

Quantum-orbit theory in strong-laser-field physics

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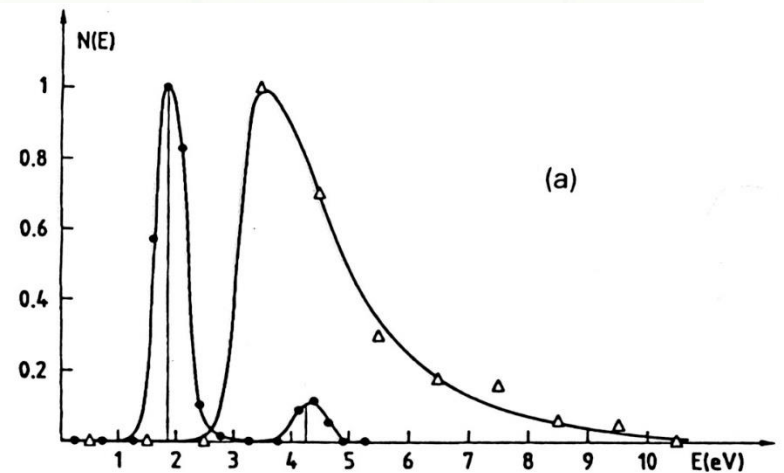
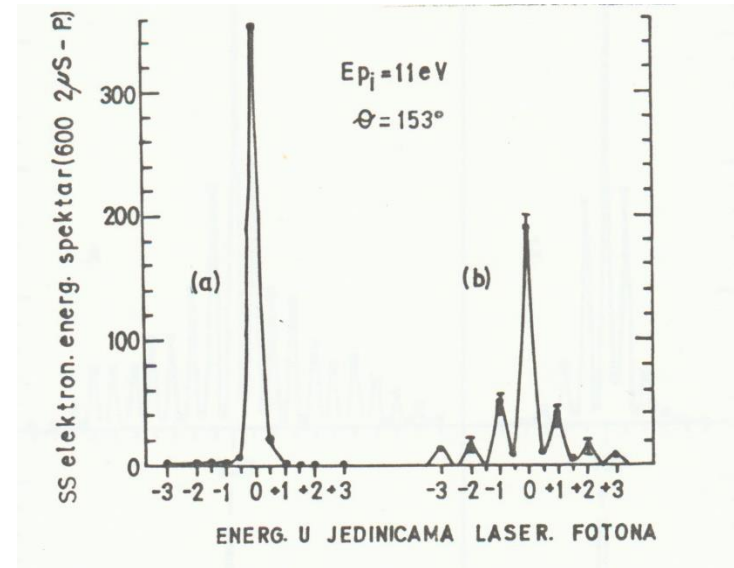
Beograd, August 30, 2022

Outline

- Strong-laser-field processes
- High-order processes, 3-step model
- Feynman path integral, strong-field approximation, quantum-orbit theory
- Examples - linear polarization
 - direct electrons, double-slit exp. in time
 - rescattered electrons, high-order ATI
- Examples - elliptical polarization
 - multiplateau structures
 - negative travel time
- Examples - tailored (complex) fields
 - bicircular, BEOTC fields (HHG, HATI)

STRONG-FIELD PROCESSES

- **Laser-assisted processes**
 - Electron-atom scattering
(Weingartshofer et al. 1977)
 - X-ray-atom scattering
 - Electron-ion recombination
- **Laser-induced processes**
 - Above-threshold ionization (ATI)
(Agostini et al. 1979)
 - Above-threshold detachment
 - High-harmonic generation (HHG)
 - Non-sequential multiple ionization



High-order ATI (Paulus et al. 1993)

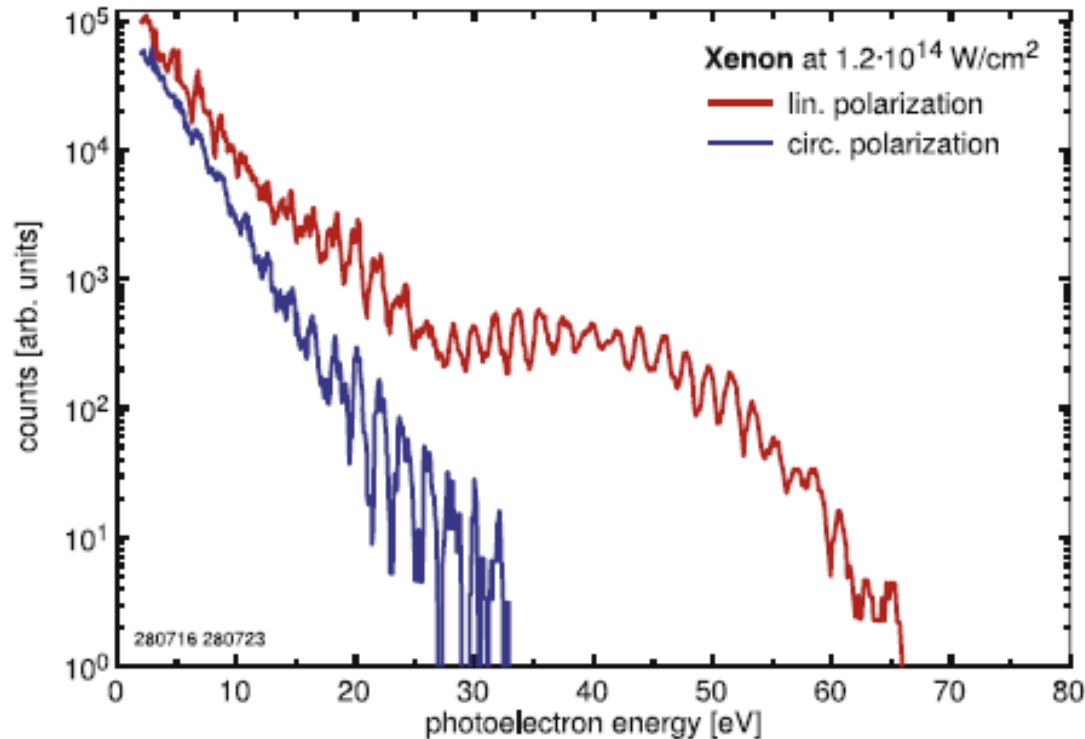
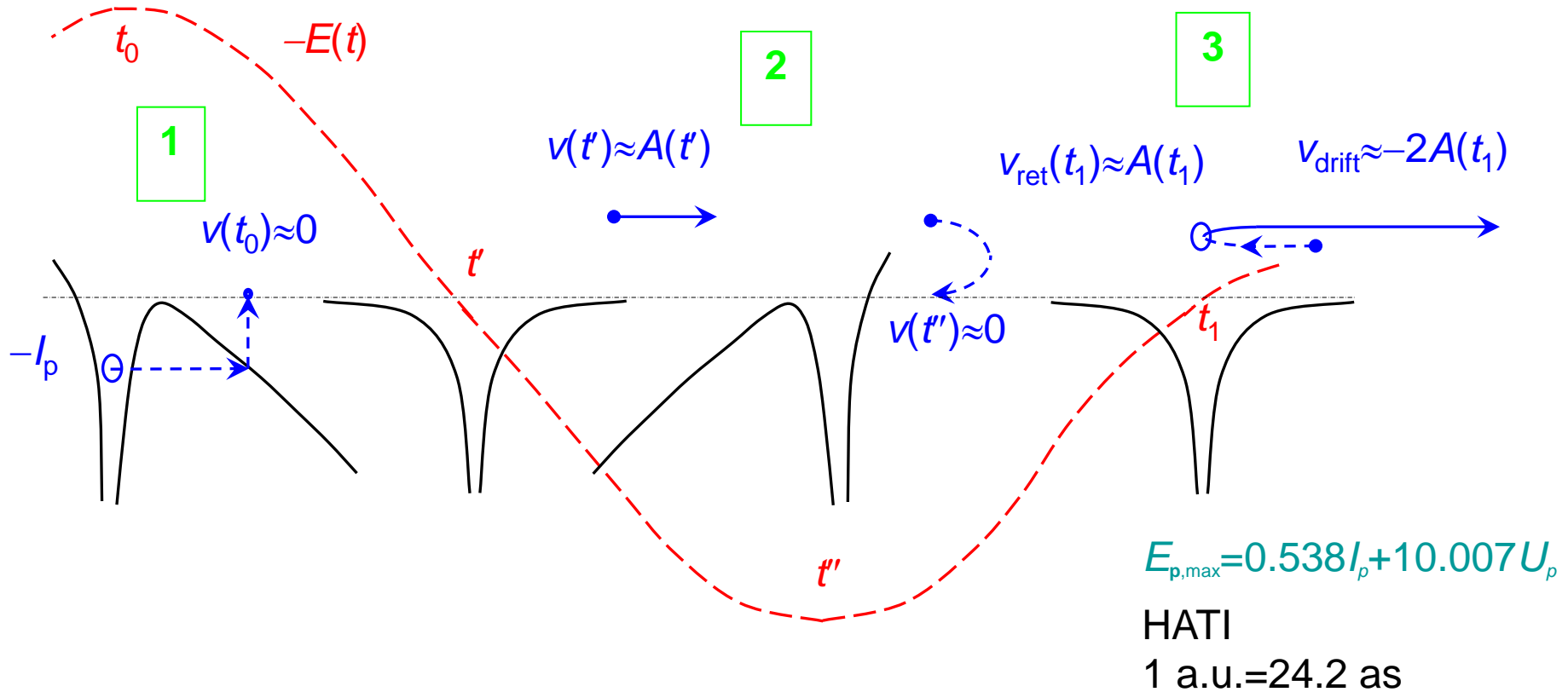


Figure 1. ATI spectra recorded in July 1993 using a femtosecond laser system consisting of a dye colliding-pulse mode-locked oscillator and a dye amplifier pumped by a copper-vapor laser. The wavelength was 630 nm, the pulse duration 40 fs, and the intensity corresponded to $U_p = 4.5$ eV. Besides the ATI plateau and its disappearance for circular polarization, also some of the effects

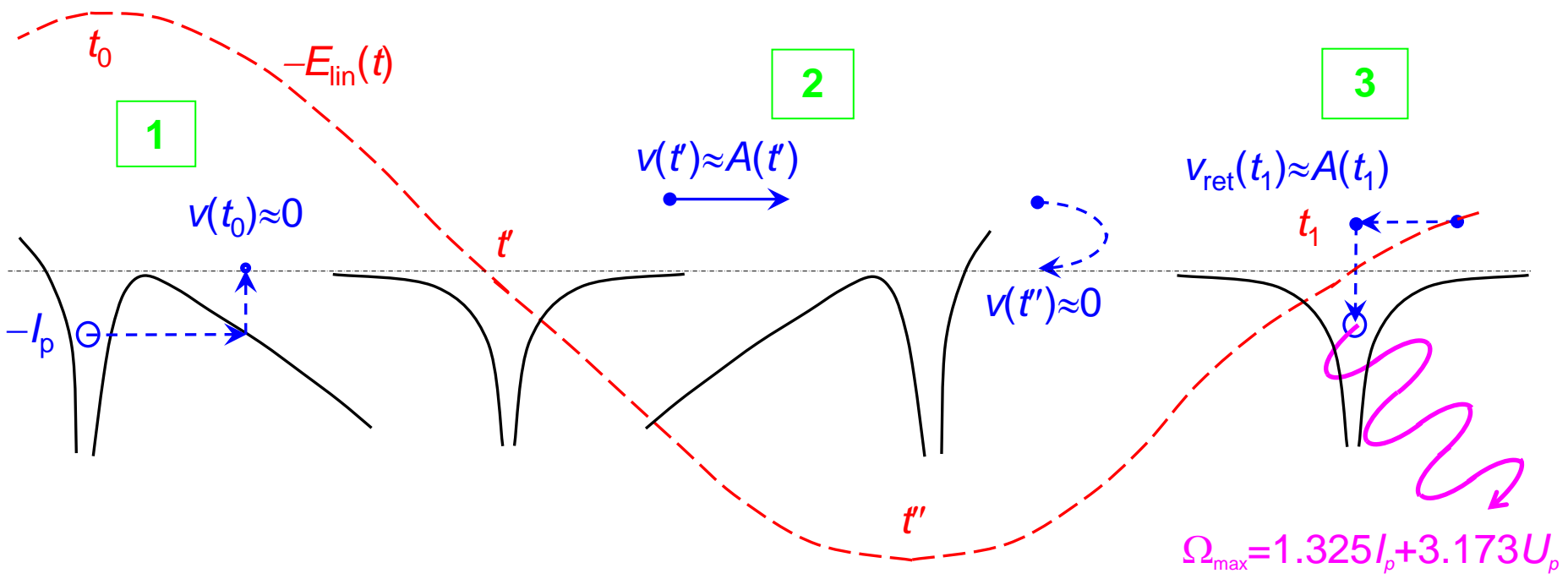
W. Becker, S. Goreslavski, D. B. Milošević, and G. G. Paulus,
The plateau in above-threshold ionization: the keystone of rescattering physics
J. Phys. B **51**, 162002 (2018) Topical Review

Three-step model (1993)



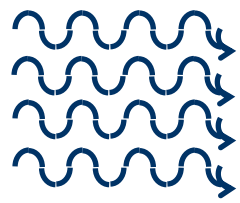
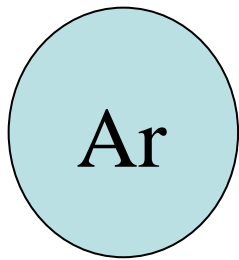
- Recollision during small part of the optical cycle → Attoscience
- Linearly polarized laser field → linear trajectories (1D), high-harmonics lin. pol.

High-order harmonic generation (1987)

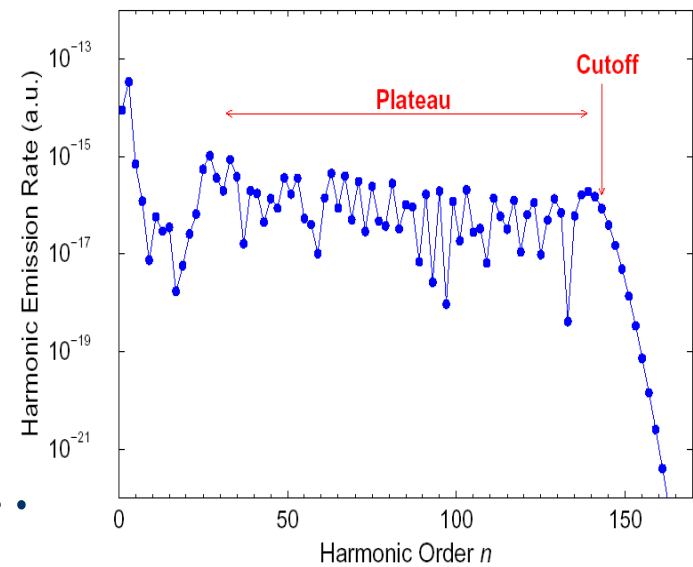


Strong Laser Field

$I > 10^{14} \text{ W/cm}^2, \omega$



$3\omega, 5\omega, 7\omega, \dots$



S-matrix theory

$$M_{fi} = \lim_{t \rightarrow \infty, t' \rightarrow -\infty} \langle \psi_f(t) | \hat{U}(t, t') | \psi_i(t') \rangle$$

SFA (Strong Field Approximation) \Rightarrow

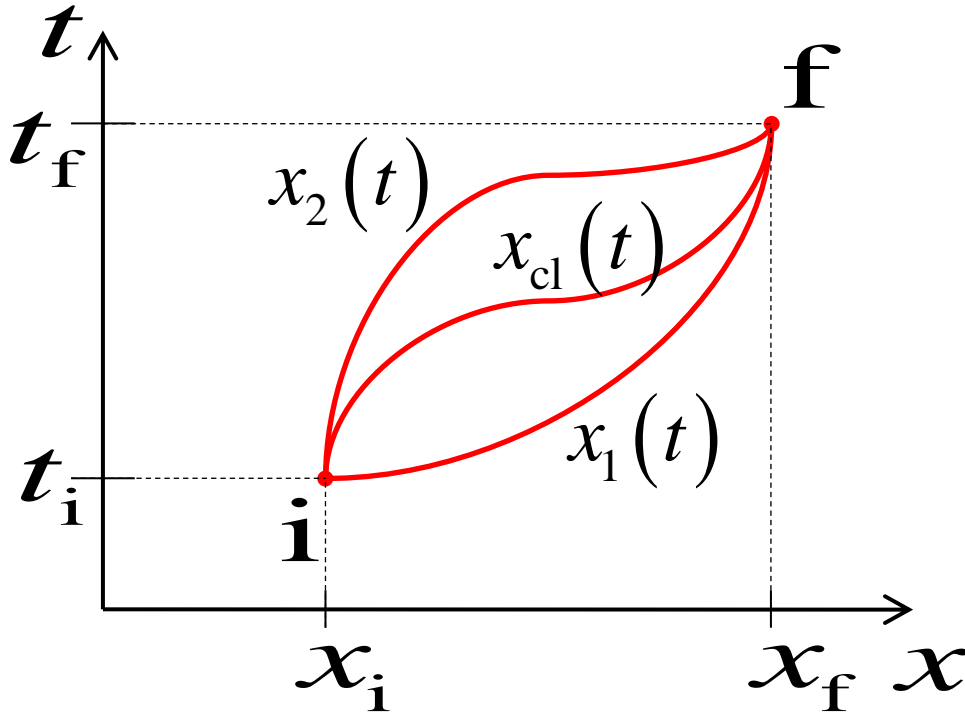
$$M_{fi} \propto \int_{-\infty}^{\infty} dt_f \int d^3 \vec{p} \int_{-\infty}^{t_f} dt_i \langle \psi_f | H_f U_{fi}^{(L)} H_i | \psi_i \rangle e^{iS(t_i, t_f; \vec{p})/\hbar}$$

SPM (Saddle Point Method) \Rightarrow

$$\frac{\partial S}{\partial t_i} = \frac{\partial S}{\partial \vec{p}} = \frac{\partial S}{\partial t_f} = 0 \Rightarrow M_{fi} \propto \sum_{\text{relevant paths: quantum orbits}} A_{fis} \exp(i\Phi_{fis})$$

t_i, t_f, \vec{p} complex

FEYNMAN'S PATH INTEGRAL



$$M_{fi} \propto \sum_{\text{all paths from } i \text{ to } f} e^{iS[x(t)]/\hbar}$$

$$S = \int_{t_i}^{t_f} dt L(x(t), \dot{x}(t), t)$$

Classical limit: $S \gg \hbar$, Hamilton principle: $\delta S = 0$

However, the problem is that the connection with path integrals is made by intuition and analogy, i.e., not by an explicit use of the path-integral formalism.

