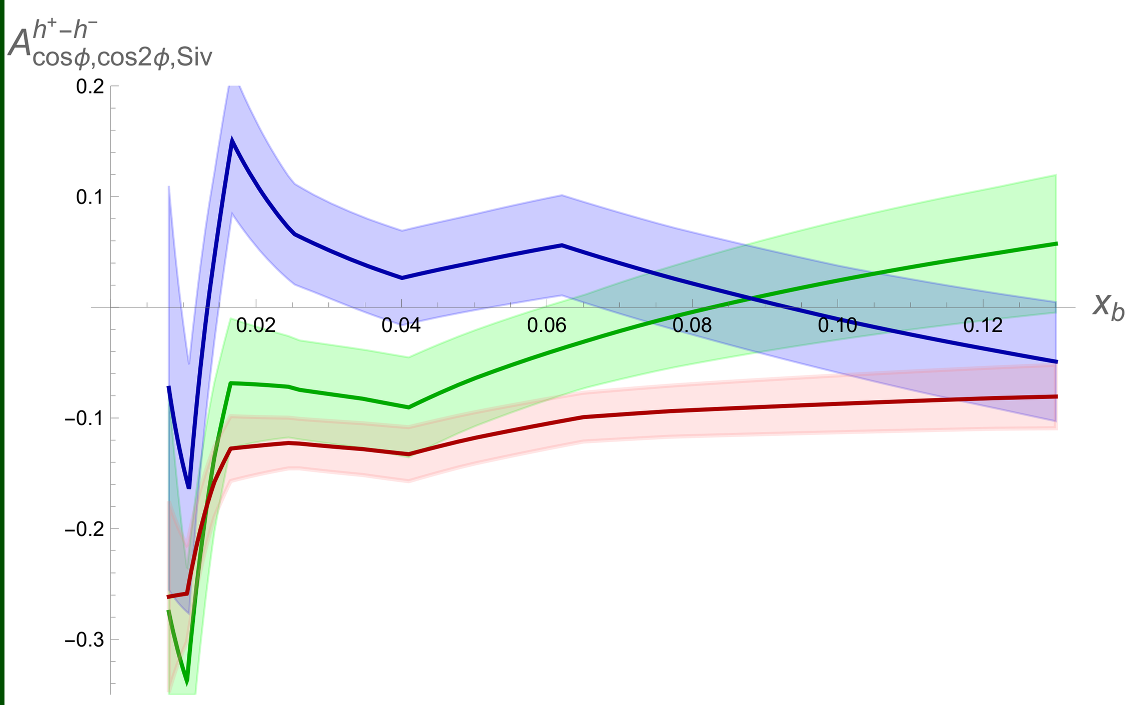


Figure 2: red — $\cos\phi$, green — $\cos 2\phi$, blue — Sivers

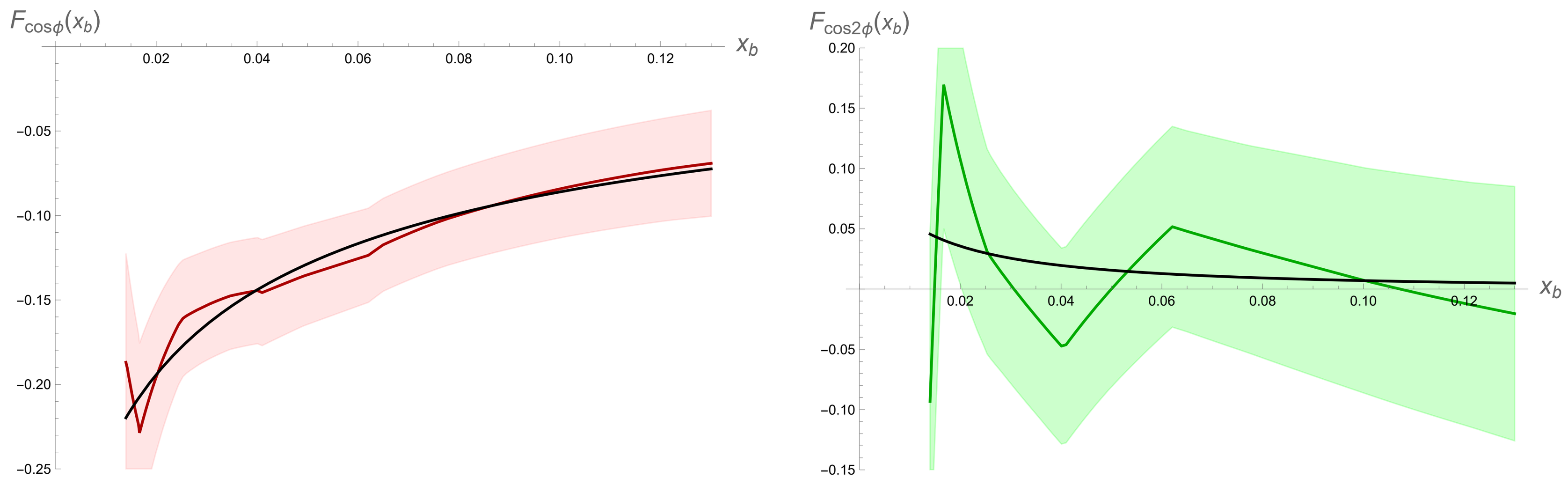
The Sivers function is connected to the term $\mathbf{S}\cdot(\mathbf{k}_\perp \times \mathbf{P})$, while the Boer – Mulders (BM) one — to $\mathbf{s_q}\cdot(\mathbf{k}_\perp \times \mathbf{P})$. We assume that they are proportional, so $\mathcal{N}^{BM}_{q_v}(x_B) \approx A^{Siv,h-\bar h}_{UT,d}(x_B)$. In this point our analysis is significantly simplified. Because we work with difference asymmetries only PDFs and FFs of valence quarks participate in eqs. (5, 6, 7) and more, because of the deutron target, only the combination $q_V = u_V + d_V$ and corresponding PDF Q_v take part. With the last assumption we get [7]:

Results

$$\underbrace{C^h_{Cahn}\sqrt{\frac{\pi}{\langle Q^2\rangle}}}_{\text{"theory"}} = \underbrace{A^{\cos\phi_h,h-\bar h}_{UU,d}(x_B) - C^h_{BM}\sqrt{\frac{\pi}{\langle Q^2\rangle}}A^{Siv,h-\bar h}_{UT,d}(x_B)}_{\text{"experiment"}}, \quad (8)$$

$$\underbrace{\hat C^h_{Cahn}\frac{MM_d}{\langle Q^2\rangle}}_{\text{"theory"}} = \underbrace{A^{\cos 2\phi_h,h-\bar h}_{UU,d}(x_B) - \hat C^h_{BM}A^{Siv,h-\bar h}_{UT,d}(x_B)}_{\text{"experiment"}}, \quad (9)$$

