BPU 11

Probing hydrodynamics in PbPb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV using higher-order cumulants

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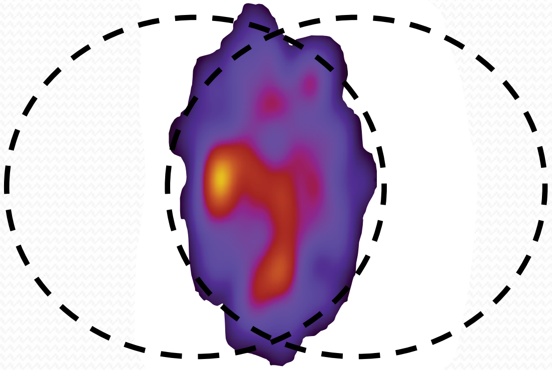
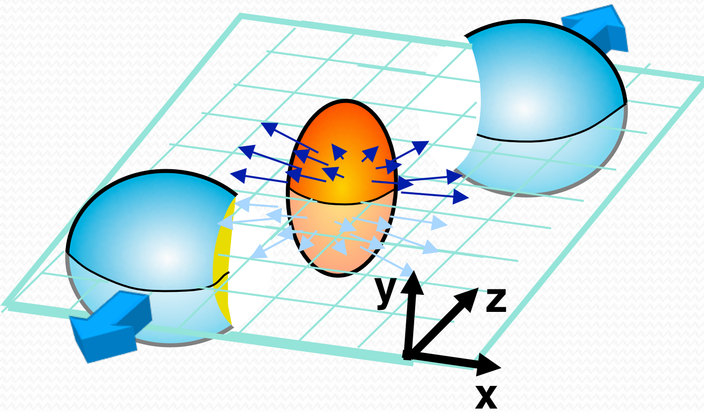
on behalf of the CMS Collaboration

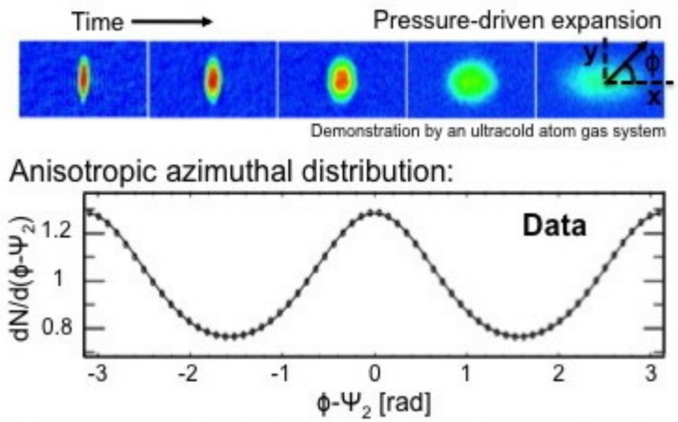
Outline



- v_2 magnitude - Introduction
- Motivation
- Collectivity in PbPb collisions – studied by Q-cumulants
- Centrality dependence of the hydrodynamic probes
- Measurement of central moments of the $\mathbf{v}_2$ distribution
- Conclusions

Azimuthal anisotropy





Ψ_n (angle of nth-order flow symmetry plane)

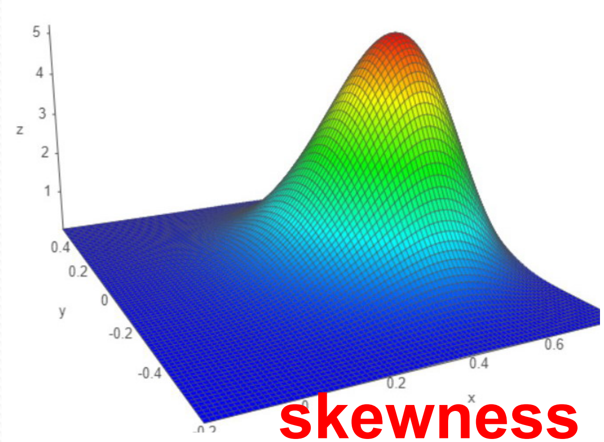
$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_n)]$$

v_n – *Fourier harmonics* depend on

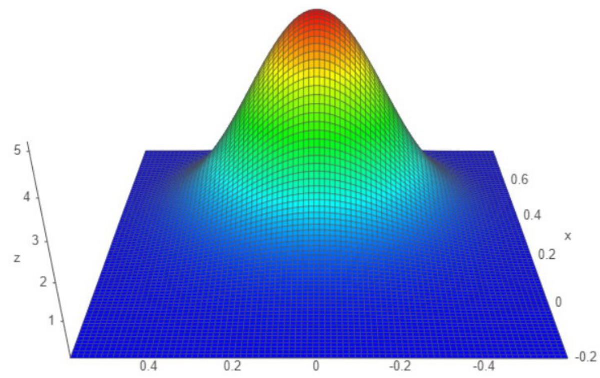
- initial state geometry
- initial state fluctuations
- medium transport properties (e.g. η/s)

$$v_n \equiv \langle \cos[n(\phi - \Psi_n)] \rangle$$

Shape of the v_2 distribution

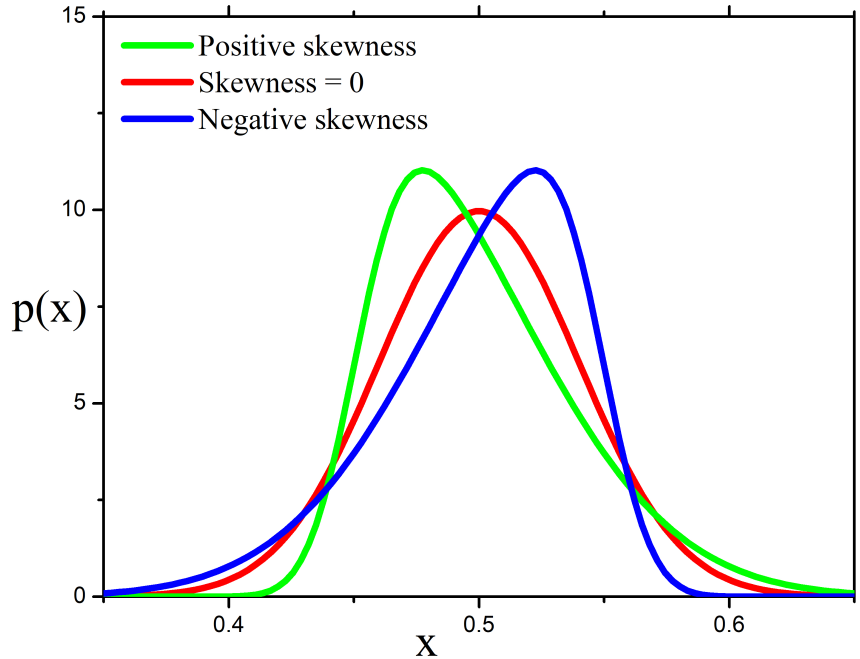


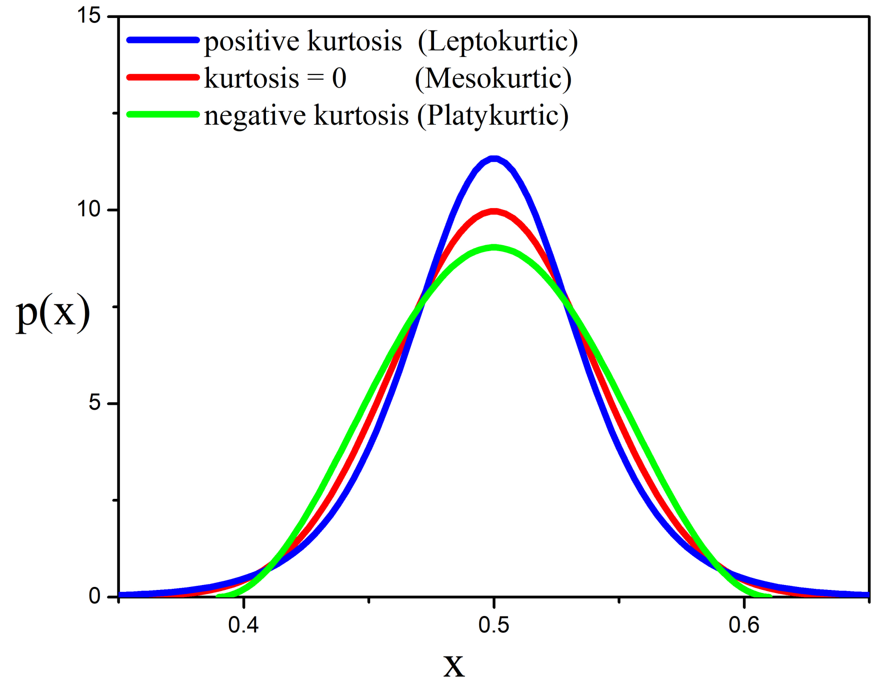
skewness



kurtosis

- Non-Gaussianities are present in the early stage.
- Partially washed out during hydro expansion