For details, see: D. B. Milošević, J. Math. Phys. **54**, 042101 (2013)

Phase space path-integral formalism

The momentum-space matrix element of the total time-evolution operator is sliced into $N + 1$ time-evolution operators, each acting across an infinitesimal time slice of width $\varepsilon = t_n - t_{n-1} = (T_p - \tau)/(N + 1)$. We denote $\mathbf{p}' = \mathbf{p}_{N+1}$, $\mathbf{p}'' = \mathbf{p}_0$, $t_{N+1} = T_p$, $t_0 = \tau$, and obtain

$$\begin{aligned}\langle \mathbf{p}' | U(T_p, \tau) | \mathbf{p}'' \rangle &= \prod_{n=1}^N \left[\int d\mathbf{p}_n \right] \prod_{n=1}^{N+1} \left[\int \frac{d\mathbf{r}_n}{(2\pi)^3} \right] e^{iA_N}, \\ &= \int_{(\mathbf{p}'', \tau) \rightarrow (\mathbf{p}', T_p)} \mathcal{D}'\mathbf{p} \int \frac{\mathcal{D}\mathbf{r}}{(2\pi)^3} e^{i\mathcal{A}[\mathbf{p}, \mathbf{r}]},\end{aligned}$$

Classical canonical action for the paths $\mathbf{r}(t)$, $\mathbf{p}(t)$ in phase space

$$\begin{aligned}\mathcal{A}[\mathbf{p}, \mathbf{r}] &= \int_{\tau}^{T_p} dt \left[-\dot{\mathbf{p}}(t) \cdot \mathbf{r}(t) - H(\mathbf{p}, \mathbf{r}, t) \right], \\ H(\mathbf{p}, \mathbf{r}, t) &= \mathbf{p}^2(t)/2 + \mathbf{r}(t) \cdot \mathbf{E}(t) + V(\mathbf{r}(t)).\end{aligned}$$

Total Hamiltonian: $H(t)=H_A+V_L(t)$, $H_A=H_0+V(\mathbf{r})$, $V_L(t)=\mathbf{r}\cdot\mathbf{E}(t)$

Expansion in powers of the atomic potential $V(\mathbf{r})$

⇒ Strong-Field Approximation (SFA)

$$\langle \mathbf{p}' | U(T_p, \tau) | \mathbf{p}'' \rangle = \sum_{m=0}^{\infty} U_{\mathbf{p}'\mathbf{p}''}^{(m)}(T_p, \tau)$$

$$U_{\mathbf{p}'\mathbf{p}''}^{(m)}(T_p, \tau) = (-i)^m \prod_{j=1}^m \left[\int d\mathbf{k}_j V(\mathbf{k}_j) \int_{\tau_{j-1}}^{T_p} d\tau_j \right] \delta \left(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}}'' + \sum_{j=1}^m \mathbf{k}_j \right) \\ \times \exp \left\{ -\frac{i}{2} \sum_{l=0}^m \int_{\tau_l}^{\tau_{l+1}} dt \left[\tilde{\mathbf{p}}' + \mathbf{A}(t) + \sum_{j=l+1}^m \mathbf{k}_j \right]^2 \right\},$$

D. B. Milošević, J. Math. Phys. **54**, 042101 (2013)

Detailed derivation in App. A in: Phys. Rev. A **96**, 023413 (2017)

$m = 0$: standard SFA

$$U_{\mathbf{p}'\mathbf{p}''}^{(0)}(T_p, \tau) = \delta(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}}'') \exp(i\mathcal{A}^{(0)}), \quad \mathcal{A}^{(0)} = - \int_{\tau}^{T_p} \frac{dt}{2} [\tilde{\mathbf{p}}' + \mathbf{A}(t)]^2.$$

$m = 1$: improved SFA (rescattering)

$$U_{\mathbf{p}'\mathbf{p}''}^{(1)}(T_p, \tau) = -i V(\tilde{\mathbf{p}}'' - \tilde{\mathbf{p}}') \int_{\tau}^{T_p} d\tau_1 e^{i\mathcal{A}^{(1)}},$$
$$\mathcal{A}^{(1)} = - \int_{\tau_1}^{T_p} \frac{dt}{2} [\tilde{\mathbf{p}}' + \mathbf{A}(t)]^2 - \int_{\tau}^{\tau_1} \frac{dt}{2} [\tilde{\mathbf{p}}'' + \mathbf{A}(t)]^2.$$

$m = 2$: double rescattering SFA

$$U_{\mathbf{p}'\mathbf{p}''}^{(2)}(T_p, \tau) = (-i)^2 \int d\mathbf{k} V(\tilde{\mathbf{p}}'' - \mathbf{k}) V(\mathbf{k} - \tilde{\mathbf{p}}') \int_{\tau}^{T_p} d\tau_1 \int_{\tau_1}^{T_p} d\tau_2 e^{i\mathcal{A}^{(2)}},$$
$$\mathcal{A}^{(2)} \equiv - \int_{\tau_2}^{T_p} \frac{dt}{2} [\tilde{\mathbf{p}}' + \mathbf{A}(t)]^2 - \int_{\tau_1}^{\tau_2} \frac{dt}{2} [\mathbf{k} + \mathbf{A}(t)]^2 - \int_{\tau}^{\tau_1} \frac{dt}{2} [\tilde{\mathbf{p}}'' + \mathbf{A}(t)]^2.$$

Phase space path-integral formalism - semiclassical approximation

Expansion in powers of fluctuations around classical trajectories

$$\delta \mathbf{r}(t) = \mathbf{r}(t) - \mathbf{r}_{\text{cl}}(t), \quad \delta \mathbf{p}(t) = \mathbf{p}(t) - \mathbf{p}_{\text{cl}}(t).$$

$$\langle \mathbf{p}' | U(T_p, \tau) | \mathbf{p}'' \rangle^{(\text{scl})} = \sum_{\text{traj}} F \exp(i \mathcal{A}^{(\text{cl})}),$$

$$\mathcal{A}^{(\text{cl})} = - \int_{\tau}^{T_p} dt \left[\frac{\mathbf{p}_{\text{cl}}^2(t)}{2} + V(\mathbf{r}_{\text{cl}}(t)) - \mathbf{r}_{\text{cl}}(t) \cdot \nabla V(\mathbf{r}_{\text{cl}}(t)) \right]$$

F - quadratic fluctuation integral

[App. B in: D. B. Milošević, Phys. Rev. A **96**, 023413 (2017)]

Final momenta $\mathbf{p}' = \mathbf{p}_{\text{cl}}(T_p)$, initial momenta $\mathbf{p}'' = \mathbf{p}_{\text{cl}}(\tau)$:

$$\mathbf{p}' - \mathbf{A}(T_p) - \mathbf{p}'' + \mathbf{A}(\tau) + \int_{\tau}^{T_p} dt \nabla V(\mathbf{r}_{\text{cl}}(t)) = \mathbf{0}$$

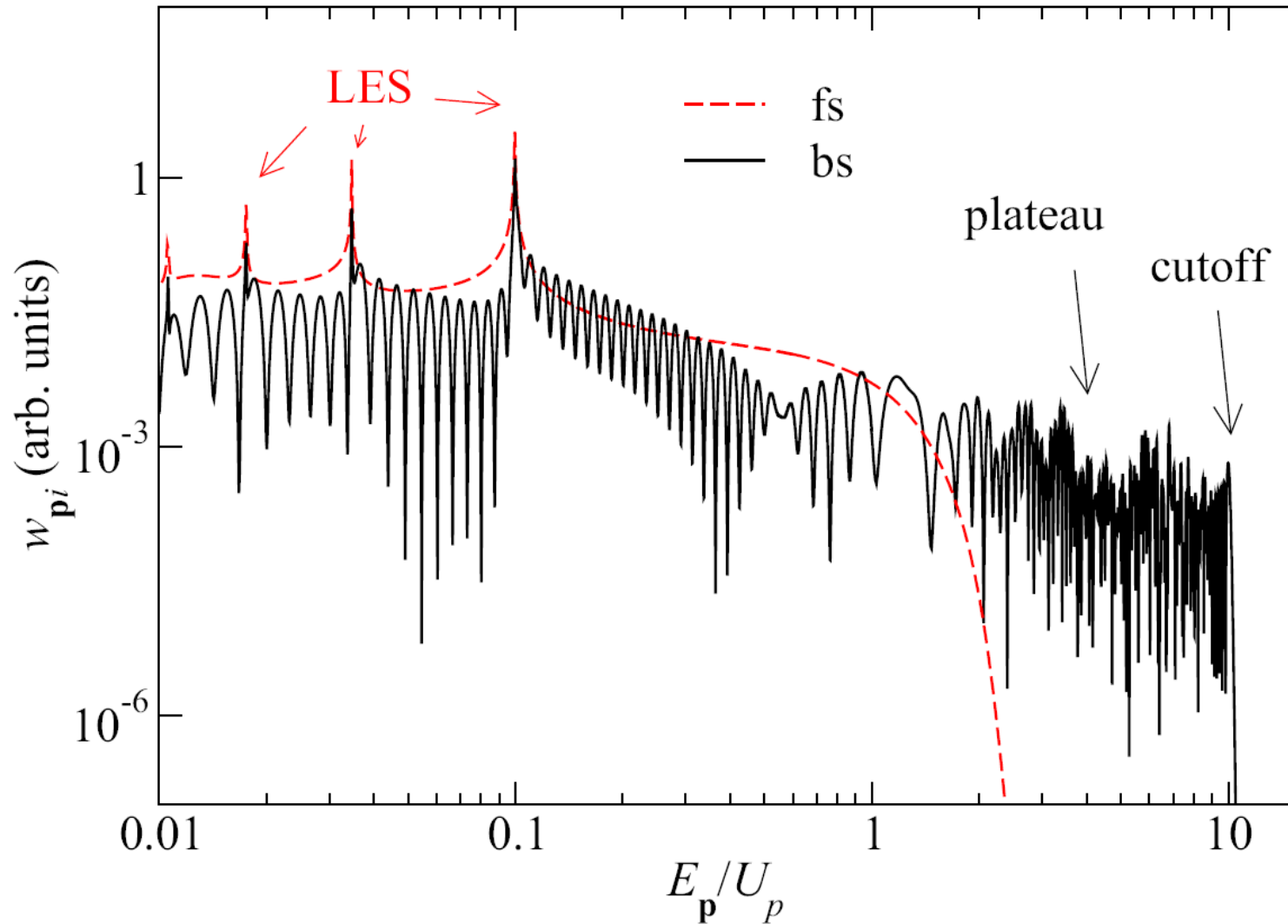
SPM: $M_{\text{fi}}^{(1)} \propto \sum_{\text{relevant paths: quantum orbits } s} A_{\text{fis}} \exp(i\Phi_{\text{fis}}), \quad t_i, t_f, \vec{k} \in \mathcal{C}$

$$(I) \quad \frac{\partial S}{\partial t_i} = 0 \quad \Rightarrow \quad -I_P = \frac{1}{2} [\vec{k} + \vec{A}(t_i)]^2$$

$$(II) \quad \frac{\partial S}{\partial \vec{k}} = \vec{0} \quad \Rightarrow \quad (t_f - t_i) \vec{k} = \int_{t_i}^{t_f} \vec{A}(\tau) d\tau \quad \Leftrightarrow \quad \vec{r}(t_f) = \vec{r}(t_i)$$

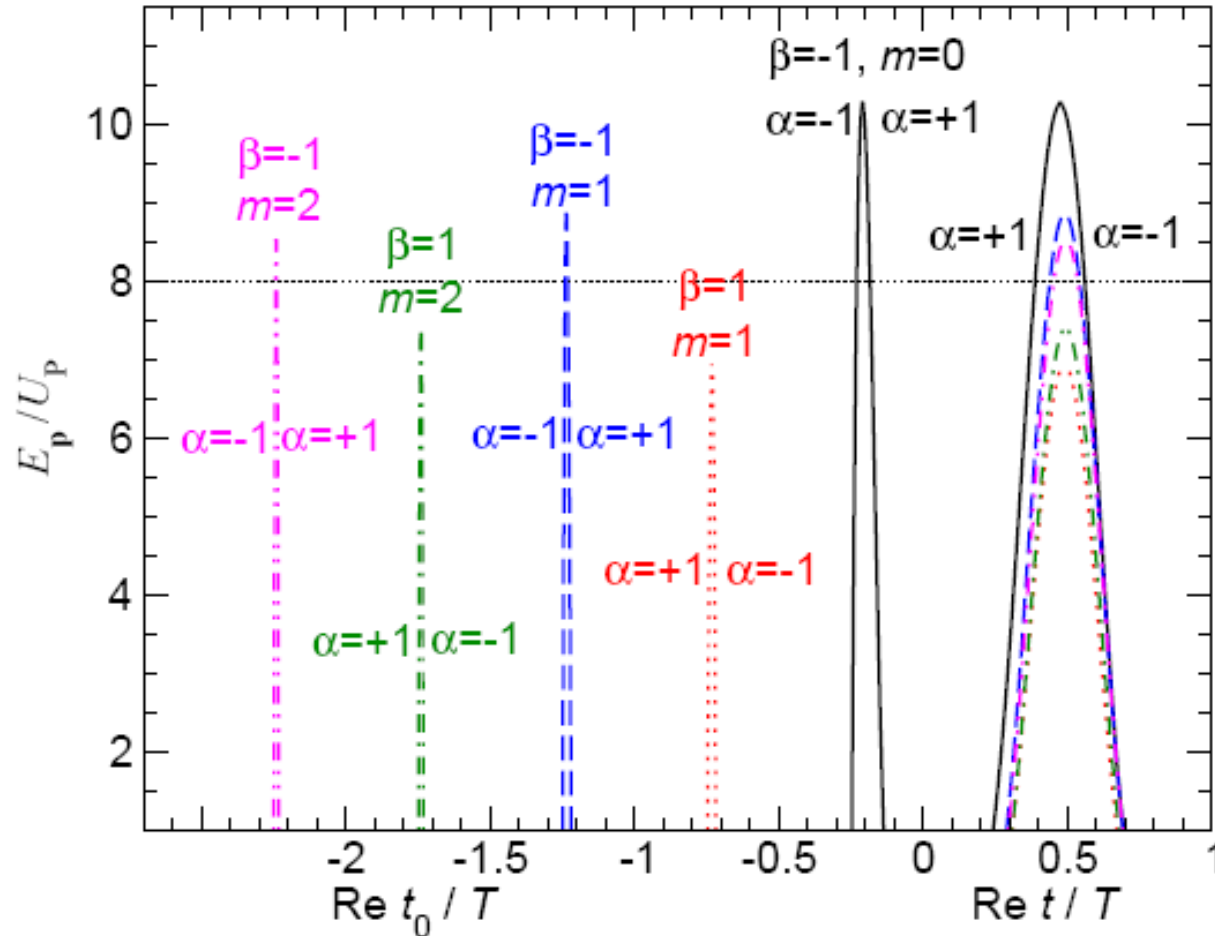
$$(III) \quad \frac{\partial S}{\partial t_f} = 0 \quad \Rightarrow \quad \frac{1}{2} [\vec{k} + \vec{A}(t_f)]^2 = \frac{1}{2} [\vec{p}_f + \vec{A}(t_f)]^2$$

Forward- and backward-scattering in strong-field ionization



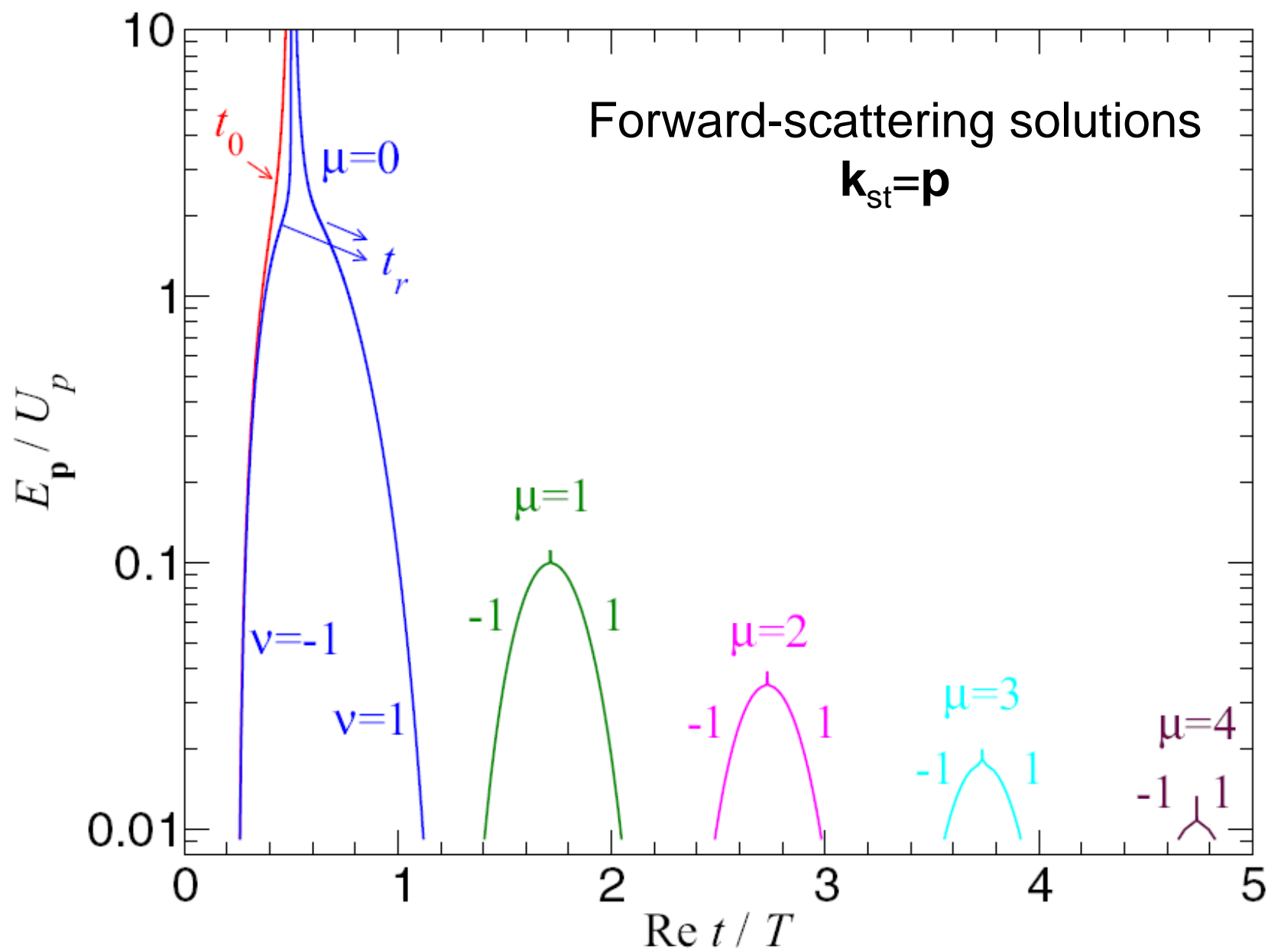
SPM for HATI – quantum-orbit theory

Classification of the saddle-point solutions: $\alpha\beta m$

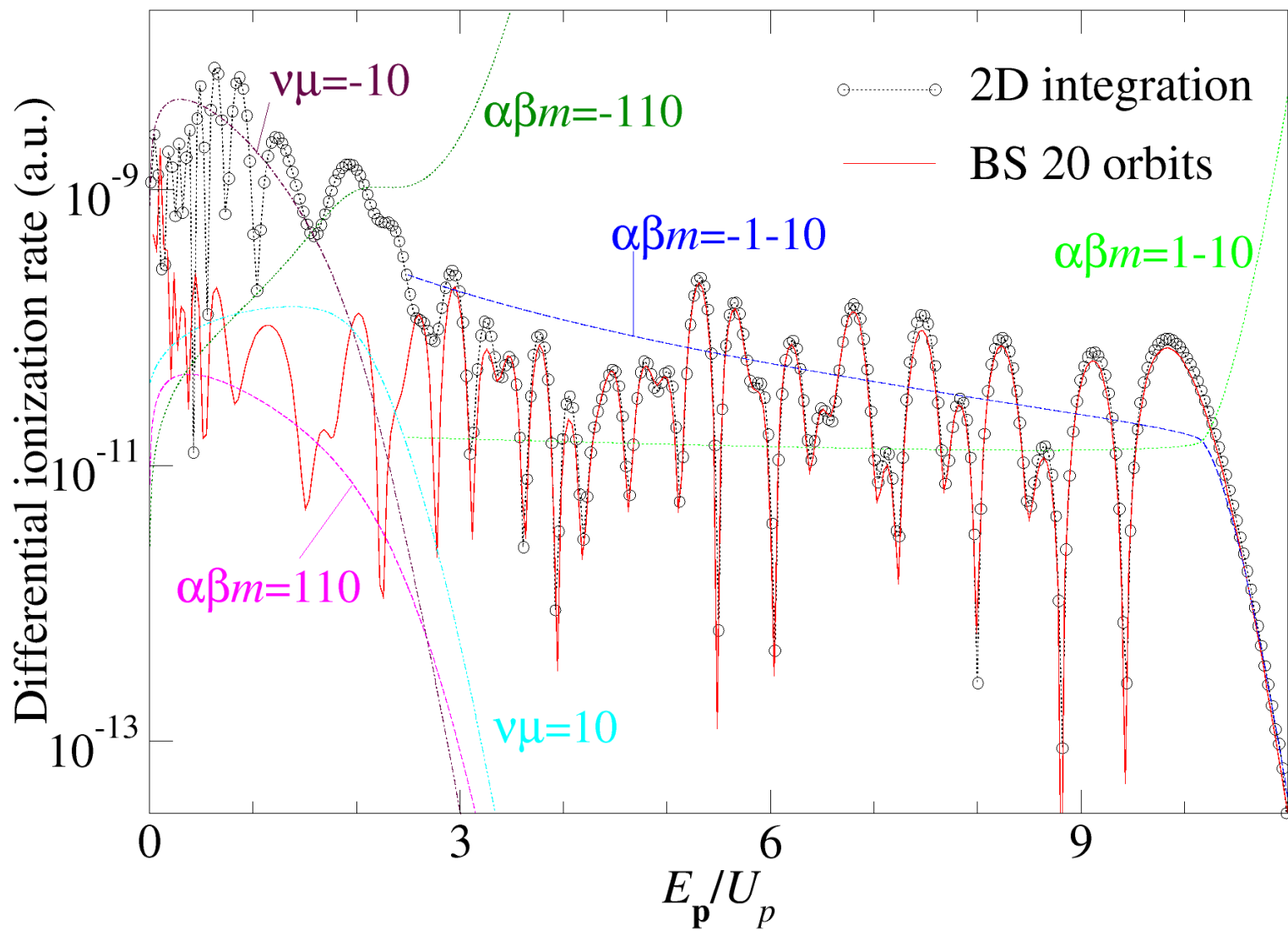


Multiindex $\alpha\beta m$: For $-(m+\frac{1}{2})T \leq t_i \equiv t_0 \leq -(m-\frac{1}{2})T$, $m=0,1,2,\dots$, there are two pairs of solutions.

The pair having the longer (shorter) travel time carries the index $\beta=-1$ ($\beta=+1$). Each pair again consists of two orbits with slightly different travel times: long and short (α) [PRA **76**, 053410 (2007)]



3100 nm, lin. pol.