The fitting of eq.(8) "experiment" (red) to "theory" (black) and eq.(9) "experiment" (green) to "theory" (black) are shown below



The coefficients, determined by the fits are

$$C^h_{BM} = 0.54 \pm 0.80,\quad C^h_{Cahn} = -0.165 \pm 0.043, \quad (10)$$

$$\hat C^h_{BM} = -1.6 \pm 1.6,\quad \hat C^h_{Cahn} = 0.045 \pm 0.124. \quad (11)$$

Note that in our case there are close formulas for Cahn coefficients:

$$C^h_{Cahn} = -\langle k_\perp^2\rangle\frac{\int dz_h\, z_h[D^h_{q_v}(z_h)]/\sqrt{\langle P_T^2\rangle}}{\int dz_h\,[D^h_{q_v}(z_h)]}, \quad (12)$$

$$\hat C^h_{Cahn} = \frac{1}{MM_d\,\langle k_\perp^2\rangle\langle p_\perp^2\rangle}\frac{\int dz_h[D^h_{q_v}(z_h)]\,J(z_h)}{\int dz_h\,[D^h_{q_v}(z_h)]}. \quad (13)$$

Here $J(z_h) = \int dP_T^2\, e^{-\frac{P_T^2}{\langle p_\perp^2\rangle}} \int dk_\perp^2 k_\perp^2 e^{-k_\perp^2\frac{\langle P_T^2\rangle}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle}} \int_0^{2\pi} d\phi\, \cos 2\phi\, e^{a\cos\phi}$, $a = (2z_h k_\perp P_T)/\langle p_\perp^2\rangle$, and D are Collins functions (CF). For each quark they are tabulated in http://lapth.cnrs.fr/ffgenerator/ . From the quarks CF and because of the Q^2-dependence can be neglected in the region of interest we can construct CF for q_v: $D^h_{q_v}(z_h) = e_u^2 D^h_{q_v}(z_h) + e_d^2 D^h_{d_v}(z_h)$,