Hydrodynamic probe as a motivation

[Phys. Rev. C **95** (2017) 014913]

Hydrodynamic probe: $\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} \approx 0.091$

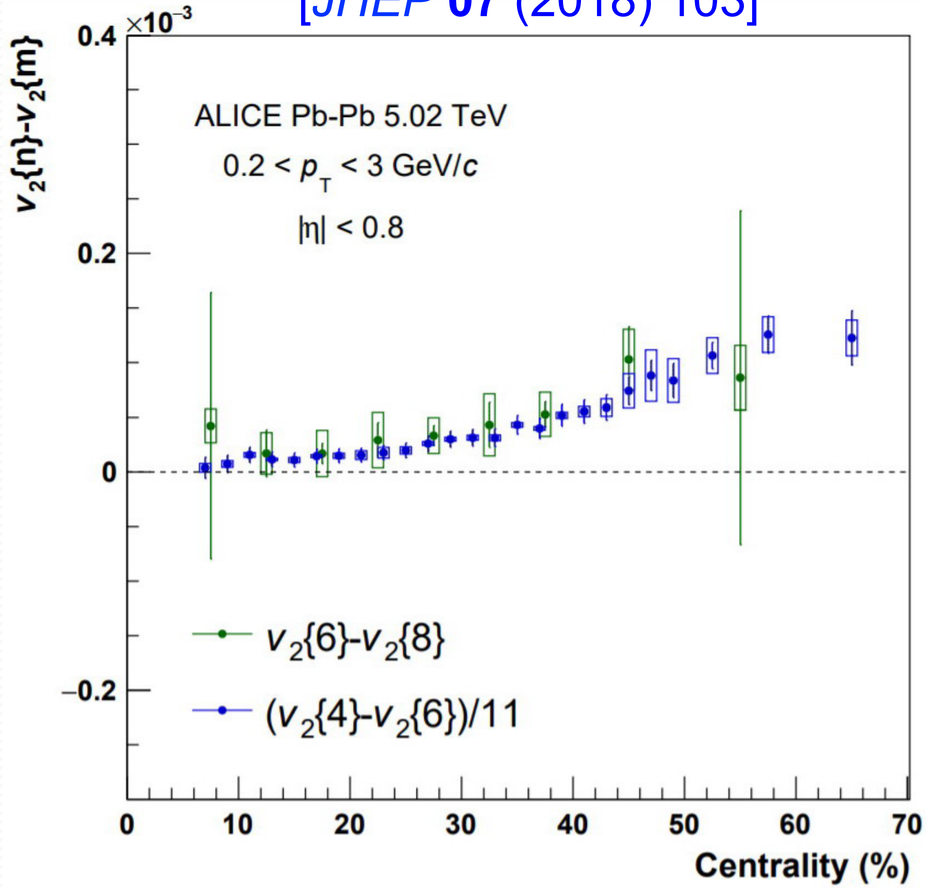
"Good for central collisions", "Higher order expansion of the v_2 distribution are required"

[Phys. Lett. B **789** (2019) 643-665]

0.143 ± 0.008(stat) ± 0.014(syst) centrality: 20–25%
0.185 ± 0.005(stat) ± 0.012(syst) centrality: 55–60%

"Higher order terms in a cumulant expansion of the v_2 distribution are required"

[*JHEP* **07** (2018) 103]



Precision was not satisfactory!

Q-cumulant method

Multi-particle correlations

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1-\phi_2)} \right\rangle \qquad \langle 4 \rangle \equiv \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle$$

Q-vector

$$Q_n = \sum_{i=1}^{M} e^{in\varphi_i}$$

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2\,\mathrm{Re}\left[Q_{2n}Q_n^*Q_n^*\right]}{M(M-1)(M-2)(M-3)} - 2\,\frac{2(M-2)|Q_n|^2 - M(M-3)}{M(M-1)(M-2)(M-3)}$$

Averaging over all events

$$\langle\langle 2 \rangle\rangle \equiv \left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle \qquad \langle\langle 4 \rangle\rangle \equiv \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\right\rangle$$

Ideal detector case

Q-cumulant method

[Phys. Rev. C 104 (2021) 034906]:

$$c_n\{2k\} = \langle\langle 2k \rangle\rangle - \sum_{m=1}^{k-1} \binom{k}{m} \binom{k-1}{m} \langle\langle 2m \rangle\rangle c_n\{2k-2m\}$$

$$c_n\{2\} = \langle\langle 2 \rangle\rangle$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle^2$$

General formulas for any order

$$v_n\{2k\} = \sqrt[2k]{\frac{(2k)!}{2^{2k}(k!)^2} \left[\frac{d^{2k}}{dl^{2k}} \ln I_0(l) \Big|_{l=0} \right]^{-1} c_n\{2k\}}$$

$$v_n\{2\} = \sqrt{c_n\{2\}}$$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

v_2 from Q-cumulants

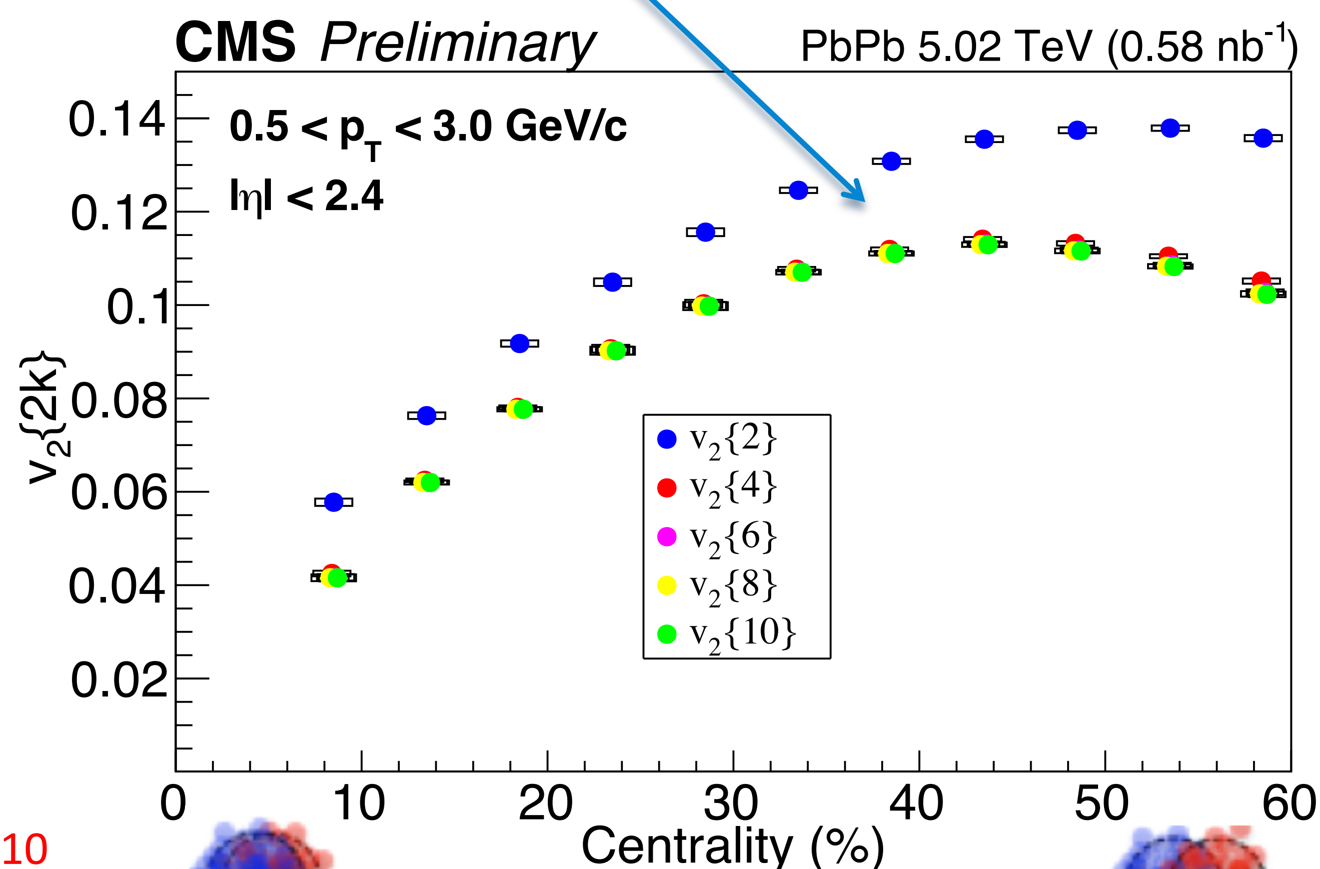
- **Flow fluctuations, σ_{v}, -> a gap between $v_2\{2\}$ and higher-order cumulants:**
  $v_2\{2\}^2 = v_2\{2k\}^2 + 2\sigma_v^2$, for (k>1)