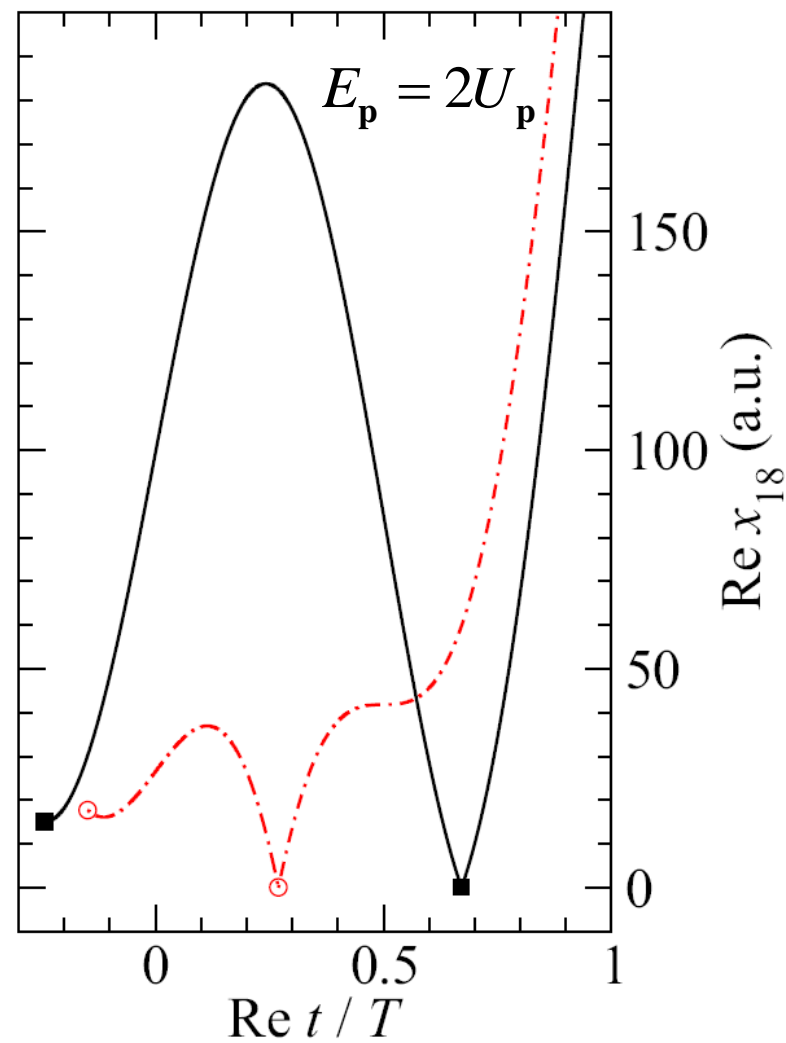
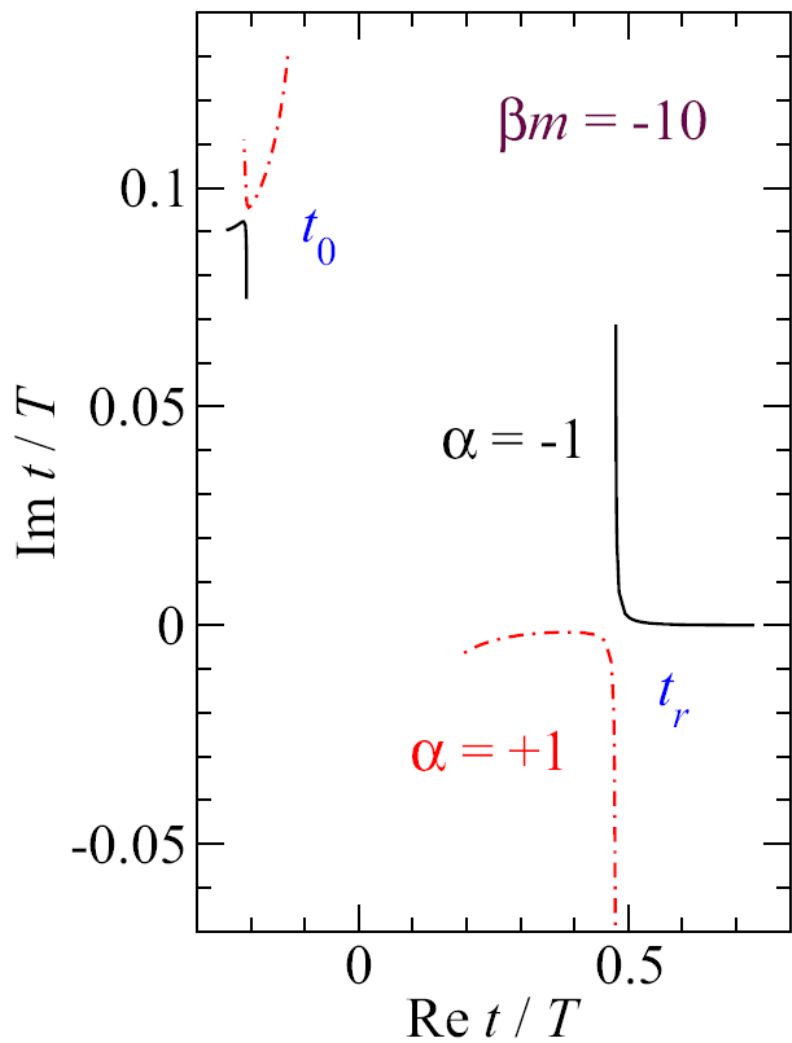


Quantum orbits

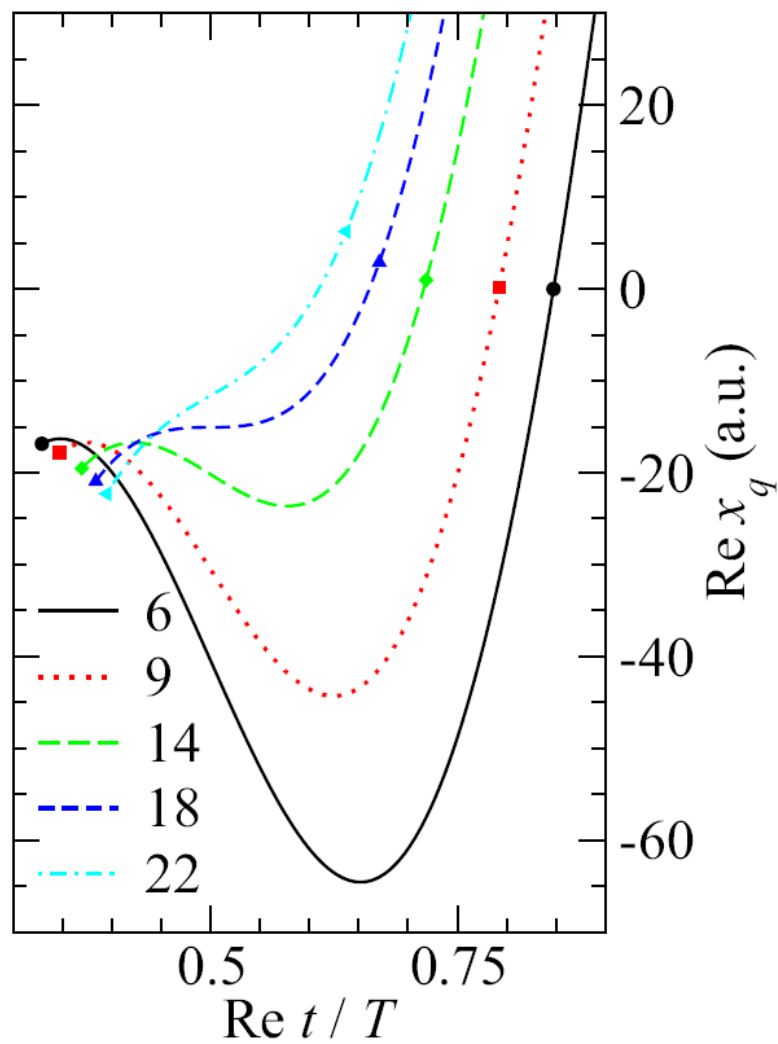
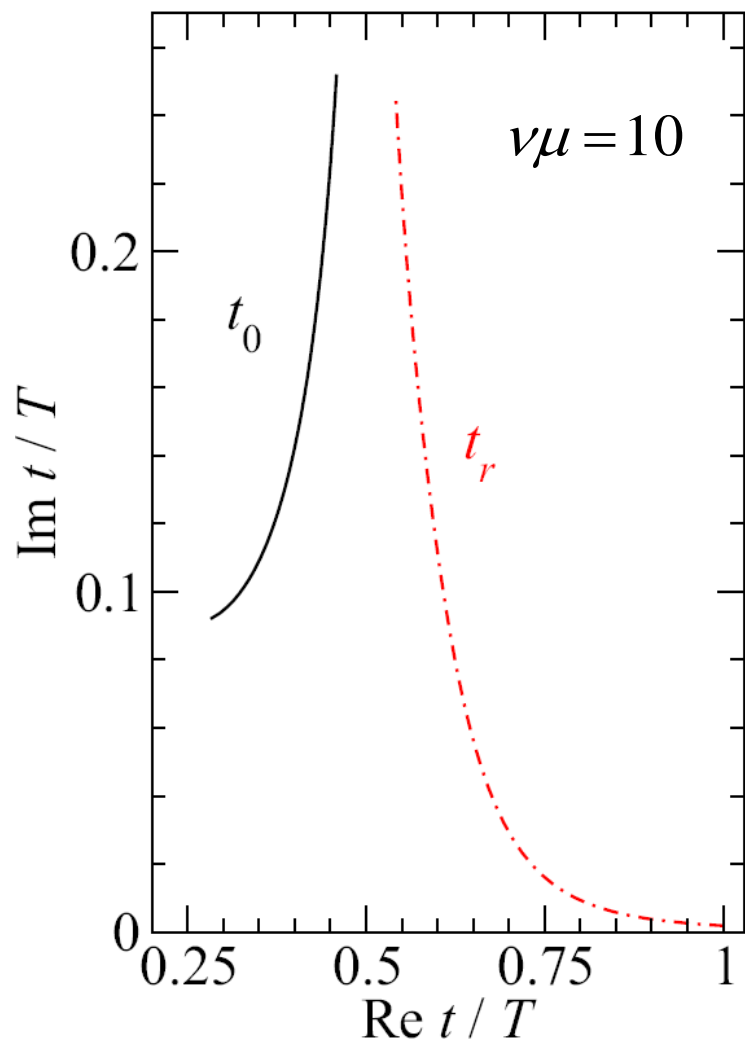
- Introducing complex solutions $\{t_0, t_r, \mathbf{k}_{st}\}$ of the saddle-point equations into Newton's equation we obtain complex trajectories. Quantum orbits are defined as complex trajectories for **real** time:

$$\mathbf{r}(t) = \begin{cases} (t - t_0)\mathbf{k}_{st} + \int_{t_0}^t \mathbf{A}(t')dt' & \text{if } \text{Re } t_0 \leq t \leq \text{Re } t_r \\ (t - t_r)\mathbf{p} + \int_{t_r}^t \mathbf{A}(t')dt' & \text{if } t > \text{Re } t_r. \end{cases}$$

- For the complex ionization time t_0 , the electron orbit departs from the origin $\mathbf{r}(t_0)=\mathbf{0}$.
- For the complex rescattering time t_r , the electron orbit returns to the origin $\mathbf{r}(t_r)=\mathbf{0}$.
- $\text{Re } \mathbf{r}(\text{Re } t_0) \neq 0 \Rightarrow$ the electron is 'born' at the 'exit of the tunnel'



H^- , 10 600 nm, 10^{11} W/cm 2 , $\theta=0^\circ$



$$E_p = \{0.67, \dots, 2.45\} U_p$$

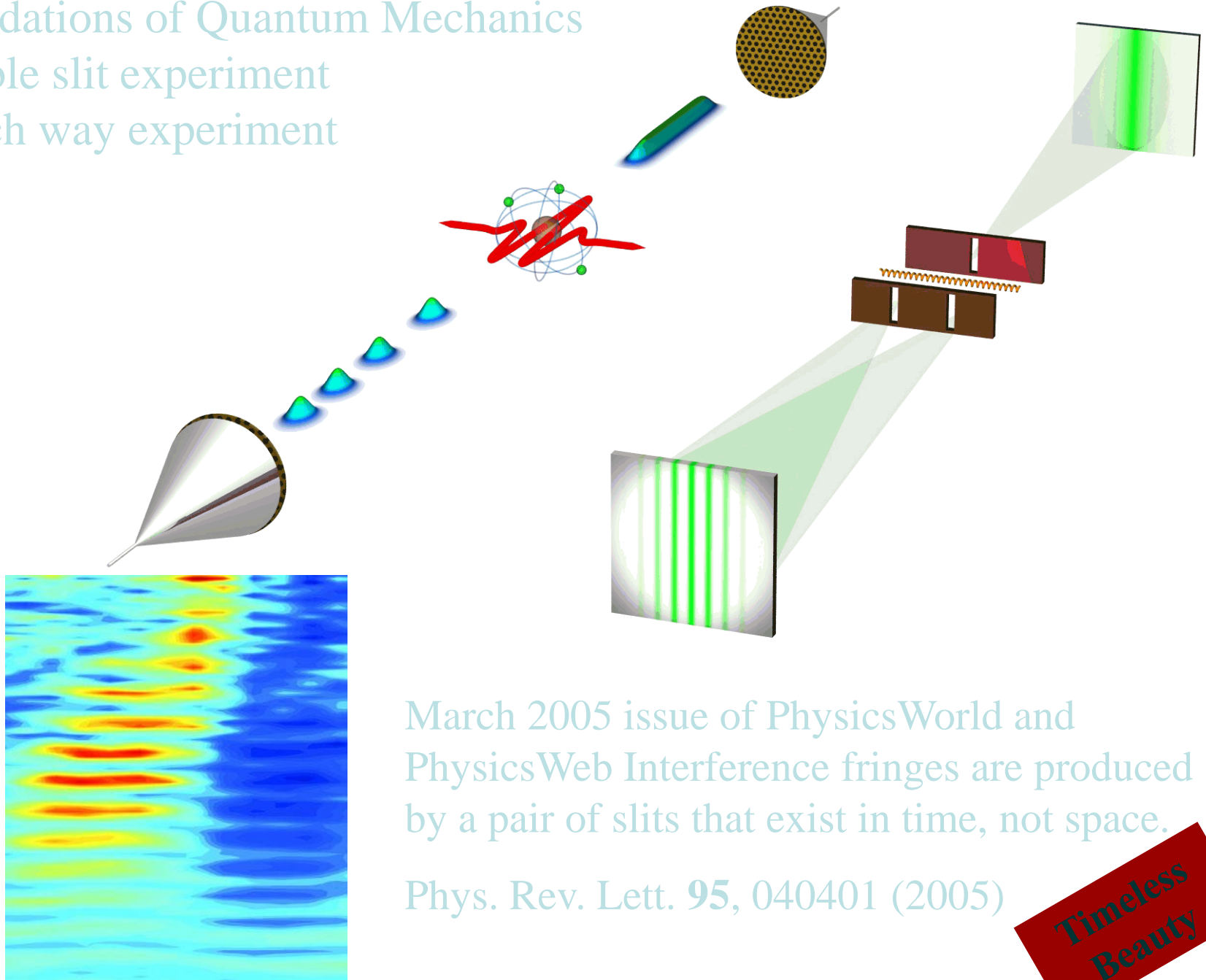
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Foundations of Quantum Mechanics

Double slit experiment

Which way experiment



March 2005 issue of PhysicsWorld and
PhysicsWeb Interference fringes are produced
by a pair of slits that exist in time, not space.

Phys. Rev. Lett. **95**, 040401 (2005)

**Timeless
Beauty**

Der Doppelspalt in etwas anderem Licht

Deutsche Forscher verwirklichen ein klassisches Experiment: Beugung mit extrem kurzen Laserpulsen

Das Doppelspalt-Experiment ist ein klassischer Interferenzversuch und jedem Physikstudenten aus dem Grundstudium vertraut. Das Experiment, bei dem ursprünglich ein Lichtstrahl in zwei sich gegenseitig überlagernde Strahlen aufgeteilt wurde, war für die Entwicklung der Lichtwellentheorie im 19. Jahrhundert ebenso wichtig wie für die Formulierung der Quantenphysik. Auch heute noch findet das Doppelspalt-Experiment in abgewandelter Form viele Anwendungen. So haben Wissenschaftler vom Max-Planck-Institut für Quantenoptik in Garching eine ungewöhnliche Version des Versuchs ersonnen.

Die Grundidee des Doppelspalt-Experiments ist etwa zweihundert Jahre alt. Füllt eine Welle durch zwei nebeneinanderliegende Schlitze, so läßt sich auf einem dahinter befindlichen Schirm ein charakteristisches Interferenzmuster aus hellen und dunklen Streifen beobachten. Es rührt von den beiden Teilwellen her, die von je einer der beiden Öffnungen ausgehen und sich gegenseitig überlagern. An den hellen Stellen des Schirms verstärken sie sich, an den dunklen löschen sie sich gegenseitig aus.

Zu Anfang des 19. Jahrhunderts gelang es dem britischen Physiker Thomas Young auf diese Weise, die Wellennatur des Lichts nachzuweisen. Später ist in abgewandelten Versionen des Experiments demonstriert worden, daß auch Materie Wellencharakter besitzt: Elektronen, Neutronen und Atome, ja sogar Moleküle zeigen ebenfalls ein charakteristisches Interferenzmuster, wenn sie einen Doppelspalt passieren. Dabei ist es nicht erforderlich, daß ein Teilchen den einen und ein zweites den anderen Weg nimmt. Experimente, bei denen einzelne Lichtquanten durch einen Doppelspalt geschickt wurden, haben gezeigt, daß ein Photon quasi mit sich selbst interferieren kann. Entscheidend ist, daß dem Teilchen zwei unterscheidbare, aber gleichwertige Wege zur Verfügung stehen, auf denen es sein Ziel – einen Schirm oder Detektor – erreichen kann.

Die Wissenschaftler vom Max-Planck-Institut für Quantenoptik in Garching haben nun ein besonders originelles Interferenzexperiment verwirklicht. Das Ungewöhnliche daran: Eine Schlitzeblende im eigentlichen Sinn gibt es nicht. Die beiden Wege, die die Teilchen – in diesem Falle sind es Elektronen – einschlagen können, werden durch einen extrem kurzen Laserpuls geschaffen, dessen sinusförmig oszillierendes elektrisches Feld aus nur wenigen Schwingungszyklen besteht. Die Wellenberge des Feldes wirken dabei gewissermaßen wie die Schlitze einer Blende, die sich nacheinander öffnen und so den Teilchen für einen kurzen Moment verschiedene Wege anbieten, auf denen sie entkommen können. Mit den Laserpulsen wurden auch die Elektronen erzeugt.

In ihrem Experiment haben die Forscher um Ferenc Krausz die Lichtblitze

auf Argonatome gerichtet, aus deren Hülle einzelne Elektronen herausgeschlagen wurden. Ein Argonatom kann man sich dabei stark vereinfacht vorstellen als einen Apfelbaum, an dem gerüttelt wird, erst zur einen, dann zur anderen Seite. Nur wenn das kräftig genug geschieht, können die Äpfel herunterfallen beziehungsweise die Elektronen der Anziehung des Atomkerns entkommen. Die freigesetzten Teilchen wurden durch das elektrische Feld des Lichtpulses in Richtung von zwei sich gegenüberstehenden Detektoren beschleunigt. Die Nachweisgeräte hatten zum Ort des Geschehens den gleichen Abstand.

Wie Krausz und seine Kollegen in der Zeitschrift „Physical Review Letters“ (Bd. 95, Nr. 040401) berichten, unterschieden sich die beiden gemessenen Elektronenspektren deutlich voneinander. Während das eine ein Interferenzmuster zeigte, wie man es von einem

Doppelspalt-Experiment erwarten würde, wies das andere Spektrum keine Interferenzstruktur auf. Die Elektronen haben sich genauso verhalten, als ob sie durch einen einfachen Spalt geflogen wären.