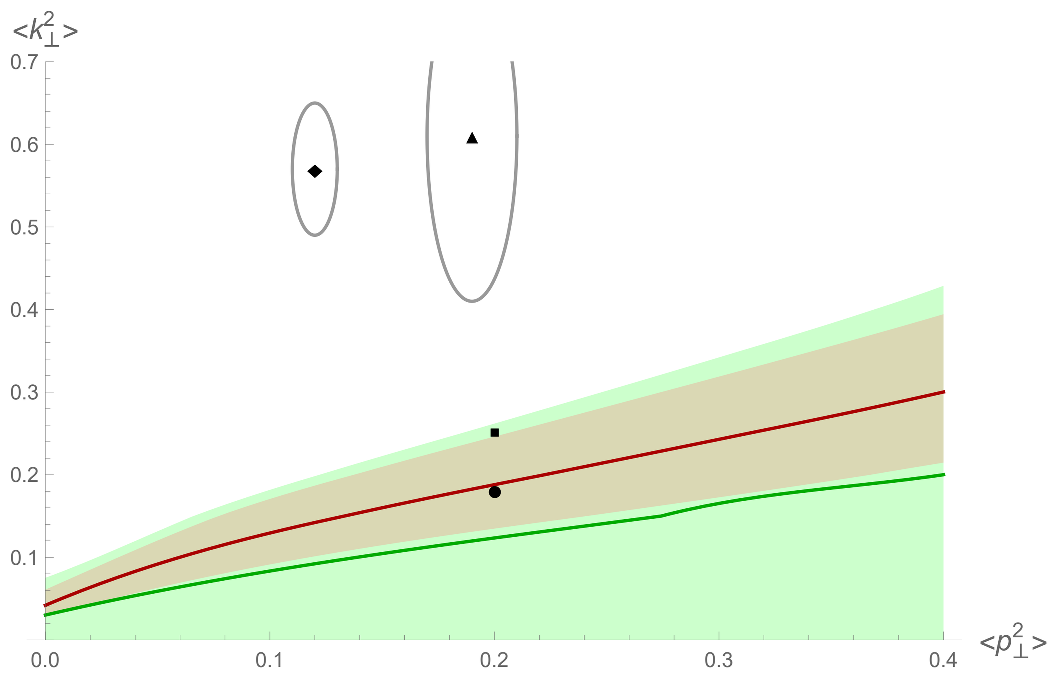


Figure 3: red — C^h_{Cahn}, green — $\hat C^h_{Cahn}$. The values of $\langle p_\perp^2\rangle$ and $\langle k_\perp^2\rangle$ given in eqs.(1–4) are indicated by black circle, box, diamond and triangle respectively.

Note that we can view eqs.(12, 13) as determining the functional dependence of Cahn coefficients on $\langle p_\perp^2\rangle$, $\langle k_\perp^2\rangle$. Having the solutions (10, 11) we can try to invert the functions and to find $\langle p_\perp^2\rangle$, $\langle k_\perp^2\rangle$. Graphically, the result is shown in Fig.4. The superposition of graphics shows the strong correlation between C^h_{Cahn} and $\hat C^h_{Cahn}$ functions which prevents $\langle p_\perp^2\rangle$ and $\langle k_\perp^2\rangle$ determination.

Conclusions: We have successfully tested twice the assumption that BM and Sivers functions of q_v (on deutron target) are proportional. Note that the corresponding fit is quite unusual with fitting parameters in data and their errors. The determination of $\langle p_\perp^2\rangle$ and $\langle k_\perp^2\rangle$ parameters of TMD fails because of reviled correlation between C^h_{Cahn} and $\hat C^h_{Cahn}$ functions which is interesting in its own ground. Nevertheless, our result selects eqs.(1, 2) from the list of announced values of $\langle p_\perp^2\rangle$, $\langle k_\perp^2\rangle$.

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