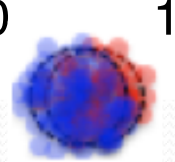
fine splitting

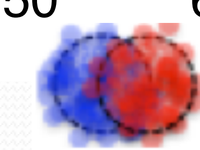
$v_2\{4\} >\approx v_2\{6\} >\approx v_2\{8\} >\approx v_2\{10\}$

- **Syst. uncertainties ~ 3 orders of magnitude greater wrt stat. ones**

Dominant source syst. uncertainties: variation on criteria for tracks





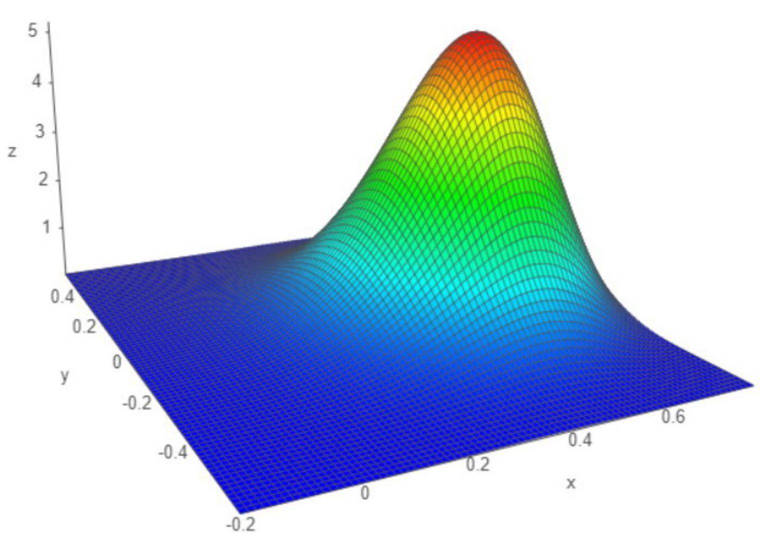


CMS-PAS-HIN-21-010

Expansion of hydro probes in central moments

$$h_1 = \frac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}} \approx \frac{1}{11} - \frac{4\kappa_{40} + \dfrac{8(p_{50}+p_{32})}{\bar{v}_2}}{11\left[2\bar{v}_2 s_{30} + 3(\kappa_{40}+\kappa_{22}) + \dfrac{3(p_{50}+2p_{32}+p_{14}) - 2(\sigma_y^2-\sigma_x^2)(5s_{30}-6s_{12})}{2\bar{v}_2}\right]}$$

$$h_2 = \frac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}} \approx \frac{3}{19} - \frac{88p_{50}}{95\left[4\bar{v}_2^2 s_{30} - 2\bar{v}_2(\kappa_{40}-3\kappa_{22}) - 13(p_{50}+10p_{32}-3p_{14}) - 2(\sigma_y^2-\sigma_x^2)(5s_{30}-6s_{32})\right]}$$



$$\frac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}} \approx \frac{1}{11} - \frac{1}{11}\,\frac{v_2\{4\}^2 - 12v_2\{6\}^2 + 11v_2\{8\}^2}{v_2\{4\}^2 - v_2\{6\}^2 + \dfrac{(\sigma_y^2-\sigma_x^2)s_{30}}{3\bar{v}_2^3}}$$

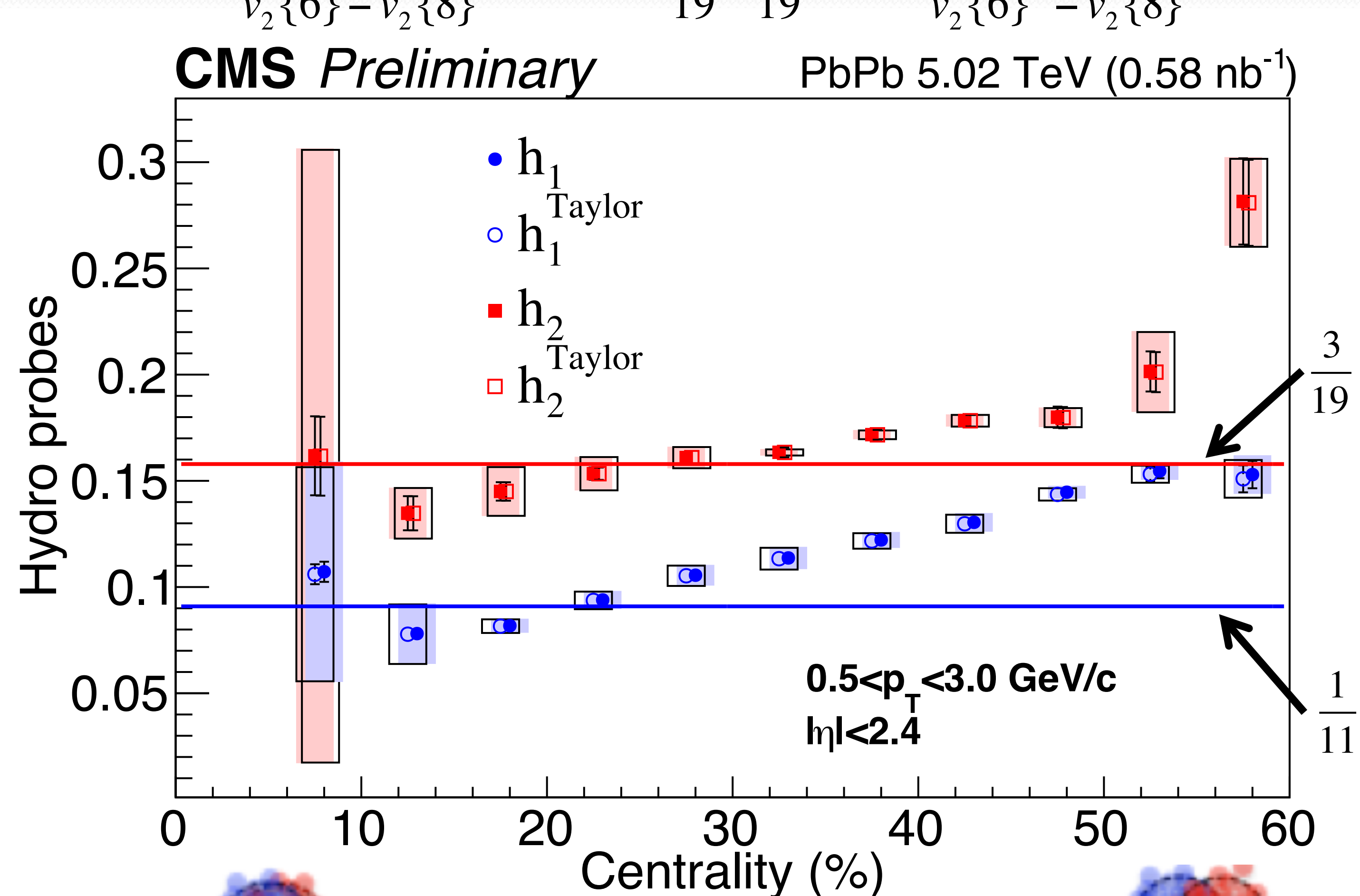
negligible

$$\frac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}} \approx \frac{3}{19} - \frac{1}{19}\,\frac{3v_2\{6\}^2 - 22v_2\{8\}^2 + 19v_2\{10\}^2}{v_2\{6\}^2 - v_2\{8\}^2 + \dfrac{(\sigma_y^2-\sigma_x^2)s_{30}}{33\bar{v}_2^3}}$$