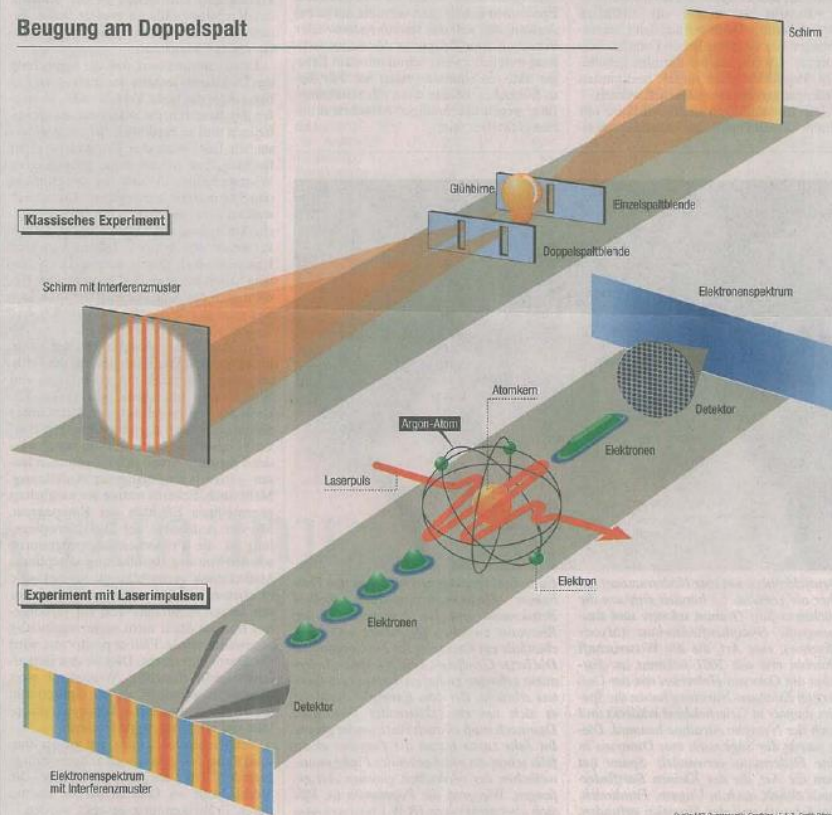
Das sinusförmig schwingende elektrische Feld hatte jedem freigesetzten Elektron – entsprechend der Zahl und der räumlichen Lage der Schwingungsmaxima – ein oder zwei Fluchtwege eröffnet, auf denen die Teilchen entweder zum einen oder zum anderen Detektor gelangen konnten. War das Elektron nur einem Schwingungsmaximum ausgesetzt, hatte es auch nur eine Möglichkeit, dem Argonatom zu entkommen. Dementsprechend trat auch kein Interferenzsignal auf. Anders bei zwei Schwingungsmaxima, die in die gleiche Richtung zeigten: Das entsprach gewissermaßen der Situation, daß ein Teilchen einen Doppelspalt passiert. Folglich war auch in einem der beiden Elektronenspektren ein

charakteristisches Streifenmuster zu erkennen.

Der Versuch stellte die Experimentierkunst der Gärchinger Physiker vor besondere Herausforderungen. Sie mußten den zeitlichen Verlauf und die Phase jeder Laserpulse bis auf wenige Attosekunden (trillionstel Sekunden) genau einstellen. Das war kein leichtes Unterfangen – selbst für Ferenc Krausz, der als Pionier auf dem Gebiet der Attosekundenphysik gilt und für seine Leistungen kürzlich mit dem renommierten Leibniz-Preis ausgezeichnet wurde. Durch die Interferenzmessungen erhoffen sich die Wissenschaftler Aufschluß darüber, wie die Elektronen mit dem schnell oszillierenden Lichtfeld wechselwirken, also ob sie gewissermaßen wie die Äpfel bei der leisen Berührung vom Baum fallen oder ob sie ziemlich fest an den Ästen hängen und sich erst nach kräftigem Schütteln vom Baum lösen.

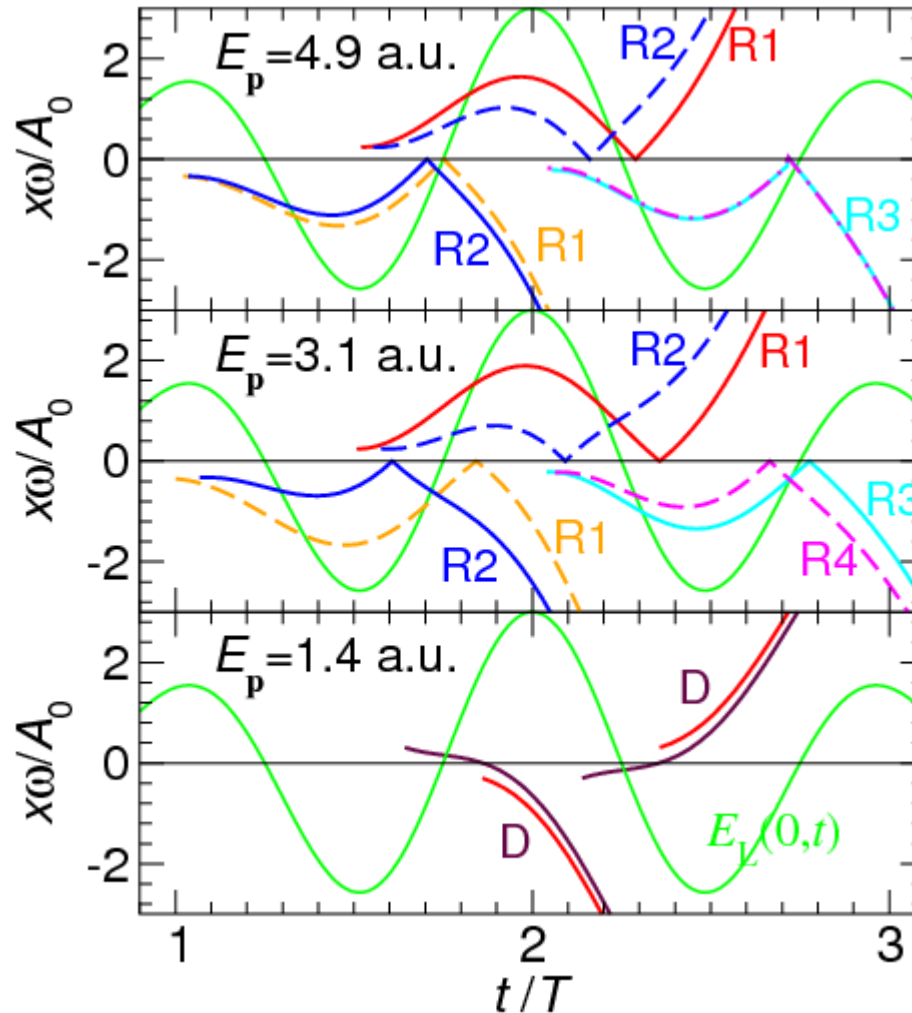
STEFANIE HENSE

Beugung am Doppelspalt



Rescattered quantum orbits in space and time

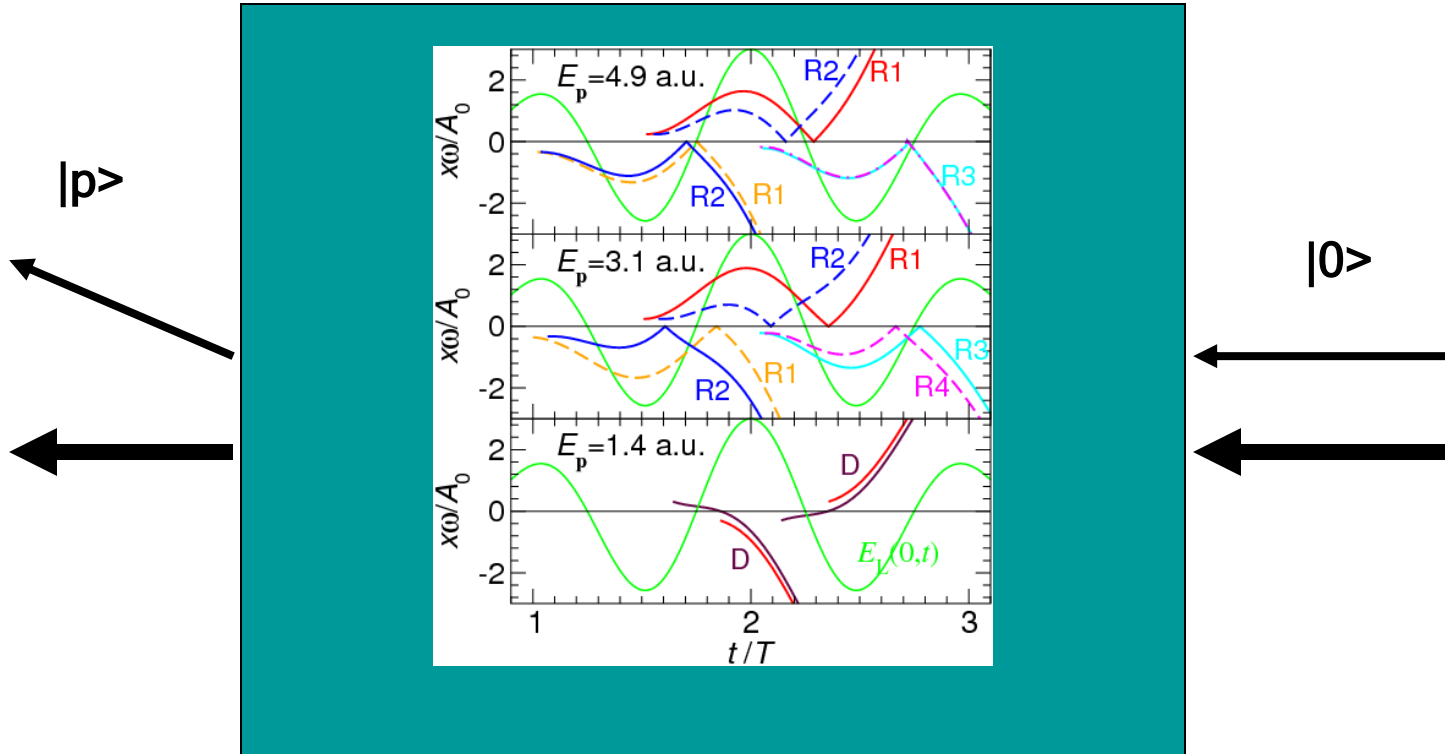
ionization time = t_1 $t =$ recollision time



The black box of S-matrix theory



... has been made transparent



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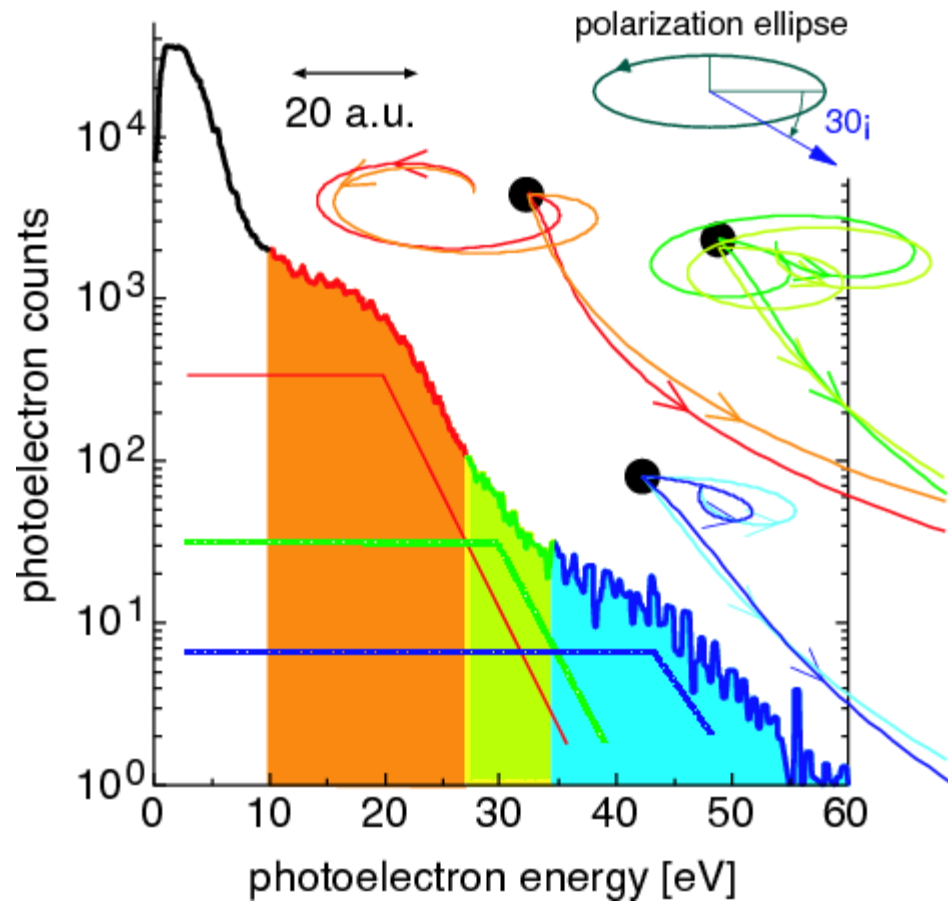
Quantum orbits for elliptical polarization: Experiment vs. theory

$$\xi = 0.36$$

xenon at $0.77 \times 10^{14} \text{Wcm}^{-2}$

The plateau becomes
a staircase

The shortest orbits are
not always the dominant
orbits

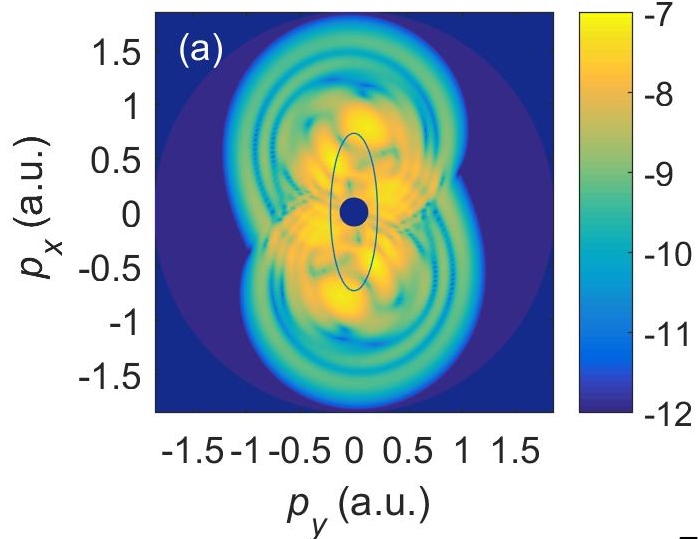


D. B. Milošević and W. Becker

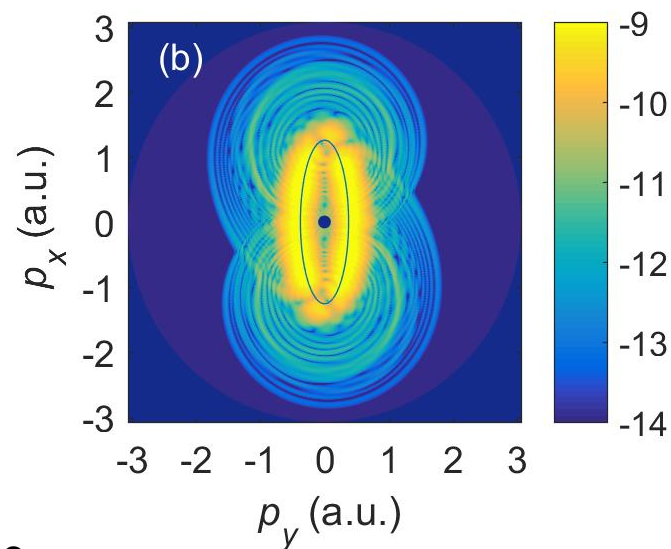
Negative-travel-time quantum orbits in strong-field ionization by an elliptically polarized laser field

Phys. Rev. A **105**, L031103 (2022)

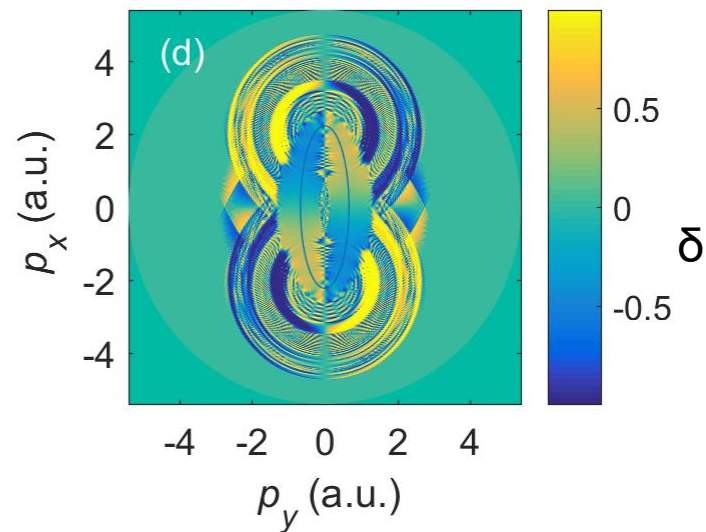
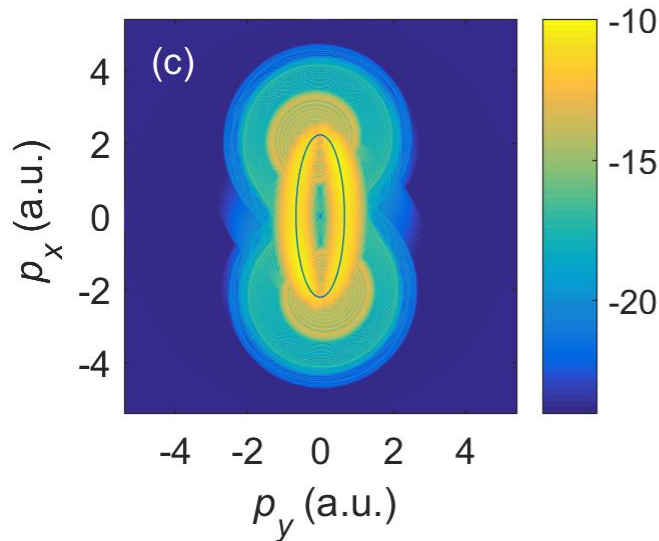
1800 nm



3100 nm



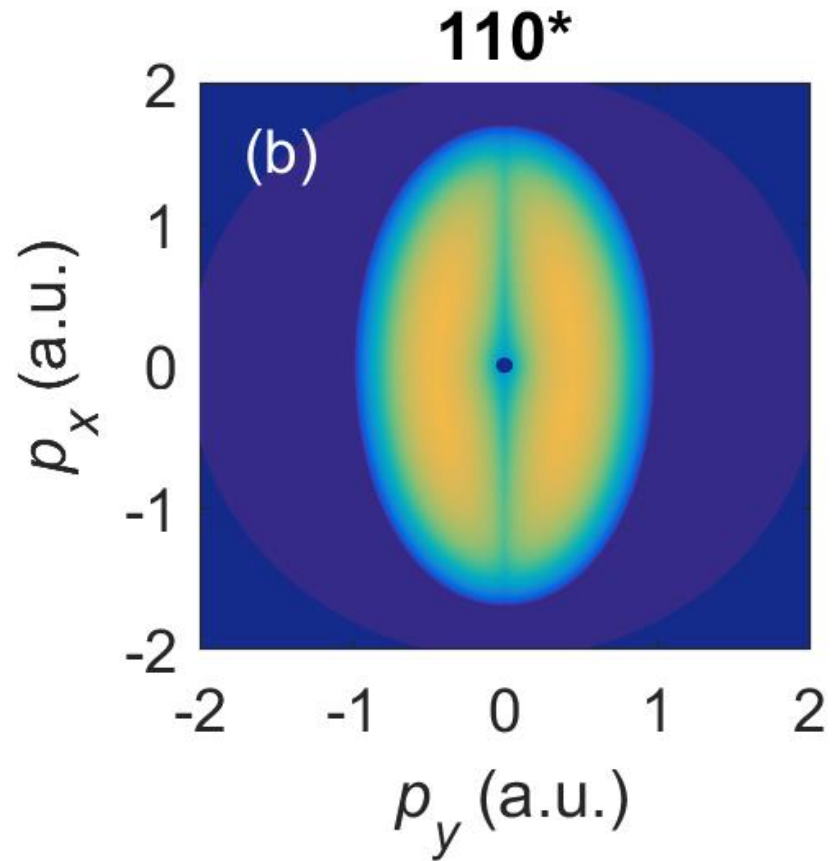
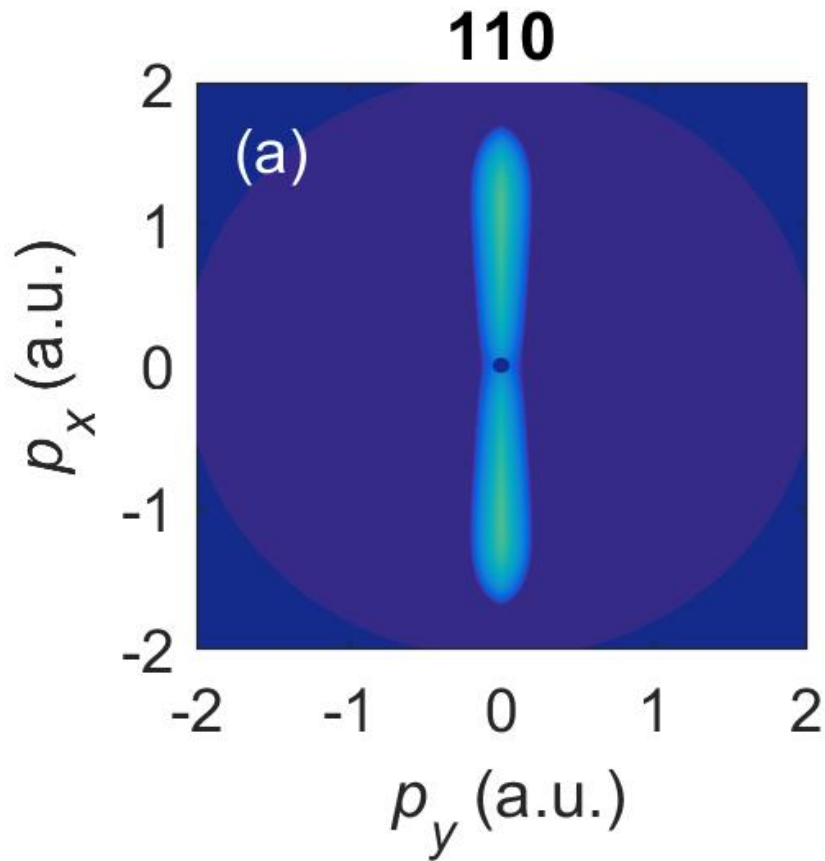
5500 nm