Hydrodynamic probes

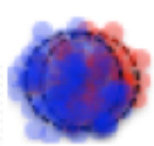
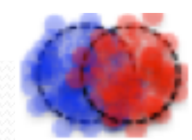
$$h_1 = \frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx h_1^{Taylor} = \frac{1}{11} - \frac{1}{11}\frac{v_2^2\{4\} - 12v_2^2\{6\} + 11v_2^2\{8\}}{v_2\{4\}^2 - v_2\{6\}^2}$$

$$h_2 = \frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx h_2^{Taylor} = \frac{3}{19} - \frac{1}{19}\frac{3v_2^2\{6\} - 22v_2^2\{8\} + 19v_2^2\{10\}}{v_2\{6\}^2 - v_2\{8\}^2}$$



CMS-PAS-HIN-21-010

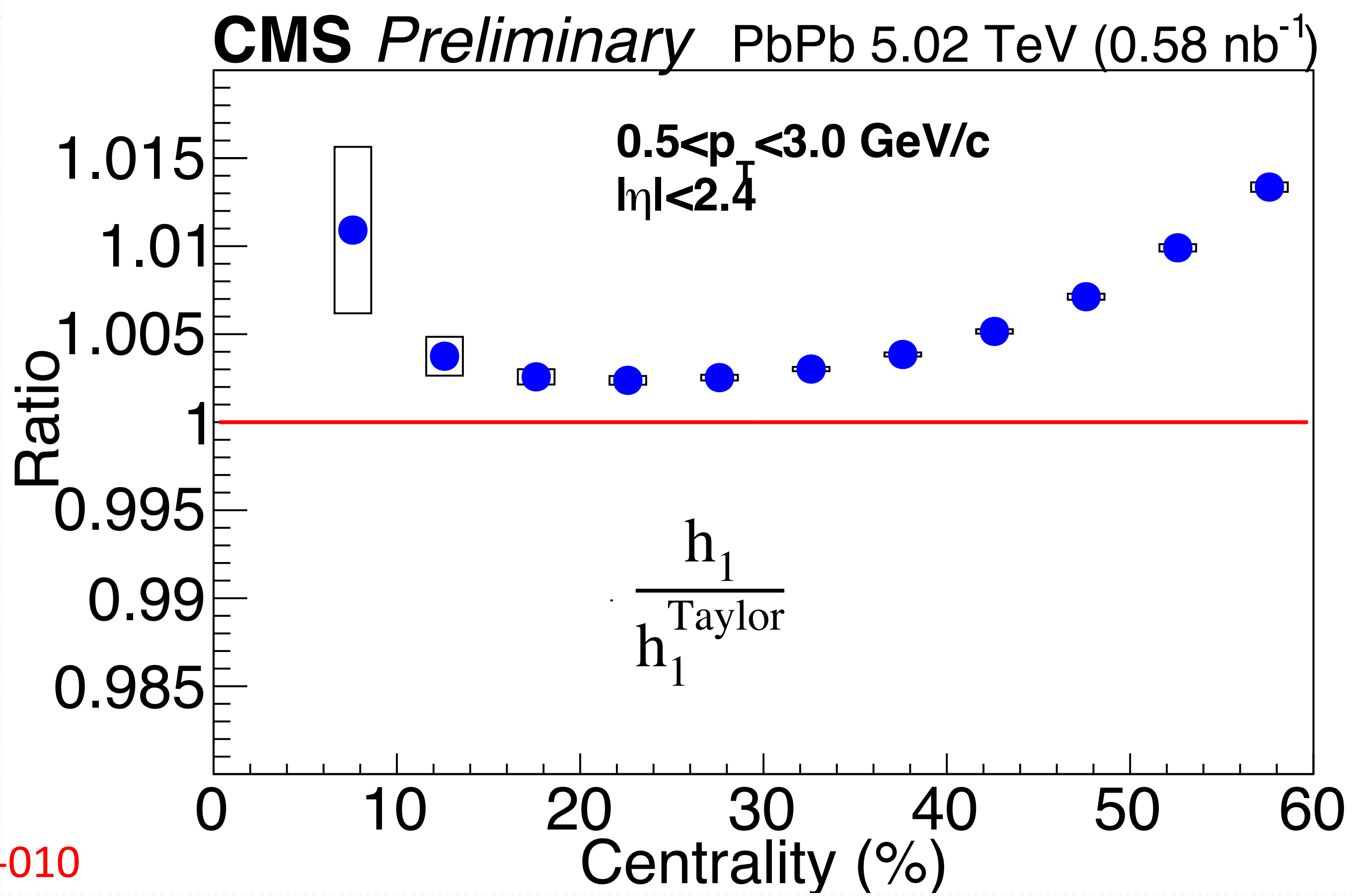
Higher-order moments necessary to describe data

Ratio between probe and its Taylor expansion

$$\frac{h_1}{h_1^{Taylor}} \approx \frac{\dfrac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}}}{\dfrac{1}{11}-\dfrac{1}{11}\dfrac{v_2^2\{4\}-12v_2^2\{6\}+11v_2^2\{8\}}{v_2\{4\}^2-v_2\{6\}^2}}$$

-Stat. uncertainties of the nominator and denominator are strongly correlated
-Syst. uncertainties dominates
-Term proportional to $(\sigma_y^2-\sigma_x^2)$ is negligible in accordance with *PRC* **95** *(2017) 014913*



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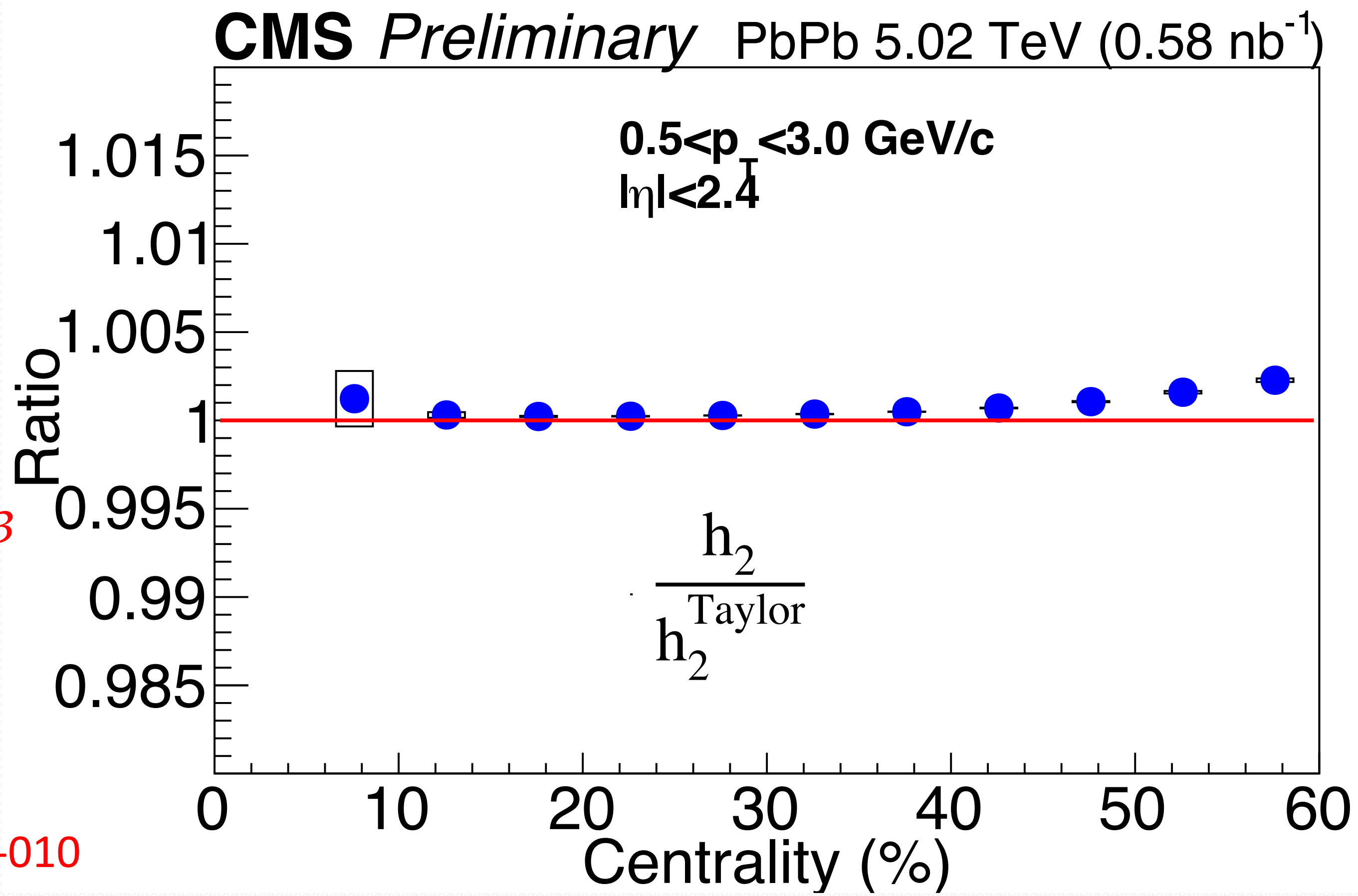
Ratio between the new probe and its Taylor expansion

$$\frac{h_2}{h_2^{Taylor}} \approx \frac{\dfrac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}}}{\dfrac{3}{19}-\dfrac{1}{19}\dfrac{3v_2^2\{6\}-22v_2^2\{8\}+19v_2^2\{10\}}{v_2\{6\}^2-v_2\{8\}^2}}$$

-Stat. uncertainties of the nominator and denominator are strongly correlated

-Syst. uncertainties dominates

-Term proportional to $(\sigma_y^2-\sigma_x^2)$ is negligible in accordance with *PRC* **95** *(2017) 014913*



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Standardized & Cleaned moments

PRC 99 (2019) 014907

$$\gamma_1^{\exp} = -2^{3/2} \frac{v_2\{4\}^3 - v_2\{6\}^3}{\left[v_2\{2\}^2 - v_2\{4\}^2\right]^{3/2}} \approx -2^{3/2} \frac{-s_{30} - O_N}{\left[2\sigma_x^2 + O_D\right]^{3/2}} \approx \frac{s_{30}}{\sigma_x^3} \equiv \gamma_1$$

$$\gamma_2^{\exp} = -\frac{3}{2} \frac{v_2\{4\}^4 - 12v_2\{6\}^4 + 11v_2\{8\}^4}{\left[v_2\{2\}^2 - v_2\{4\}^2\right]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} \equiv \gamma_2$$

Kurtosis

$$\gamma_3^{\exp} = 6\sqrt{2} \frac{3v_2\{6\}^5 - 22v_2\{8\}^5 + 19v_2\{10\}^5}{\left[v_2\{2\}^2 - v_2\{4\}^2\right]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} \equiv \gamma_3$$

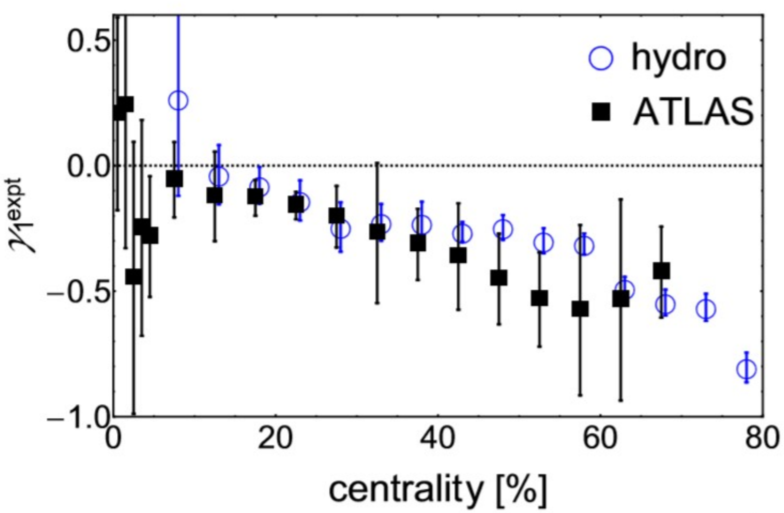
Superskewness

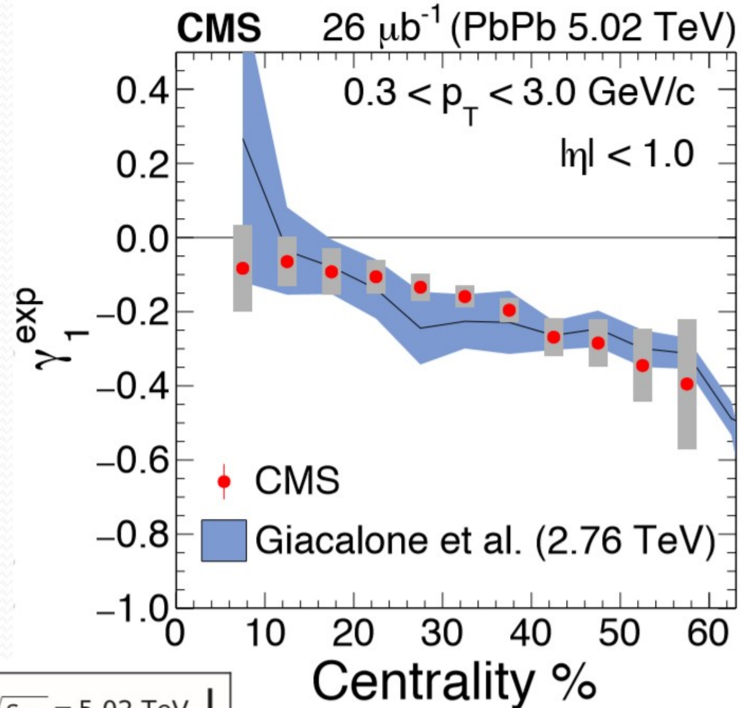
Conditions: $s_{12} \approx \frac{s_{30}}{3}$ $\kappa_{22} \approx \frac{\kappa_{40}}{3}$ $p_{32} \approx p_{14} \approx \frac{p_{50}}{5}$ Ell. pow. distr. param. $\varepsilon_0 \leq 0.15$

$$\gamma_{1,corr}^{\exp} = -2^{3/2} \frac{187v_2\{8\}^3 - 16v_2\{6\}^3 - 171v_2\{10\}^3}{\left[v_2\{2\}^2 - 40v_2\{6\}^2 + 495v_2\{8\}^2 - 456v_2\{10\}^2\right]^{3/2}}$$

Skewness, kurtosis and superskewness

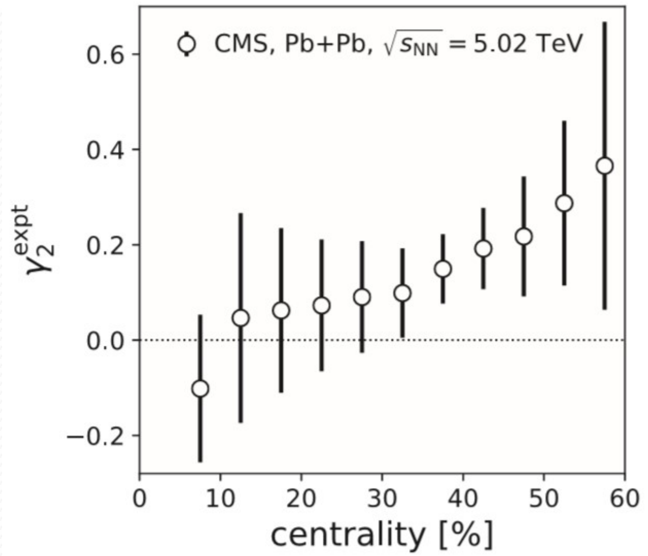
PHYSICAL REVIEW C **95**, 014913 (2017)