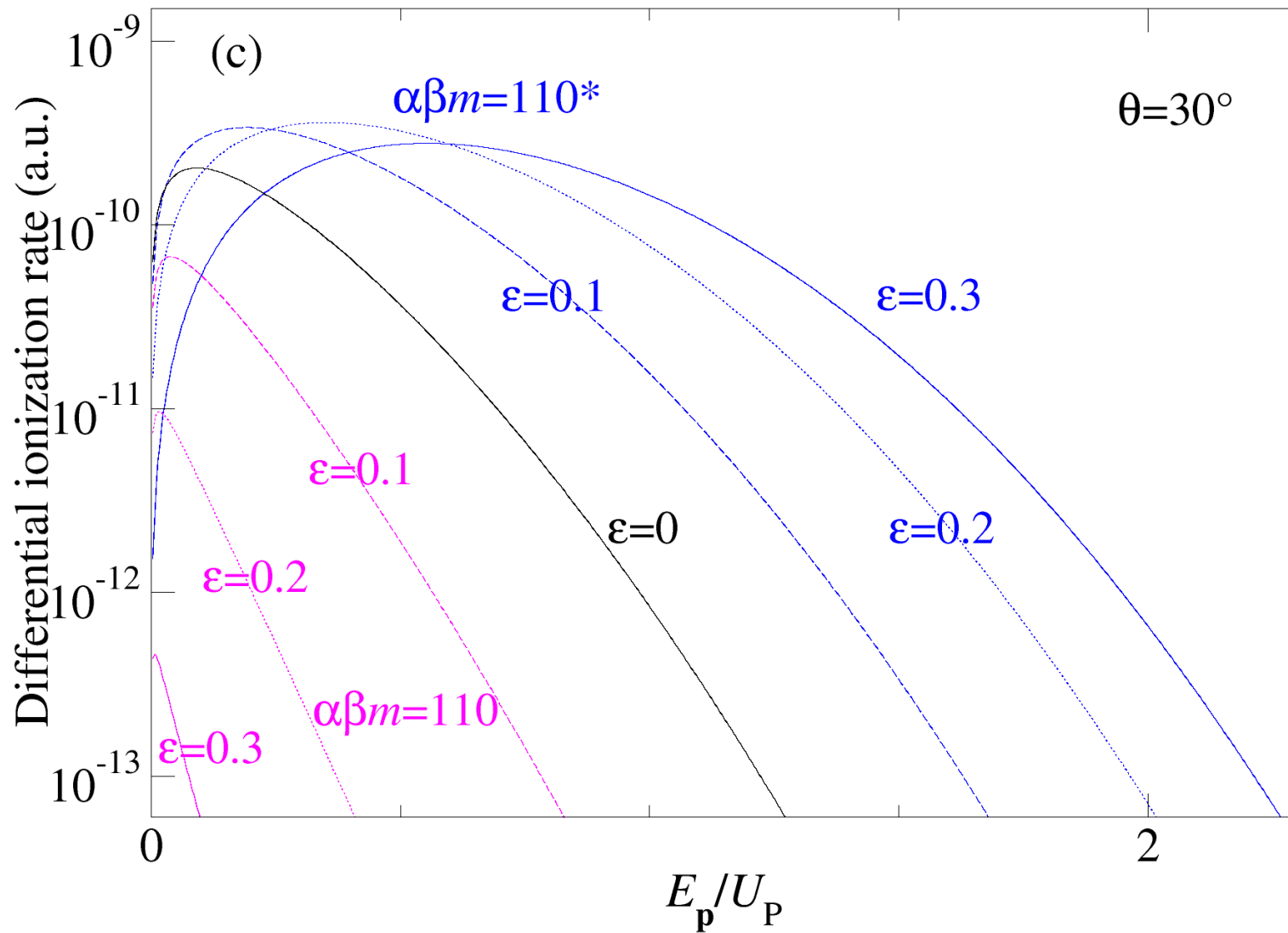

 $F^-, \varepsilon=0.3, 1.3 \times 10^{13} \text{ W/cm}^2$

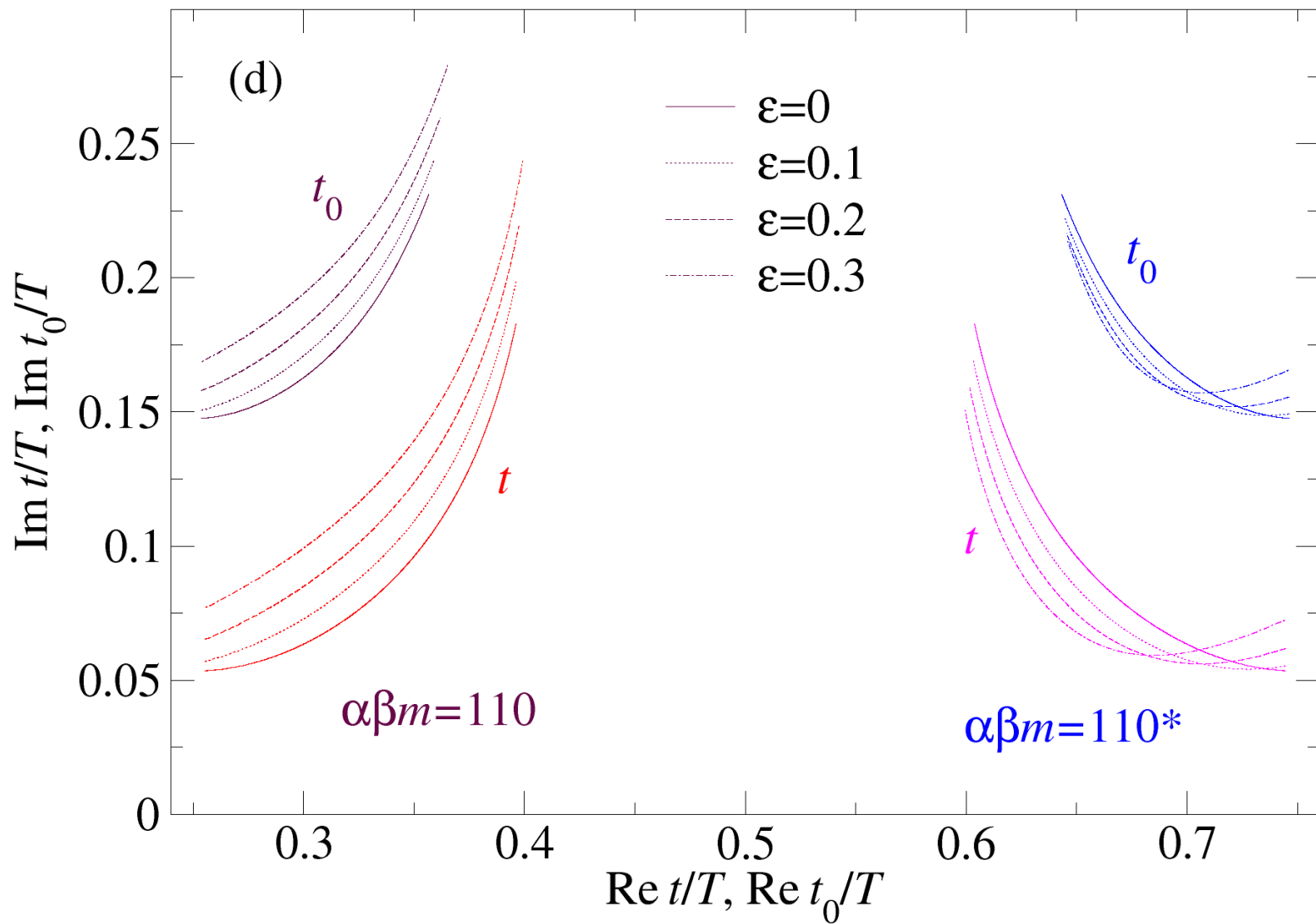
$$\delta(\mathbf{p}, \varepsilon) \equiv \frac{\bar{w}(\mathbf{p}, \varepsilon) - \bar{w}(\mathbf{p}, -\varepsilon)}{\bar{w}(\mathbf{p}, \varepsilon) + \bar{w}(\mathbf{p}, -\varepsilon)}$$

Linear polarization: $t_{0s}, t_s \leftrightarrow T - t_{0s}^*, T - t_s^*$

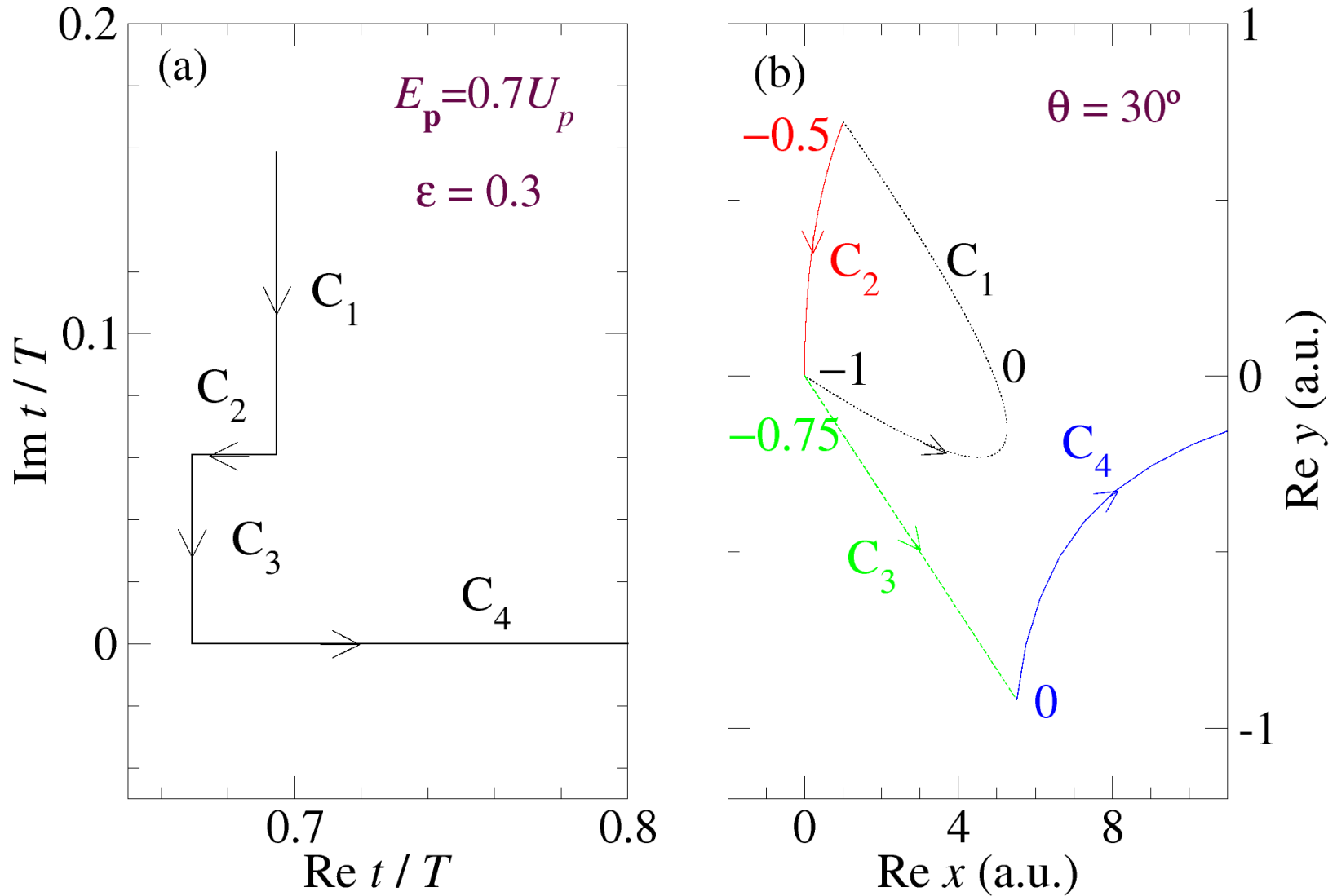
Elliptical polarization - bifurcation: $(\alpha, \beta, m), (\alpha, \beta, m)^*$







Complex-time quantum orbits



Outline

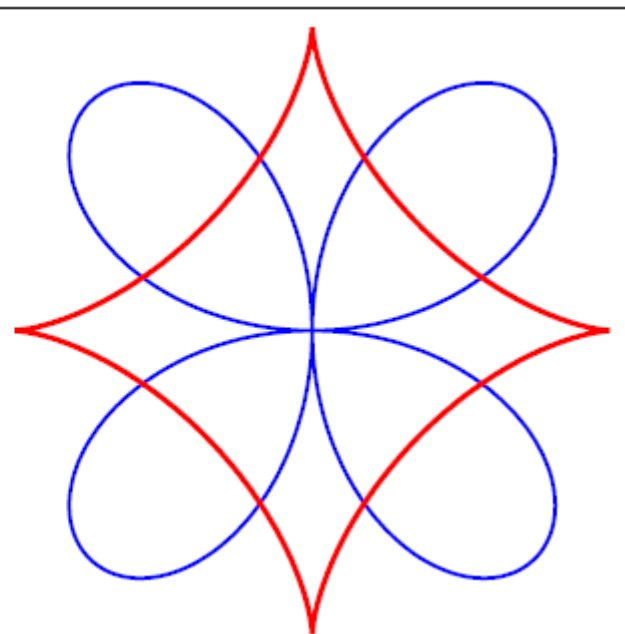
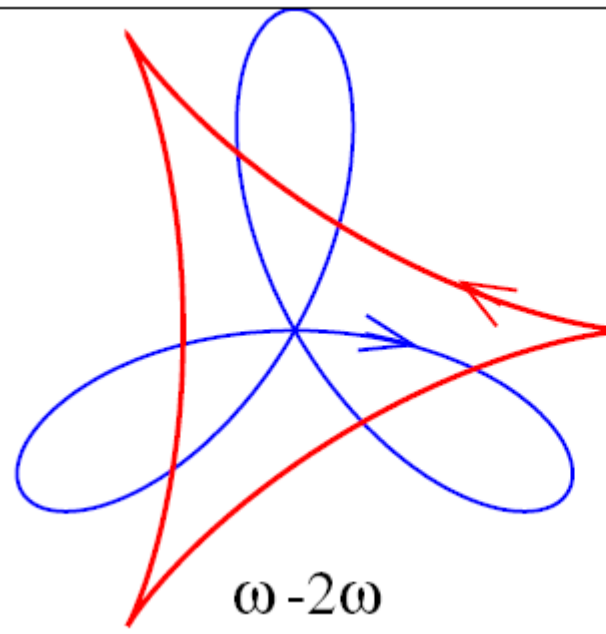
- Strong-laser-field processes
- High-order processes, 3-step model
- Feynman path integral, strong-field approximation, quantum-orbit theory
- Examples - linear polarization
 - direct electrons, double-slit exp. in time
 - rescattered electrons, high-order ATI
- Examples - elliptical polarization
 - multiplateau structures
 - negative travel time
- **Examples - tailored (complex) fields**
 - **bicircular, BEOTC fields (HHG, HATI)**

Tailored/complex fields

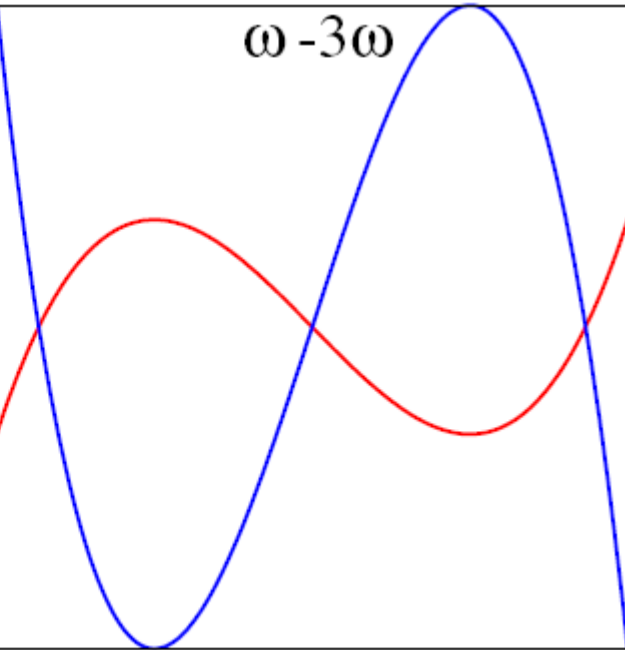
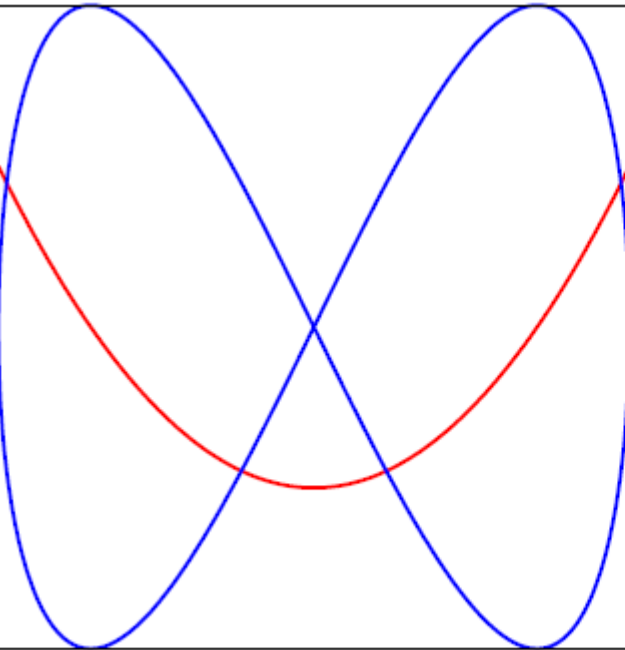
- Linearly polarized:
 - Monochromatic
 - Bichromatic
 - Few-cycle pulses:
 \sin^2 envelope, total pulse duration n_p o.c.
3-colors: $\omega, \omega \pm \omega/n_p$
carrier-envelope phase
- Elliptically polarized monochromatic
- Bichromatic $r\omega - s\omega$ elliptically polarized
 $\omega_r, \omega_s, \varepsilon_r, \varepsilon_s, I_r, I_s, \varphi_{rs}$
- pulses, envelopes, CEPs, delays
- Bicircular field
- OTC field

- The dynamics of electrons driven by a **linearly** polarized field is **one-dimensional**
- Find better way to explore structure / dynamics of more complex targets such as molecules
- Find more appropriate field configurations that possess particular **symmetry** properties and the possibility that the laser field driven liberated electron **returns** to the parent ion. Solutions:
- **Bicircular field** which consists of two coplanar counter-rotating circularly polarized fields of frequencies $r\omega$ and $s\omega$
- Orthogonally polarized two-color (**OTC**) field with frequencies $r\omega$ and $s\omega$ and relative phase φ