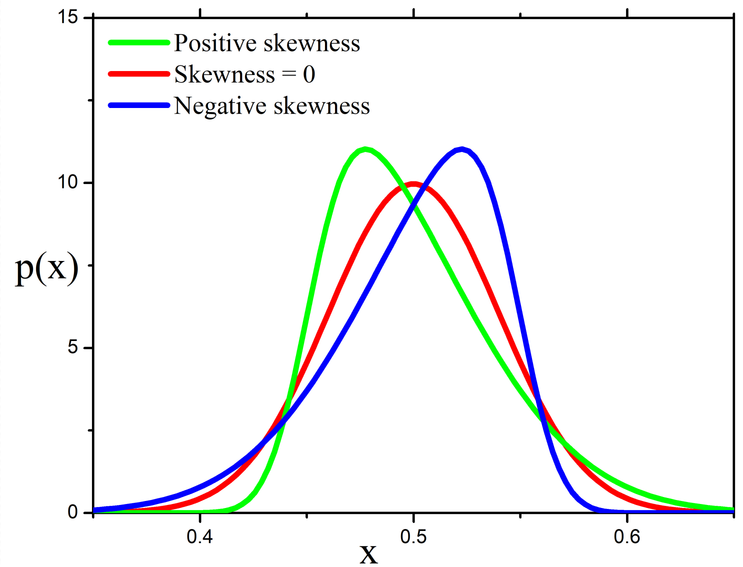


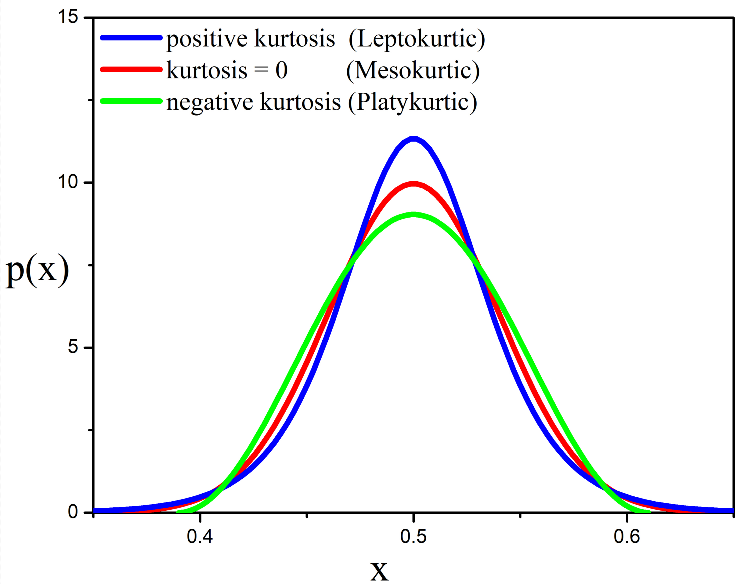
Phys. Lett. B **789** 643 (2019):

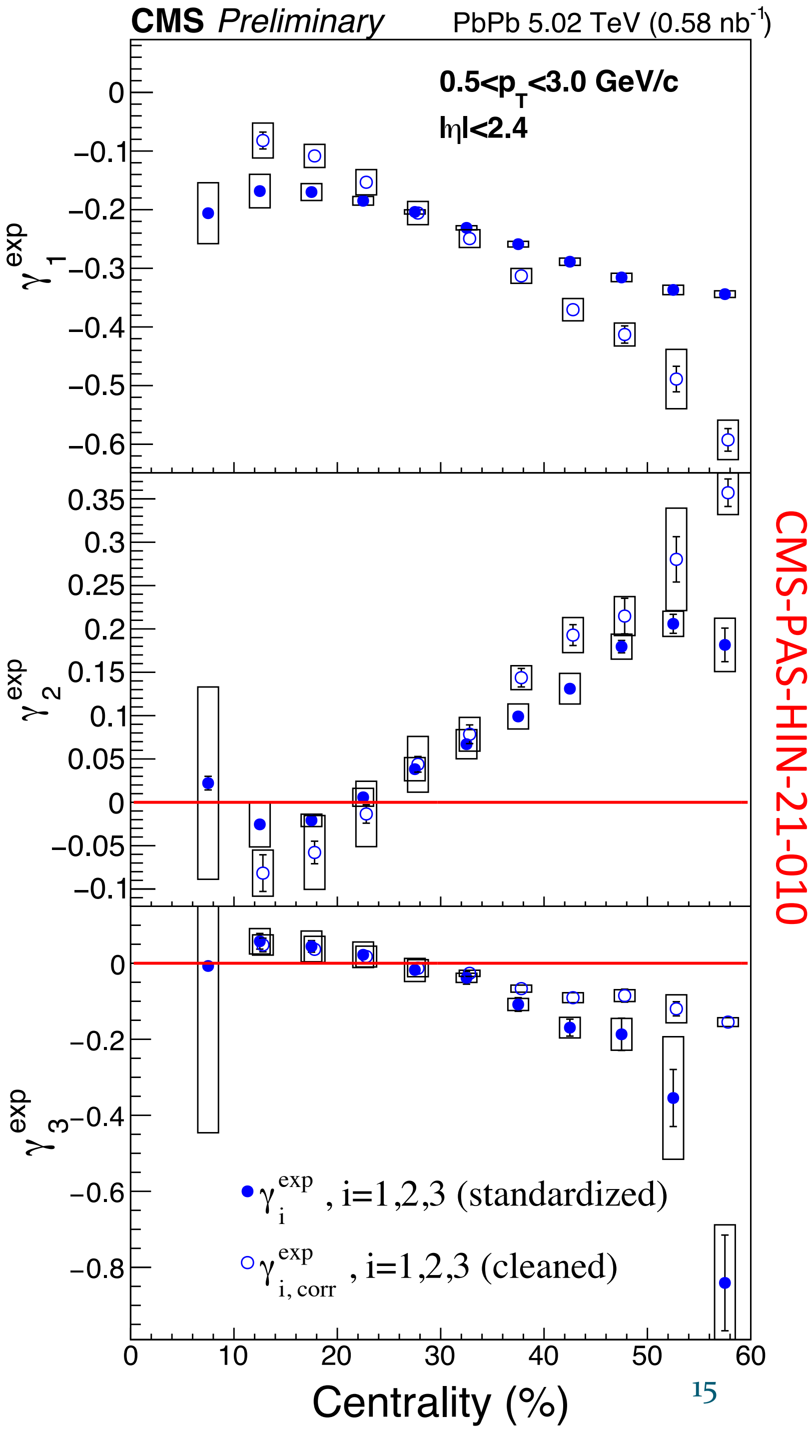
Physical Review C **99** 014907 (2019):



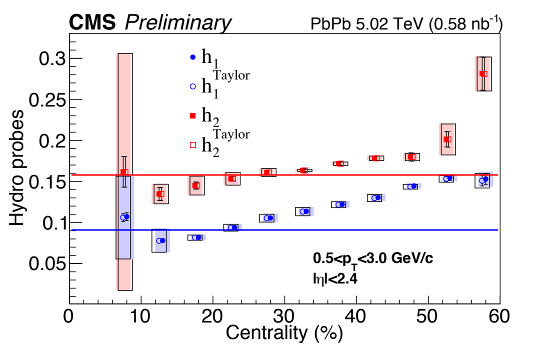
Skewness, kurtosis and superskewness

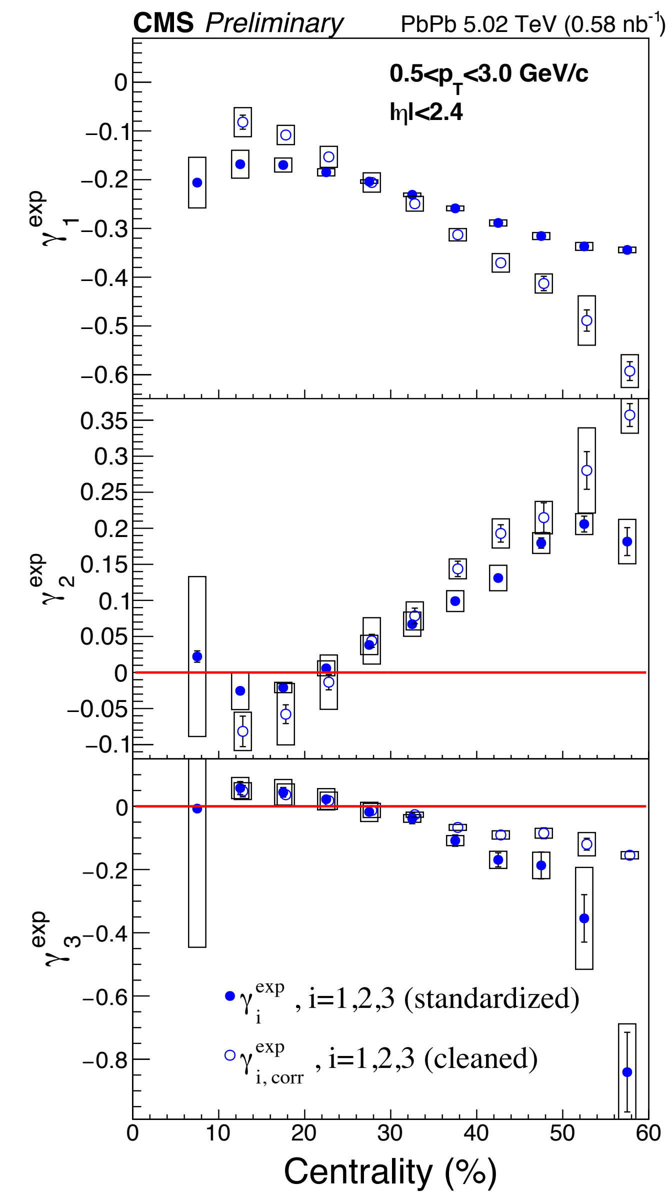






Conclusions





- Two hydrodynamic probes performed on CMS PbPb data at 5.02 TeV energy
- High precision measurement of skewness, kurtosis and superskewness of the v_2 distribution.
- These results can provide novel constraints on the initial state geometry used in hydrodynamic calculations of the medium expansion in high energy nuclear collisions