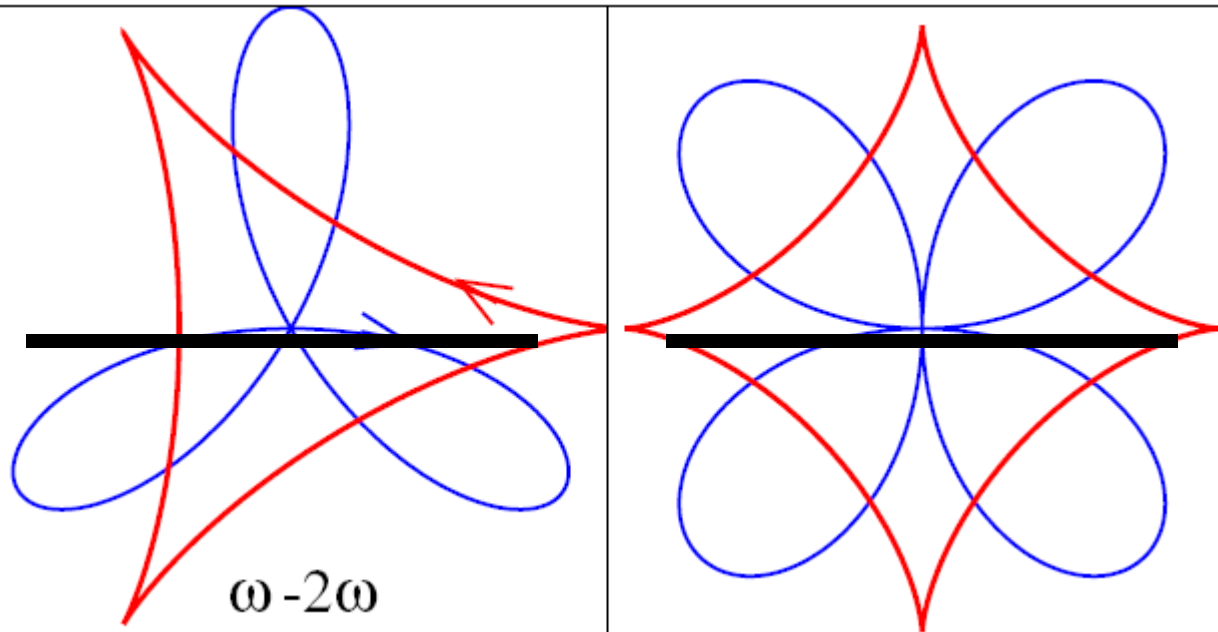
bicircular



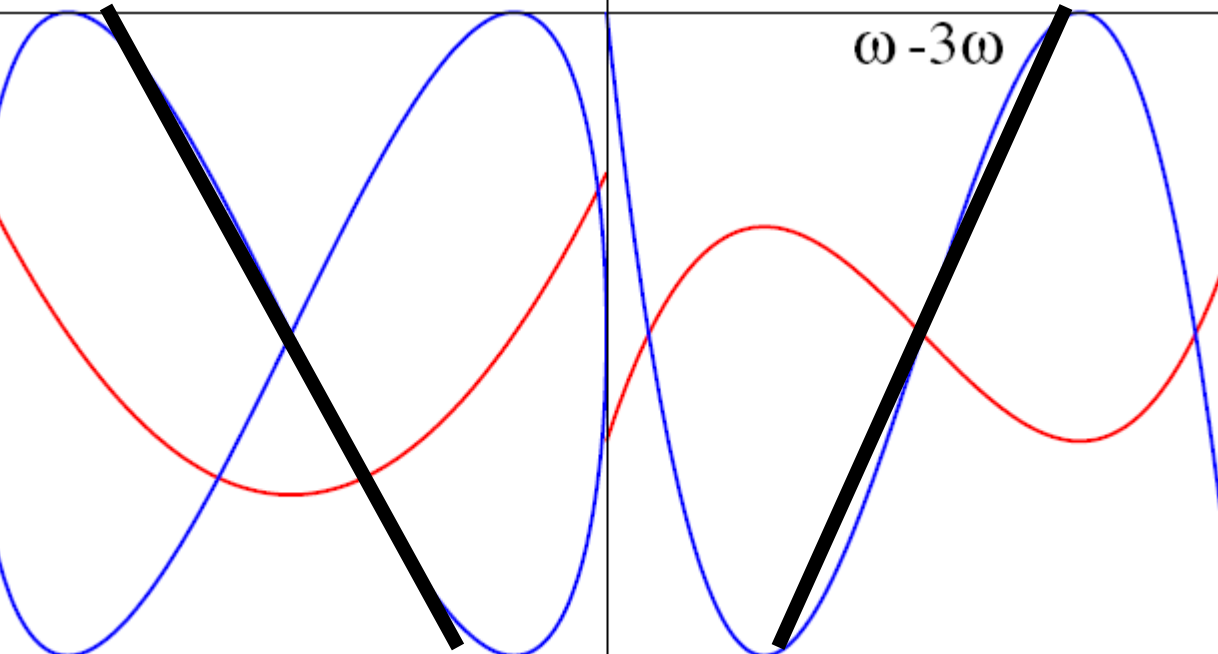
OTC



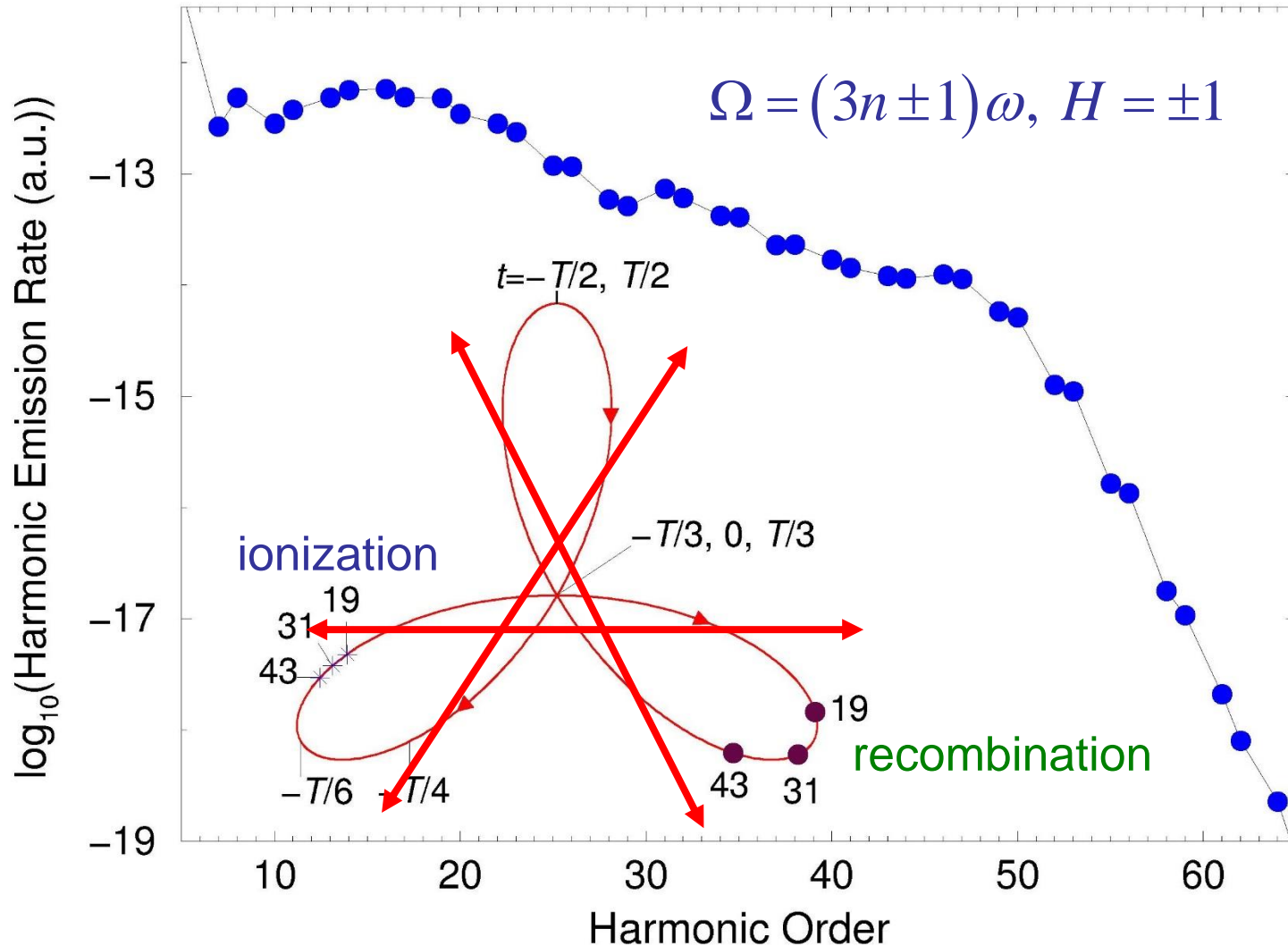
bicircular



OTC

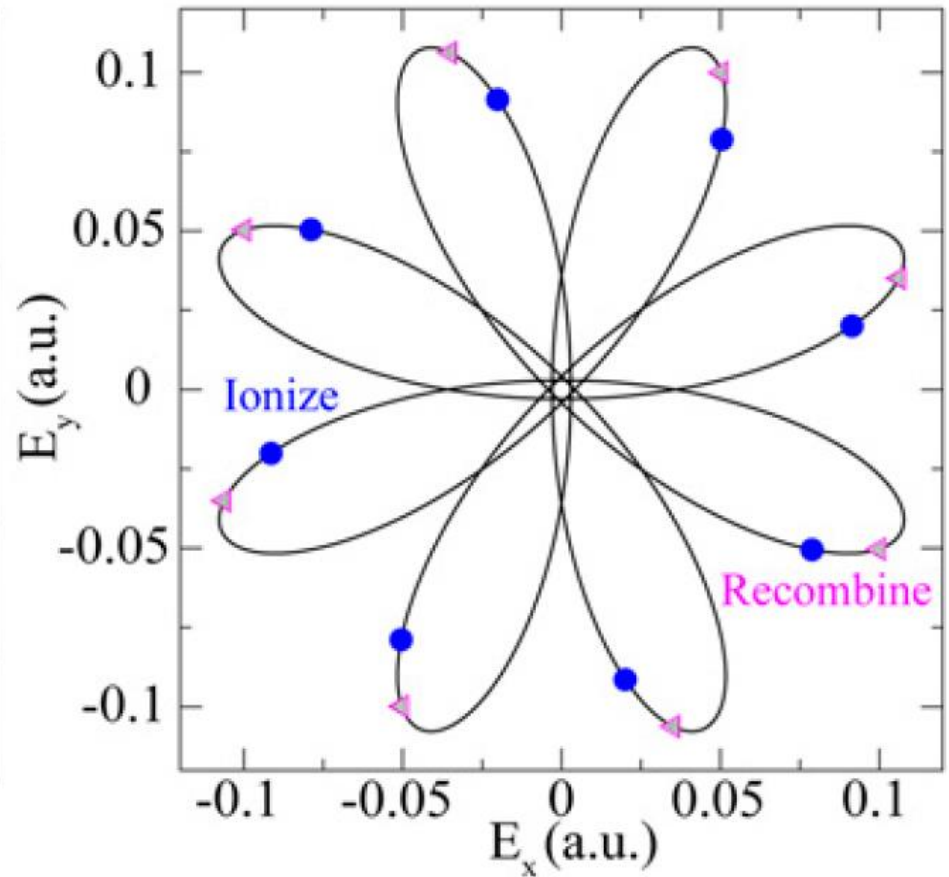
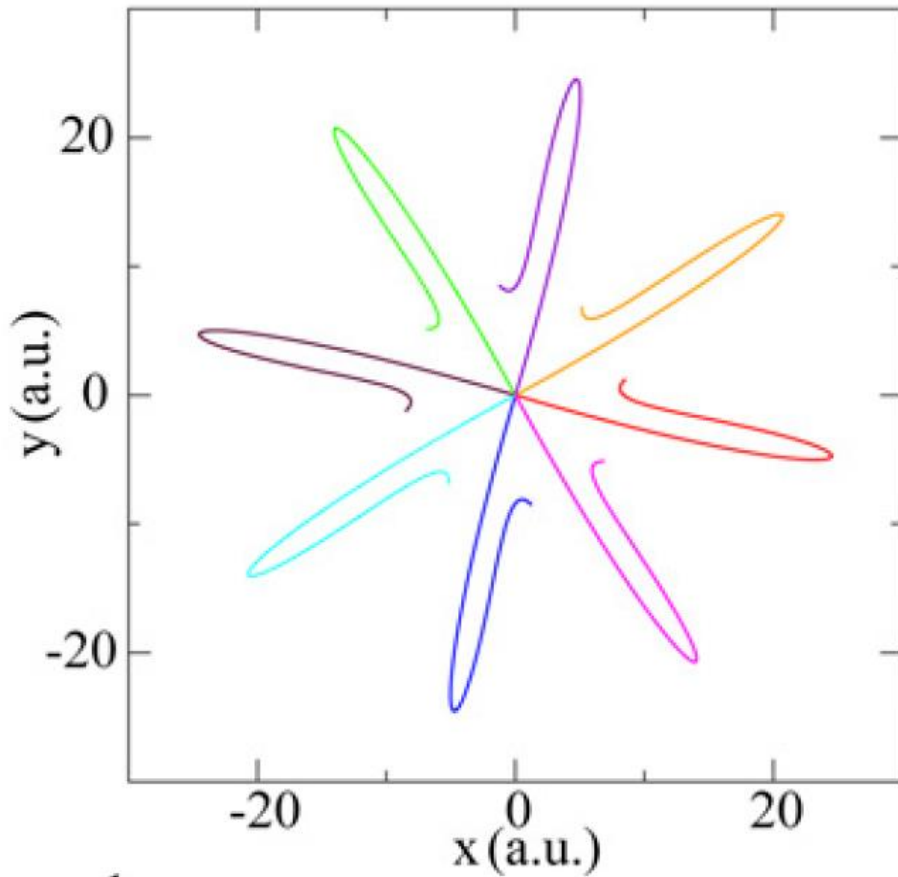


HHG by bicircular field



Along these three segments, between ionization and recombination the field is approximately linearly polarized

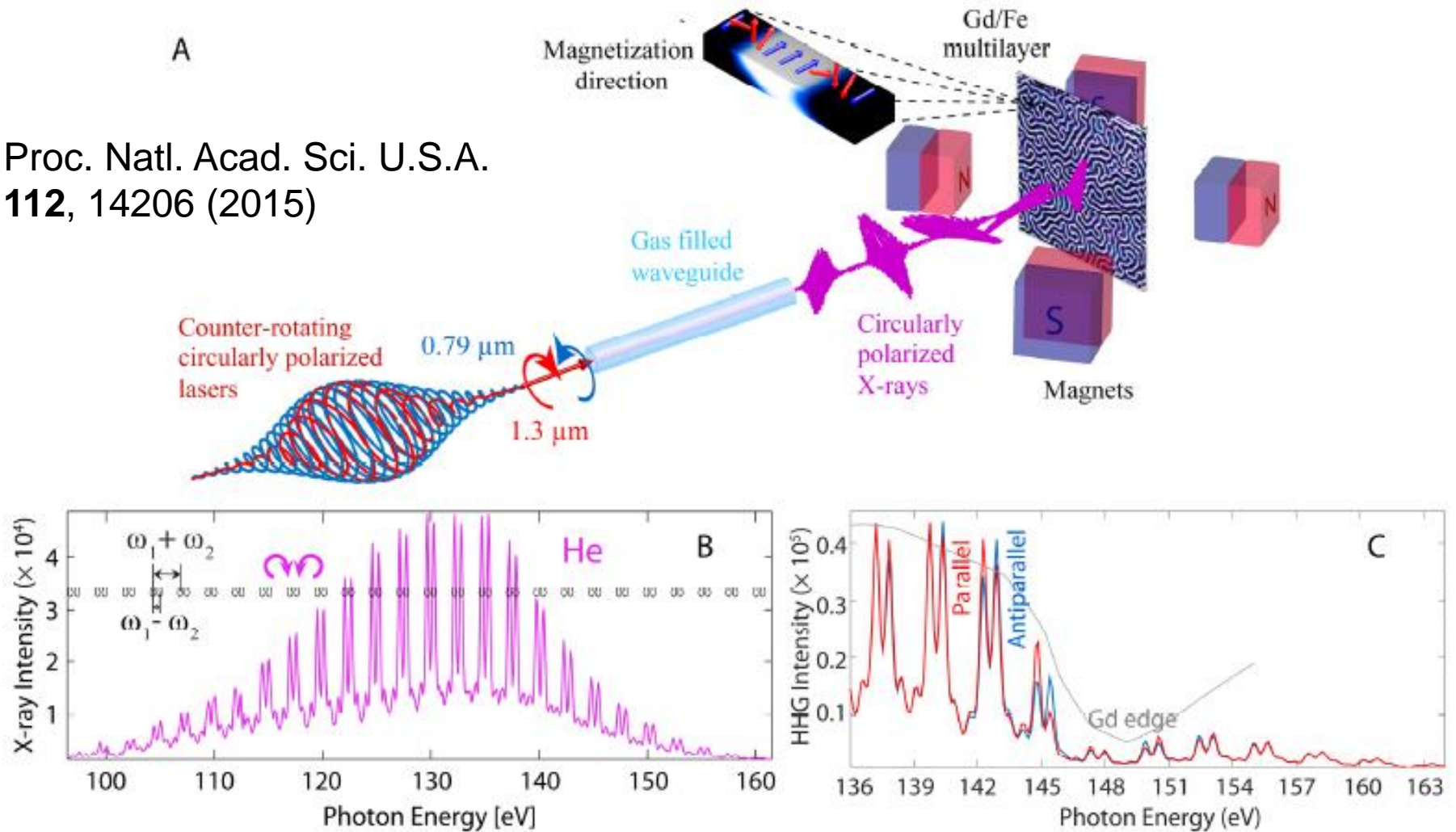
$3\omega-5\omega$



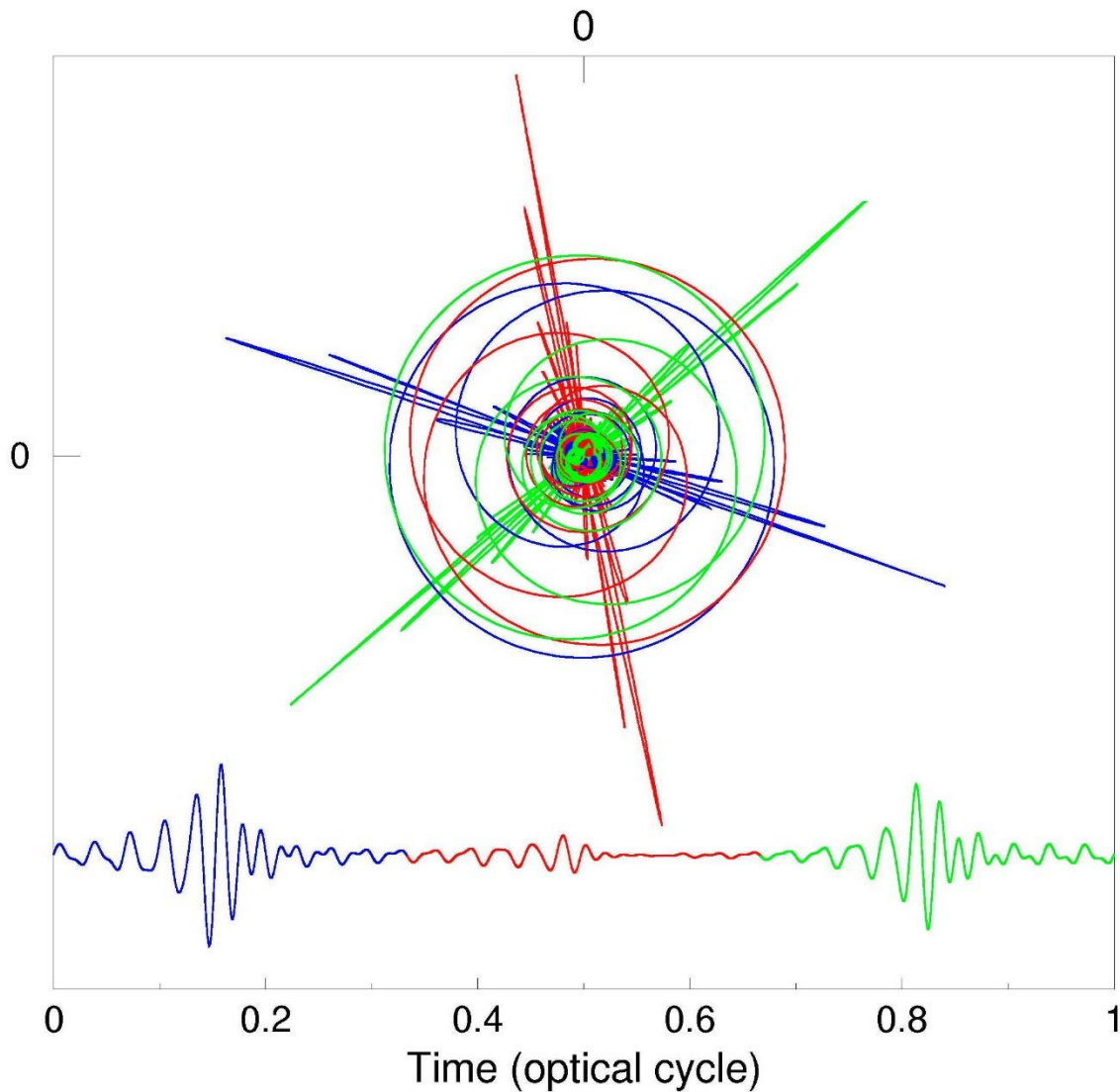
- Proc. Natl. Acad. Sci. U.S.A. **112**, 14206 (2015)

- J. Mod. Opt. **64**, 971 (2017)

Proc. Natl. Acad. Sci. U.S.A.
112, 14206 (2015)



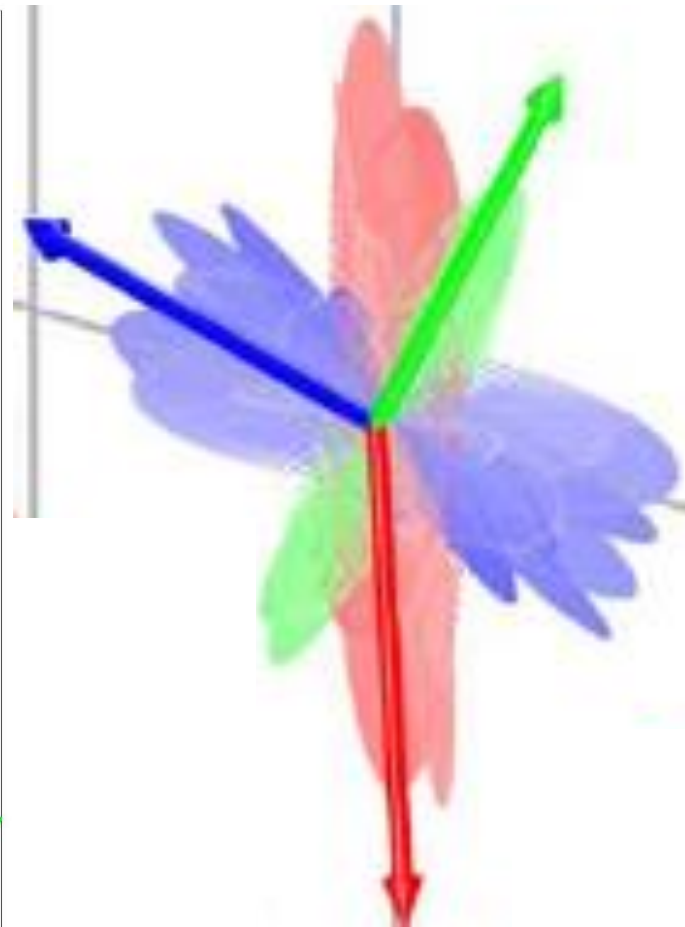
A. Circularly polarized soft X-ray beams are generated by focusing counter-rotating circularly polarized bi-chromatic lasers at $0.79\mu\text{m}$ and $1.3\mu\text{m}$ into a gas-filled waveguide. This source is then used for XMCD in transmission geometry. **B.** Circular HHG spectrum exhibits a peak-pair structure, with a separation within each pair of $\omega_1 - \omega_2$, while different pairs are separated by $\omega_1 + \omega_2$. **C.** HHG spectra transmitted through a Gd/Fe multilayer as the magnetization direction is parallel (red) and antiparallel (blue) to HHG propagation direction.