Backup

Ten-particle azimuthal angle correlation

- **The size of formula increase with the order *k***
- **Formula for the corresponding statistical uncertainties are even much bigger**
- **This is a kind of a disadvantage of the method because it is not easy to implement**

$$c_n\{10\} = \langle\langle 10\rangle\rangle - 25\cdot\langle\langle 2\rangle\rangle\langle\langle 8\rangle\rangle - 100\cdot\langle\langle 4\rangle\rangle\langle\langle 6\rangle\rangle + 400\cdot\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle^2 + 900\cdot\langle\langle 2\rangle\rangle\langle\langle 4\rangle\rangle^2 - 3600\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^3 + 2880\cdot\langle\langle 2\rangle\rangle^5$$

- **For the first time $v_n\{10\}$:**

$$v_n\{10\} = \sqrt[10]{\frac{1}{456}c_n\{10\}}$$

$$\begin{aligned}
<10> = &\frac{|Q_n|^{10} - 20Re[Q_{2n}|Q_n|^6 Q_n^* Q_n^*] + 100|Q_{2n}|^2|Q_n|^6 + 30Re[Q_{2n}Q_{2n}|Q_n|^2 Q_n^*Q_n^*Q_n^*Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{225|Q_{2n}|^4|Q_n|^2 - 300Re[Q_{2n}|Q_{2n}|^2|Q_n|^2 Q_n^*Q_n^*] + 40Re[Q_{3n}|Q_n|^4 Q_n^*Q_n^*Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{600Re[Q_{3n}Q_n|Q_n|^2 Q_{2n}^*Q_{2n}^*] - 400Re[Q_{3n}|Q_n|^4 Q_n^*Q_{2n}^*] + 400|Q_{3n}|^2|Q_n|^4}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{400Re[Q_{3n}|Q_{2n}|^2 Q_n^*Q_n^*Q_n^*] - 40Re[Q_{3n}Q_{2n}Q_n^*Q_n^*Q_n^*Q_n^*Q_n^*] - 600Re[Q_{3n}|Q_{2n}|^2 Q_n^*Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{400|Q_{3n}|^2|Q_{2n}|^2 - 800Re[|Q_{3n}|^2 Q_{2n}Q_n^*Q_n^*] - 60Re[Q_{4n}|Q_n|^2 Q_n^*Q_n^*Q_n^*Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{600Re[Q_{4n}|Q_n|^2 Q_n^*Q_n^*Q_{2n}^*] - 900Re[Q_{4n}|Q_n|^2 Q_{2n}^*Q_{2n}^*] - 1200Re[Q_{4n}|Q_n|^2 Q_n^*Q_{3n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{1200Re[Q_{4n}Q_nQ_{2n}^*Q_{3n}^*] + 900|Q_{4n}|^2|Q_n|^2 + 48Re[Q_{5n}Q_n^*Q_n^*Q_n^*Q_n^*Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{720Re[Q_{5n}Q_n^*Q_{2n}^*Q_{2n}^*] - 480Re[Q_{5n}Q_n^*Q_n^*Q_n^*Q_{2n}^*] + 960Re[Q_{5n}Q_n^*Q_n^*Q_{3n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{576|Q_{5n}|^2 - 960Re[Q_{5n}Q_{2n}^*Q_{3n}^*] - 1440Re[Q_{5n}Q_n^*Q_{4n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\
&+\frac{300Re[Q_{2n}|Q_n|^4 Q_n^*Q_n^*] - 25|Q_n|^8 - 900|Q_{2n}|^2|Q_n|^4 - 150Re[Q_{2n}Q_{2n}Q_n^*Q_n^*Q_n^*Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\
&+\frac{900Re[Q_{2n}|Q_{2n}|^2 Q_n^*Q_n^*] - 225|Q_{2n}|^4 - 400Re[Q_{3n}|Q_n|^2 Q_n^*Q_n^*Q_n^*] + 2400Re[Q_{3n}|Q_n|^2 Q_n^*Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\
&+\frac{300Re[Q_{4n}Q_n^*Q_n^*Q_n^*Q_n^*] - 1200Re[Q_{3n}Q_nQ_{2n}^*Q_{2n}^*] - 1600|Q_{3n}|^2|Q_n|^2 - 1800Re[Q_{4n}Q_n^*Q_n^*Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\
&+\frac{900Re[Q_{4n}Q_{2n}^*Q_{2n}^*] + 2400Re[Q_{4n}Q_n^*Q_{3n}^*] - 900|Q_{4n}|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\
&+\frac{200|Q_n|^6 - 1200Re[Q_{2n}|Q_n|^2 Q_n^*Q_n^*] + 1800|Q_{2n}|^2|Q_n|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-7)(M-8)} + \\
&+\frac{800Re[Q_{3n}Q_n^*Q_n^*Q_n^*] - 2400Re[Q_{3n}Q_n^*Q_{2n}^*] + 800|Q_{3n}|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-7)(M-8)} + \\
&+\frac{1200Re[Q_{2n}Q_n^*Q_n^*] - 600|Q_n|^4 - 600|Q_{2n}|^2}{M(M-1)(M-2)(M-3)(M-5)(M-6)(M-7)} + \\
&+\frac{600|Q_n|^2}{M(M-1)(M-3)(M-4)(M-5)(M-6)} - \frac{120}{(M-1)(M-2)(M-3)(M-4)(M-5)}
\end{aligned}$$

Statistical uncertainty of the $v_n\{10\}$

- Statistical uncertainties of the $v_n\{10\}$ cumulant is calculated analytically using the data [the same approach as in Phys. Rev. C 104 (2021) 034906, arXiv: 2104.00588]

$$s^2[v_n\{10\}]\cdot 4560^2 (v_n\{10\})^{18} = A^2\sigma^2_{\langle\langle 2\rangle\rangle} + B^2\sigma^2_{\langle\langle 4\rangle\rangle} + C^2\sigma^2_{\langle\langle 6\rangle\rangle} + D^2\sigma^2_{\langle\langle 8\rangle\rangle} + \sigma^2_{\langle\langle 10\rangle\rangle} + 2AB\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 4\rangle\rangle} + 2AC\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 6\rangle\rangle} + 2AD\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 8\rangle\rangle} + 2A\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 10\rangle\rangle} + 2BC\sigma_{\langle\langle 4\rangle\rangle,\langle\langle 6\rangle\rangle} + 2BD\sigma_{\langle\langle 4\rangle\rangle,\langle\langle 8\rangle\rangle} + 2B\sigma_{\langle\langle 4\rangle\rangle,\langle\langle 10\rangle\rangle} + 2CD\sigma_{\langle\langle 6\rangle\rangle,\langle\langle 8\rangle\rangle} + 2C\sigma_{\langle\langle 6\rangle\rangle,\langle\langle 10\rangle\rangle} + 2D\sigma_{\langle\langle 8\rangle\rangle,\langle\langle 10\rangle\rangle}$$

$$A = 14400\langle\langle 2\rangle\rangle^4 - 10800\langle\langle 2\rangle\rangle^2\langle\langle 4\rangle\rangle + 800\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle + 900\langle\langle 4\rangle\rangle^2 - 25\langle\langle 8\rangle\rangle$$

$$B = 1800\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle - 3600\langle\langle 2\rangle\rangle^3 - 100\langle\langle 6\rangle\rangle$$

$$C = 400\langle\langle 2\rangle\rangle^2 - 100\langle\langle 4\rangle\rangle$$

$$D = -25\langle\langle 2\rangle\rangle$$

- With variances

$$\sigma^2_{\bar{x}_w} = \frac{N}{N-1}\frac{\sum_{i=1}^{N} w_i^2\left[x_i - \bar{x}_w\right]^2}{\left(\sum_{i=1}^{N} w_i\right)^2}$$