Theory: D. B. Milošević and W. Becker

Phys. Rev. A **62**, 011403(R) (2000)

Phys. Rev. A **61**, 063403 (2000)

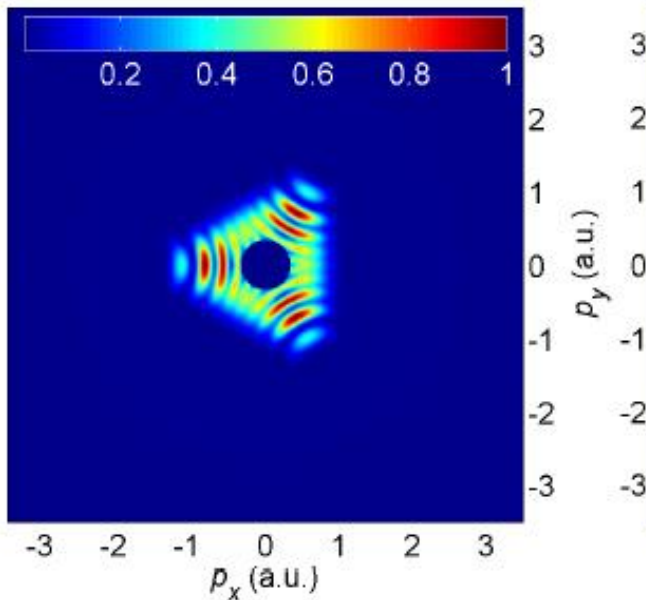


Experiment: 3D atto
metrology, Murnane et al
Science Advances **2**,
e1501333 (2016)

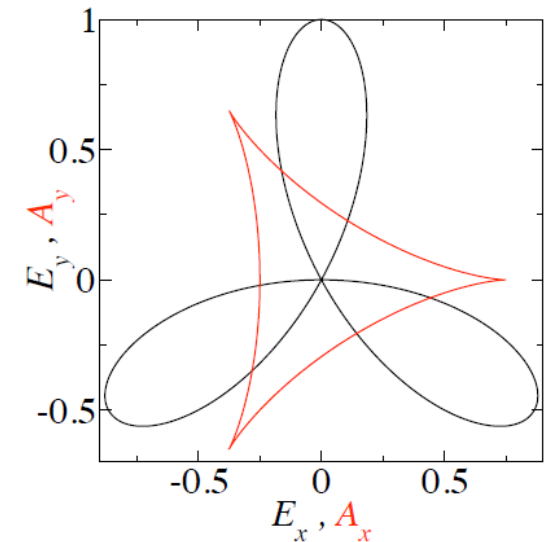
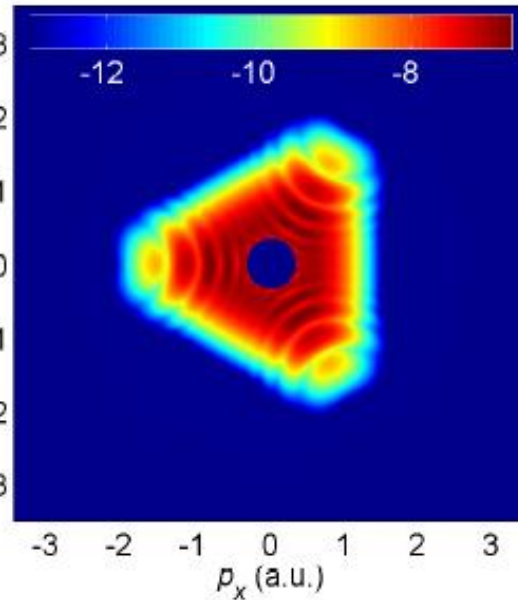
(H)ATI by bicircular field

ATI - direct electrons only

linear color code



log. color code



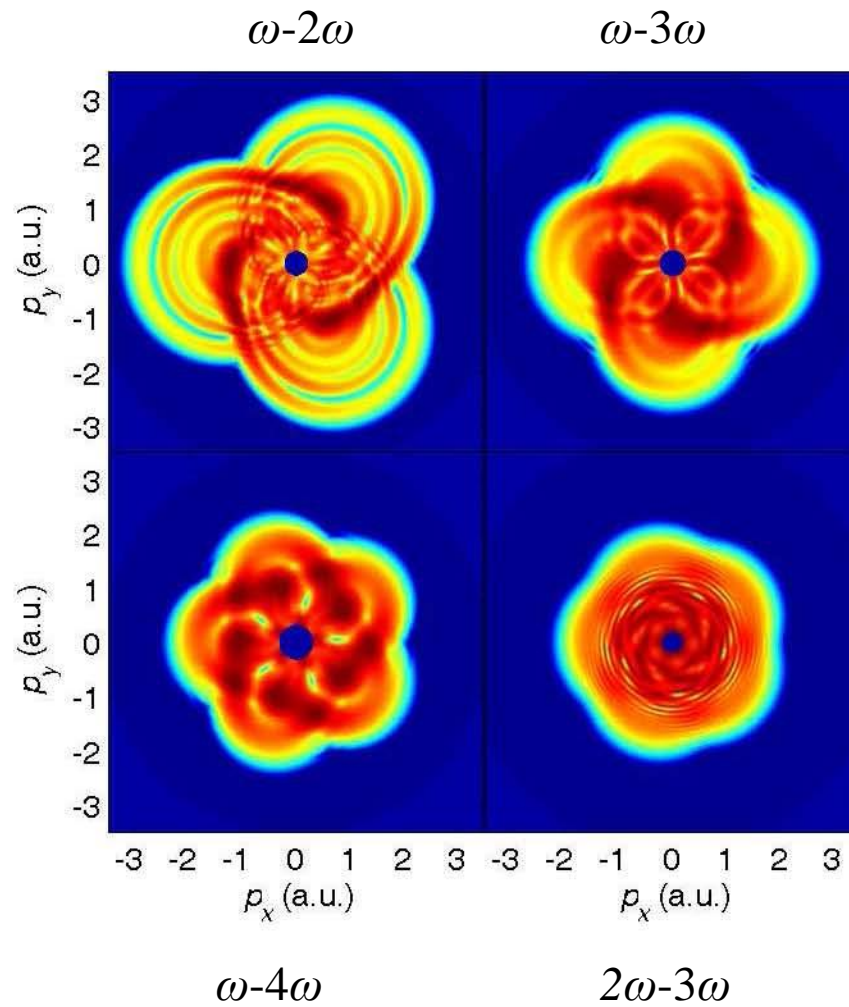
Vector potential: red line

The distribution follows the vector potential: $\mathbf{p} = -\mathbf{A}(t)$

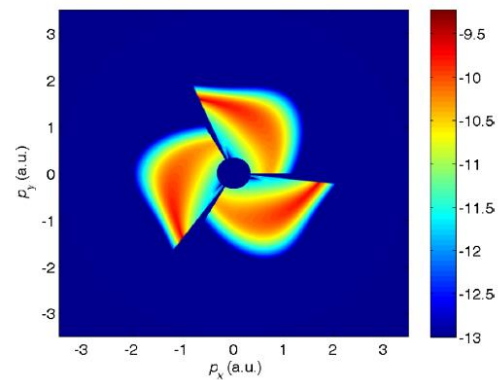
A. Kramo, E. Hasović, D. B. Milošević, and W. Becker, Laser Phys. Lett. **4**, 279 (2007)

E. Hasović, W. Becker, and D. B. Milošević, Opt. Express **24**, 6413 (2016)

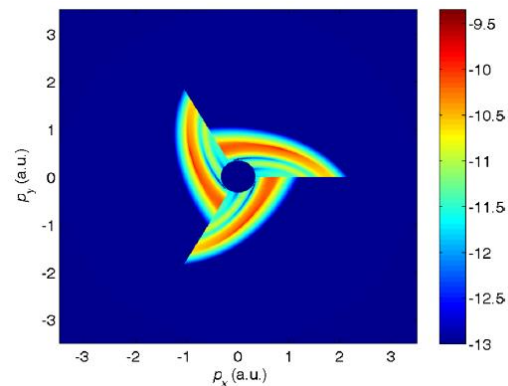
ATI, Ne, 2×10^{14} W/cm², 800 nm + 400 nm



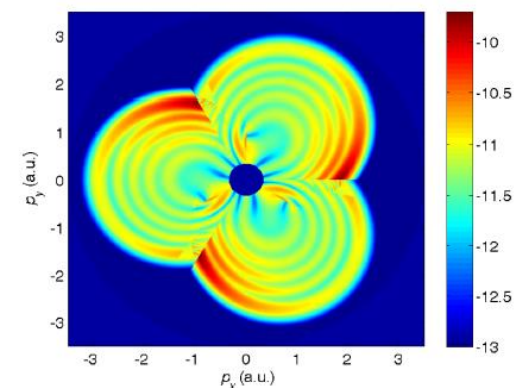
D. B. Milošević, W. Becker, Phys. Rev. A **93**, 063418 (2016)



forward scattering
 $(\nu, \rho, \mu) = (1, 0, 0)$

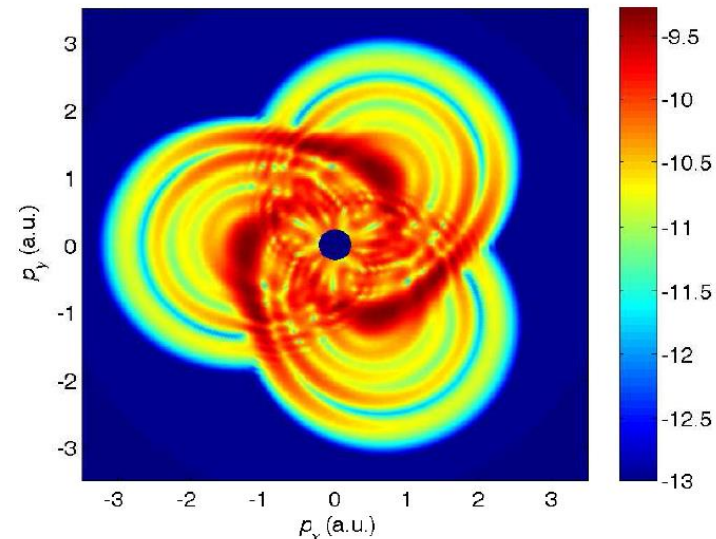


forward scattering
 $(\nu, \rho, \mu) = (\pm 1, 2, 0)$



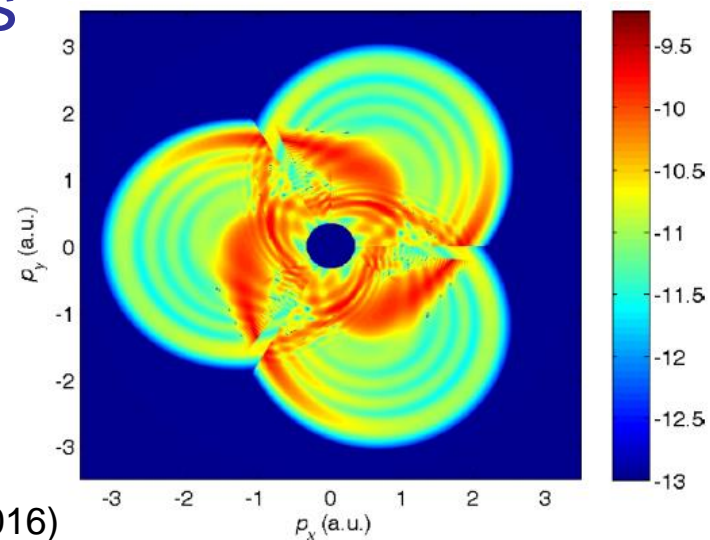
$I_1 = I_2 = 2 \times 10^{14} \text{ W/cm}^2$
 $\lambda_1 = \lambda_2/2 = 800 \text{ nm}$
 neon

backward scattering
 $(\alpha, \beta, m) = (\pm 1, 1, 0)$

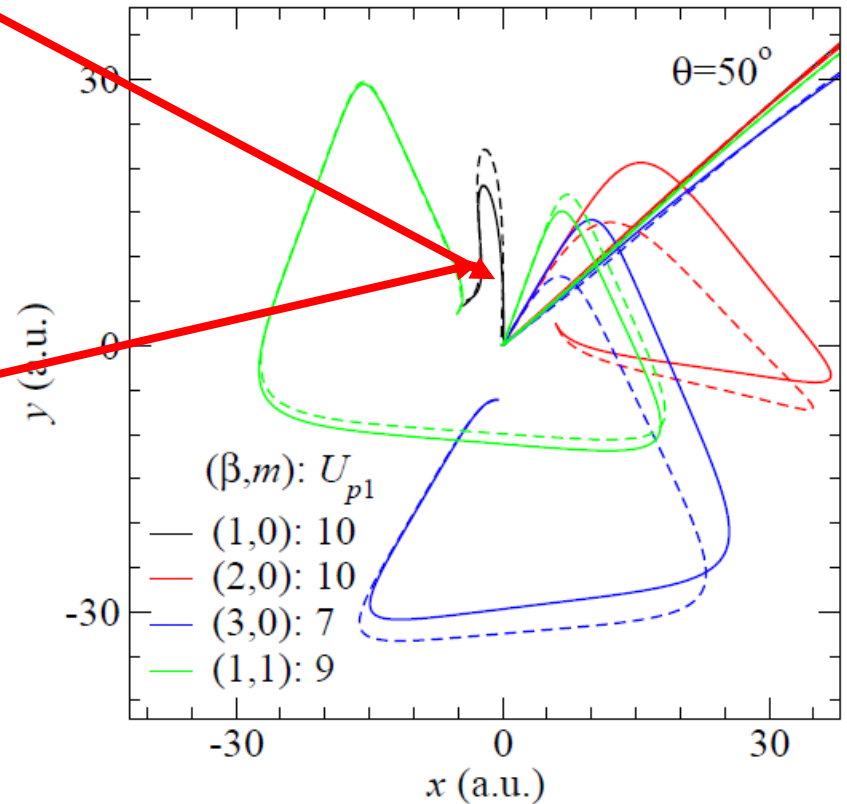
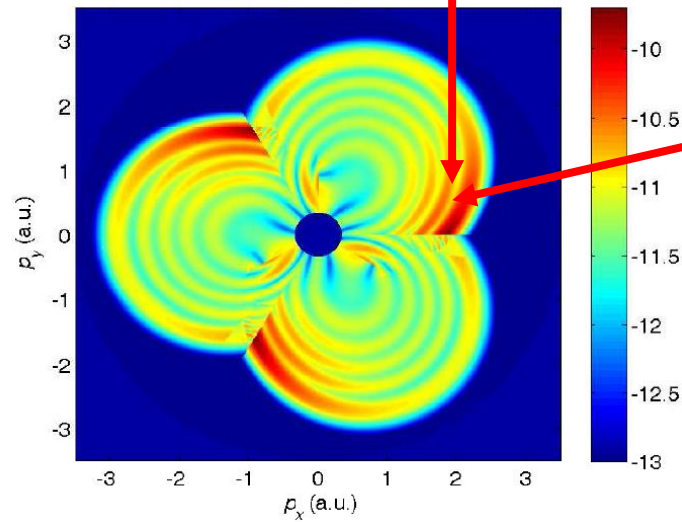
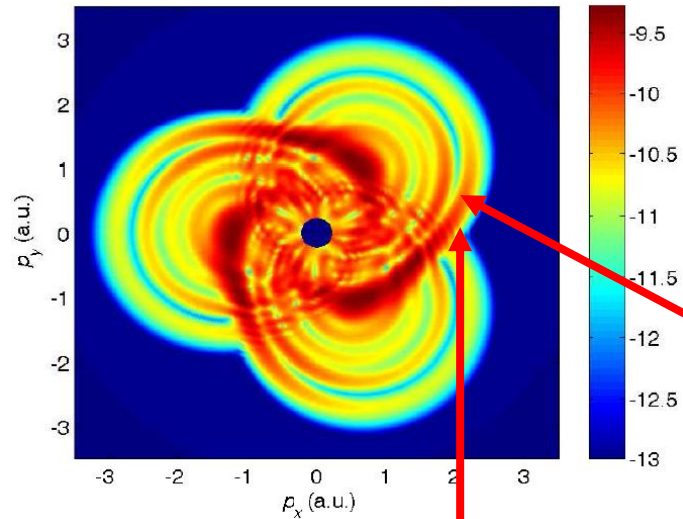


exact numerical

3 pairs of orbits as
 given on the left



Associating features of the velocity map with orbits of liberated electrons in space



Orthogonally polarized two-color OTC laser field

$$\mathbf{E}(t) = E_r \cos(r\omega t) \hat{\mathbf{e}}_x + E_s \cos(s\omega t + \phi) \hat{\mathbf{e}}_y$$

OTC $r\omega$ - $s\omega$ laser field with the relative phase ϕ

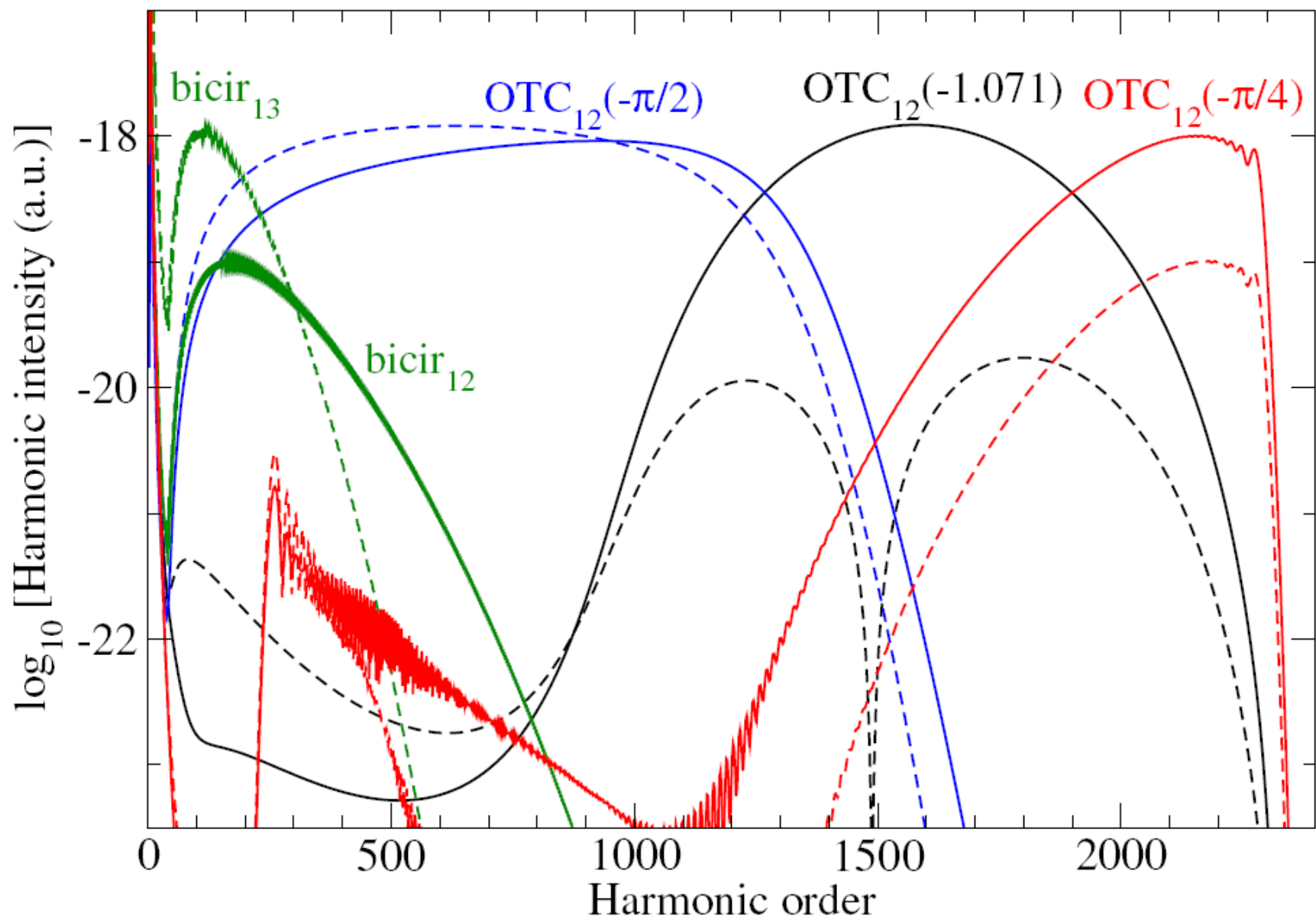
Milošević, Becker, Phys. Rev. A **100**, 031402(R) (2019)
 ω - 3ω , unusual shape, HH ellipticity.