← weights have to be squared!

and covariances

$$\sigma_{\bar{x}_w,\bar{y}_w} = \frac{N}{N-1}\frac{\sum_{i=1}^{N} w_i^x w_i^y\left[x_i - \bar{x}_w\right]\left[y_i - \bar{y}_w\right]}{\sum_{i=1}^{N} w_i^x \sum_{i=1}^{N} w_i^y}$$

The simplest case for the efficiency corrections within Q-cumulants

Formalism developed by A. Bilandzic

[Phys. Rev. C 83 (2011) 044913, and PhD thesis, Utrecht Uni. 2012]

Example of the simplest case of two particle correlation:

Without efficiency corrections

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

With efficiency corrections

$$\langle 2 \rangle = \frac{|Q_{n,1}|^2 - S_{1,2}}{S_{2,1} - S_{1,2}}$$

Where M is the number of the tracks, while $S_{p,k} = [\sum_{i=1}^{M} w_i^k]^p$

$$Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$

$$Q_{n,k} = \sum_{i=1}^{M} w_i^k e^{in\phi_i}$$

Taylor expansion of the hydrodynamics probe

Expansion up to 4[th] moment i.e. up to kurtosis

- Moments of the second order $\sigma_x^2 = \left\langle \left(v_x - \langle v_x \rangle\right)^2 \right\rangle$ $\sigma_y^2 = \left\langle v_y^2 \right\rangle$
- Moments of the third order $s_{30} = \left\langle \left(v_x - \langle v_x \rangle\right)^3 \right\rangle$ $s_{12} = \left\langle \left(v_x - \langle v_x \rangle\right) v_y^2 \right\rangle$
- Moments of the fourth order $\kappa_{40} = \left\langle \left(v_x - \langle v_x \rangle\right)^4 \right\rangle - 3\sigma_x^4$ $\kappa_{22} = \left\langle \left(v_x - \langle v_x \rangle\right)^2 v_y^2 \right\rangle - \sigma_x^2 \sigma_y^2$

$$v_2\{4\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_{30} + s_{12}}{\bar{v}_2^2} - \frac{\kappa_{40} + 2\kappa_{22} + \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(3s_{30} + 3s_{12}\right)}{2\bar{v}_2^4}$$

$$v_2\{6\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{2}{3}s_{30} + s_{12}}{\bar{v}_2^2} + \frac{\kappa_{40} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{p_{50} + 2p_{32} + p_{14}}{4\bar{v}_2^4} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(4s_{30} + 15s_{12}\right)}{6\bar{v}_2^4}$$

$$v_2\{8\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{7}{11}s_{30} + s_{12}}{\bar{v}_2^2} + \frac{\frac{31}{33}\kappa_{40} + \frac{2}{11}\kappa_{22} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{\frac{5}{3}p_{50} + \frac{14}{3}p_{32} + 3p_{14}}{11\bar{v}_2^4} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(13s_{30} + 57s_{12}\right)}{22\bar{v}_2^4}$$

$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11}\left(1 - \frac{4\kappa_{40} + \frac{8(p_{50} + p_{32})}{\bar{v}_2}}{2\bar{v}_2 s_{30} + 3(\kappa_{40} + \kappa_{22}) + \frac{3(p_{50} + 2p_{32} + p_{14}) - 2(\sigma_y^2 - \sigma_x^2)(5s_{30} - 6s_{12})}{2\bar{v}_2}}\right)$$

- The hydro probe is then

$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} = \frac{1}{11} - \frac{1}{11}\frac{v_2\{4\}^2 - 12v_2\{6\}^2 + 11v_2\{8\}^2}{v_2\{4\}^2 - v_2\{6\}^2 + \frac{(\sigma_y^2 - \sigma_x^2)s_{30}}{3\bar{v}_2^3}}$$

negligible

Taylor expansion of the new hydrodynamics probe

In this case, expansion up to 5th moment

- Moments of the fifth order $p_{50} = \left\langle \left(v_x - \langle v_x \rangle\right)^5 \right\rangle - 10\sigma_x^2 s_{30}$

$$p_{32} = \left\langle \left(v_x - \langle v_x \rangle\right)^3 v_y^2 \right\rangle - \sigma_y^2 s_{30} - 3\sigma_x^2 s_{12} \qquad p_{14} = \left\langle \left(v_x - \langle v_x \rangle\right) v_y^4 \right\rangle - 6\sigma_y^2 s_{12}$$

$$v_2\{10\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{12}{19}s_{30} + s_{12}}{\bar{v}_2^2} + \frac{\frac{53}{57}\kappa_{40} + \frac{4}{19}\kappa_{22} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{\frac{163}{60}p_{50} + \frac{47}{6}p_{32} + \frac{21}{4}p_{14}}{19\bar{v}_2^4} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(11s_{30} + \frac{99}{2}s_{12}\right)}{19\bar{v}_2^4}$$

$$\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx \frac{3}{19} - \frac{88p_{50}}{95\left[4\bar{v}_2^2 s_{30} - 2\bar{v}_2(\kappa_{40} - 3\kappa_{22}) - 13(p_{50} + 10p_{32} - 3p_{14}) - 2(\sigma^2 - \sigma_x^2)(5s_{30} - 6s_{32})\right]}$$

- The new hydro probe is then given as

$$\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} = \frac{3}{19} - \frac{1}{19}\frac{3v_2\{6\}^2 - 22v_2\{8\}^2 + 19v_2\{10\}^2}{v_2\{6\}^2 - v_2\{8\}^2 + \frac{(\sigma_y^2 - \sigma_x^2)s_{30}}{33\bar{v}_2^3}}$$

negligible

Cleaned moments

- Kurtosis:

$$O_N = \frac{16(p_{50}+p_{32})}{3\bar{v}_2}$$

$$\gamma_2^{\exp} \equiv -\frac{3}{2}\frac{v_2^{\,4}\{4\}-12v_2^{\,4}\{6\}+11v_2^{\,4}\{8\}}{\left[v_2\{2\}^2-v_2\{4\}^2\right]^2} \approx -\frac{3}{2}\frac{-\frac{8\kappa_{40}}{3}-O_N}{\left[2\sigma_x^2+O_D\right]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} = \gamma_2$$

- Cleaned kurtosis:

$$\gamma_{2,corr}^{\exp} \equiv -\frac{3}{2}\frac{v_2^{\,4}\{4\}+24v_2^{\,4}\{6\}-253v_2^{\,4}\{8\}+228v_2^{\,4}\{10\}}{\left[v_2\{2\}^2-40v_2\{6\}^2+495v_2\{8\}^2-456v_2\{10\}^2\right]^2} \approx -\frac{3}{2}\frac{-\frac{8\kappa_{40}}{3}}{\left[2\sigma_x^2+\bar{O}_D\right]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} = \gamma_2$$

$$\bar{O}_D = -\frac{2(\sigma_y^2-\sigma_x^2)(s_{30}-s_{12})}{\bar{v}_2^3} + \cdots$$

- Superskewness:

$$\gamma_3^{\exp} \equiv 6\sqrt{2}\,\frac{3v_2\{6\}^5-22v_2\{8\}^5+19v_2\{10\}^5}{\left[v_2\{2\}^2-v_2\{4\}^2\right]^{5/2}} \approx 6\sqrt{2}\,\frac{\frac{2}{3}p_{50}}{\left[2\sigma_x^2+O_D\right]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} = \gamma_3$$

- Cleaned Superskewness:

$$\gamma_{3,corr}^{\exp} = 6\sqrt{2}\,\frac{3v_2^{\,5}\{6\}-22v_2^{\,5}\{8\}+19v_2^{\,5}\{10\}}{\left[v_2\{2\}^2-40v_2\{6\}^2+495v_2\{8\}^2-456v_2\{10\}^2\right]^{5/2}} \approx 6\sqrt{2}\,\frac{\frac{2}{3}p_{50}}{\left[2\sigma_x^2+\bar{O}_D\right]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} = \gamma_3$$