PRA **102**, 023107 (2020): BEOTC, small $\varepsilon \rightarrow$ large ε_H

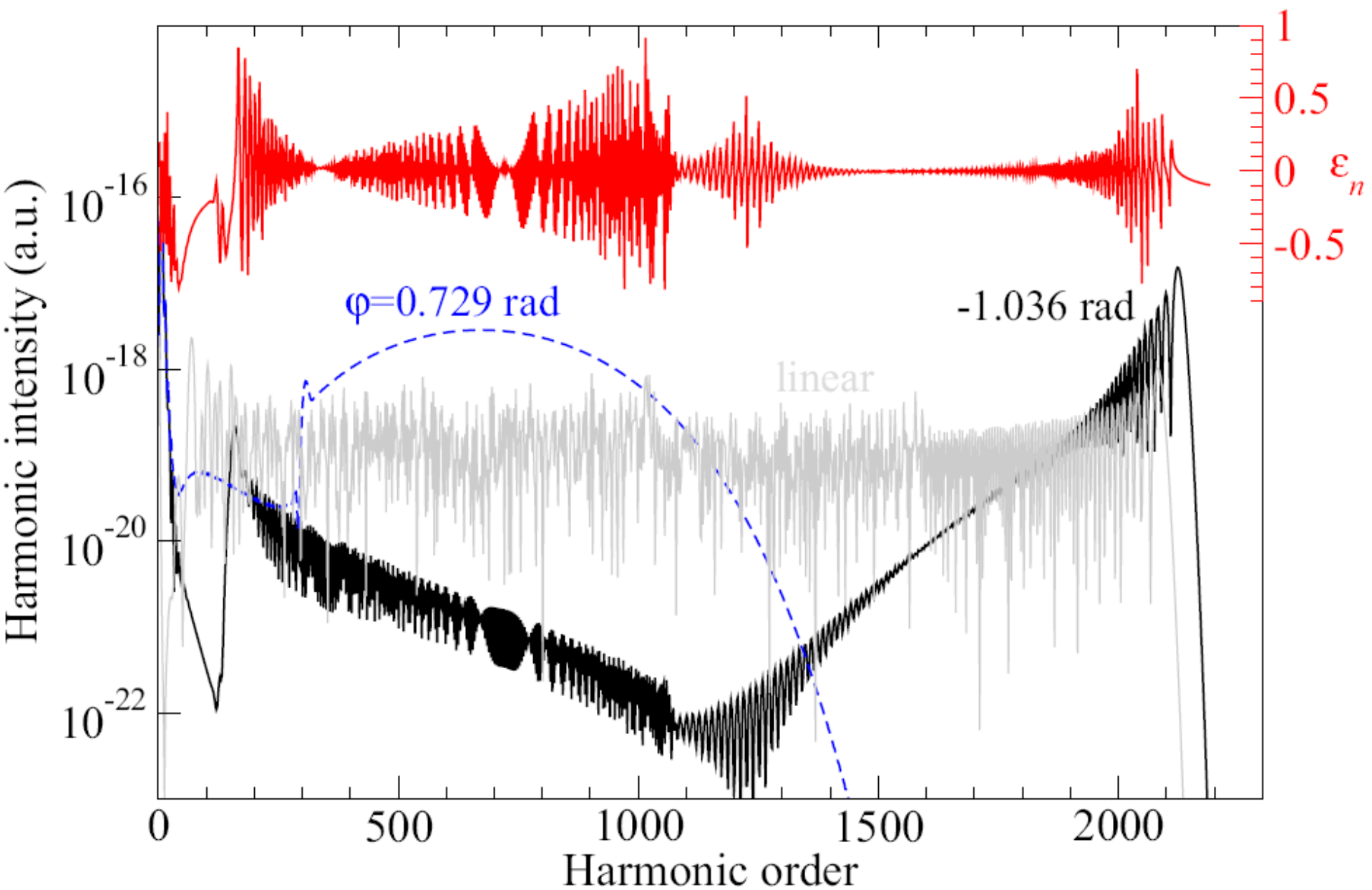
Habibović, Becker, Milošević: molecules

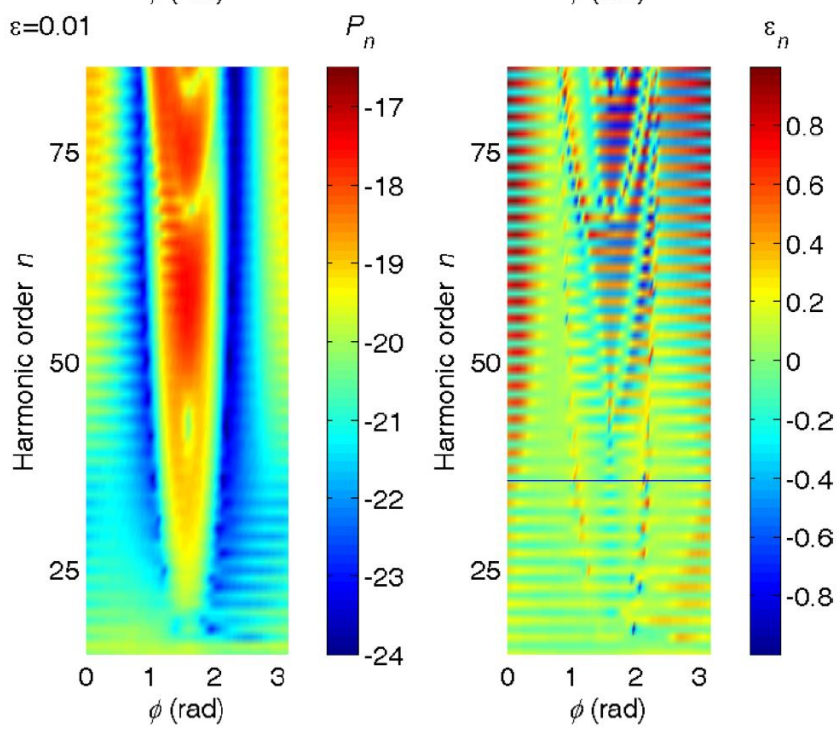
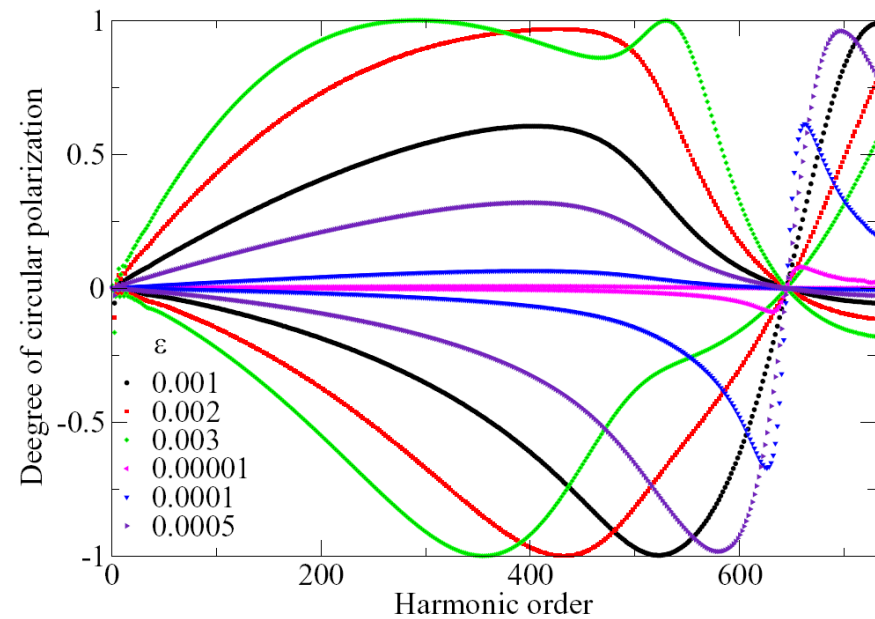
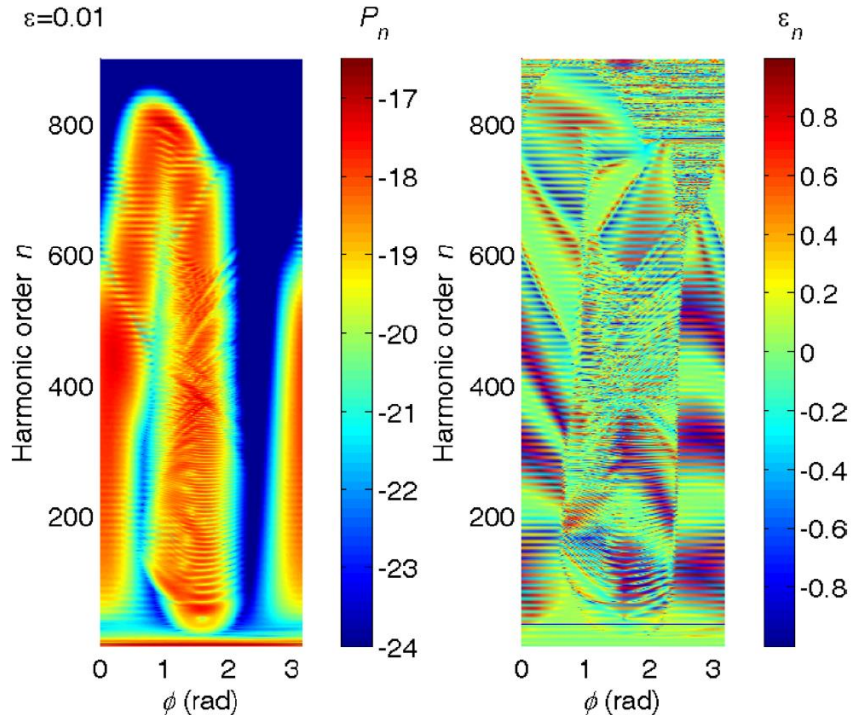
$$\mathbf{E}_1(t) = \frac{E_1}{\sqrt{1 + \varepsilon_1^2}} [\hat{\mathbf{e}}_x \sin(r\omega t) - \varepsilon_1 \hat{\mathbf{e}}_y \cos(r\omega t)]$$

$$\mathbf{E}_2(t) = \frac{E_2}{\sqrt{1 + \varepsilon_2^2}} [\hat{\mathbf{e}}_y \sin(s\omega t + \phi) - \varepsilon_2 \hat{\mathbf{e}}_x \cos(s\omega t + \phi)]$$



He, 2200 nm, $\omega-3\omega$, $I_1=8 \times 10^{14}$ W/cm², $I_3=7 \times 10^{14}$ W/cm²





$$T_{n\pm} \propto \int dt \int d\tau f(t, \tau) e^{\mp i\phi_q(t, \tau)}$$

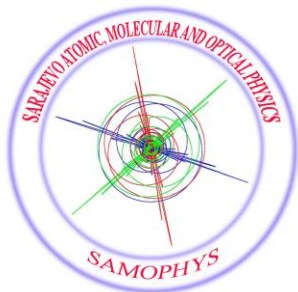
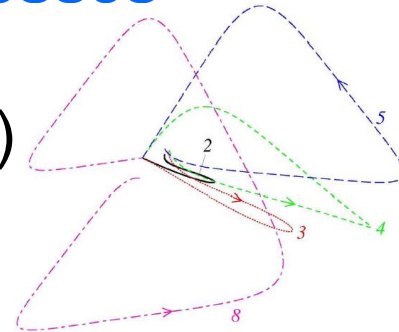
$$\xi_n = \frac{|T_{n+}|^2 - |T_{n-}|^2}{|T_{n+}|^2 + |T_{n-}|^2}$$

Small ε large difference
between T_{n+} and T_{n-}

Conclusions and perspectives

Bicircular and BEOTC field driven processes

- 1D (linear) \rightarrow 2D (trajectories unfold in a plane)



Elliptically polarized high harmonics (selection rules, chiral molecules, magnetic materials, etc.)

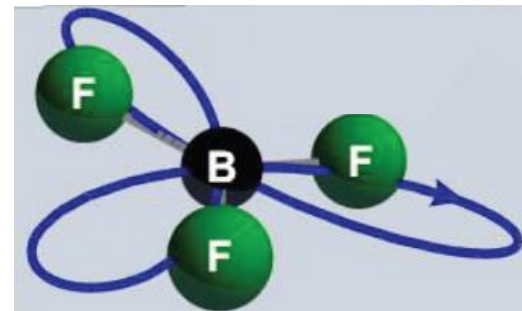
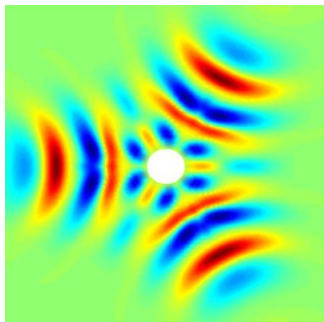
- Exploration of molecular symmetries using dynamical symmetry of the fields

PRA 94, 033419 (2016);

Mol. Phys. 115, 1750 (2017)

Spin polarized electrons – attospin

PRA 93, 051402(R) (2016); 98, 053420 (2018)

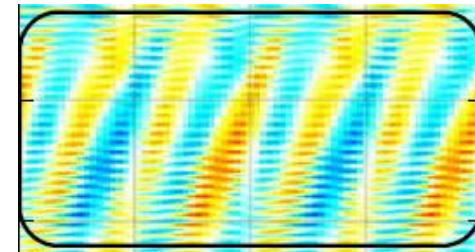


Conclusions and perspectives

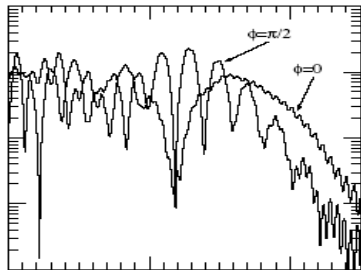
Various field / pulse combinations

- Few-cycle pulses, trifurcation

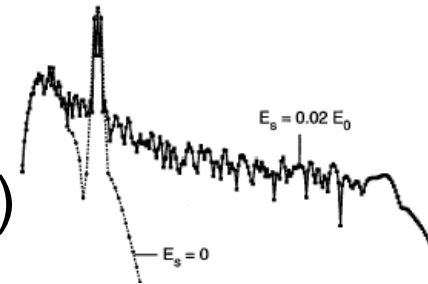
PRL 126, 113201 (2021)



- HHG – PRL 81, 1837 (1998)

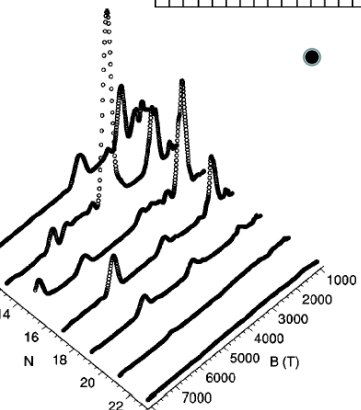


- Static electric field, PRL 81, 5097 (1998)

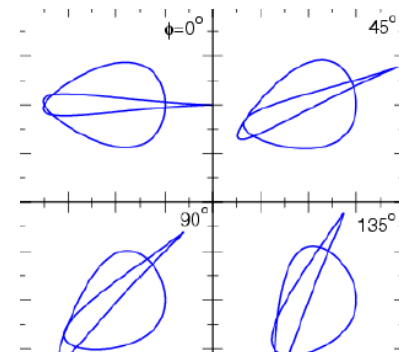


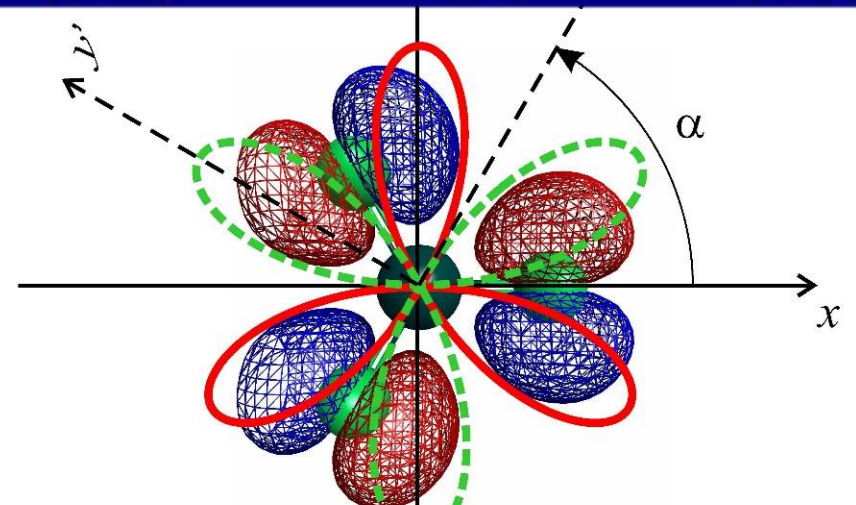
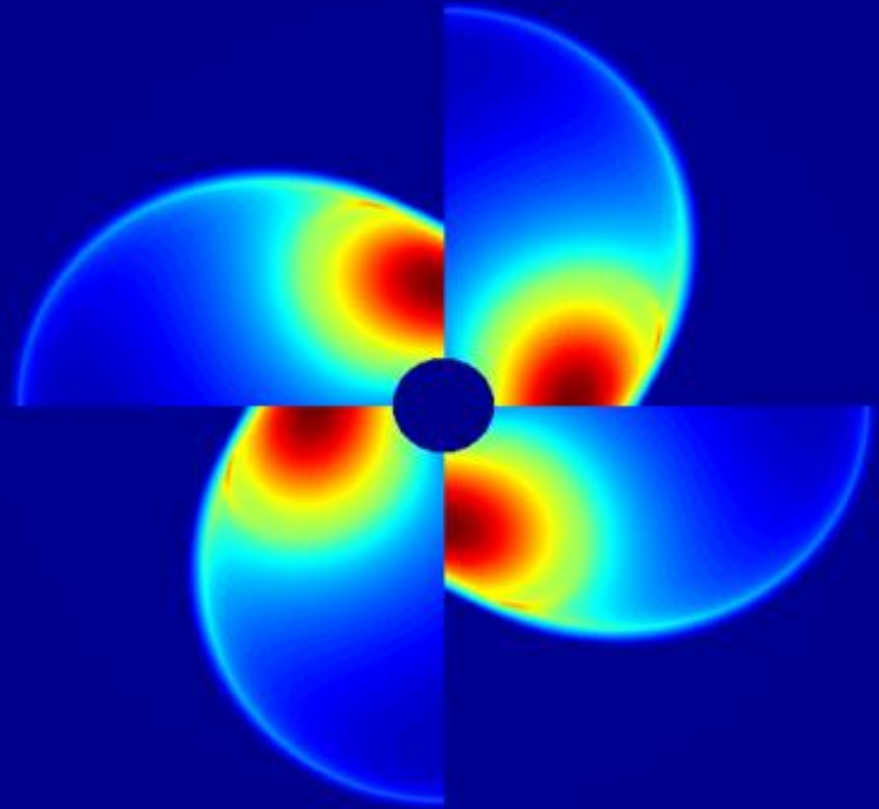
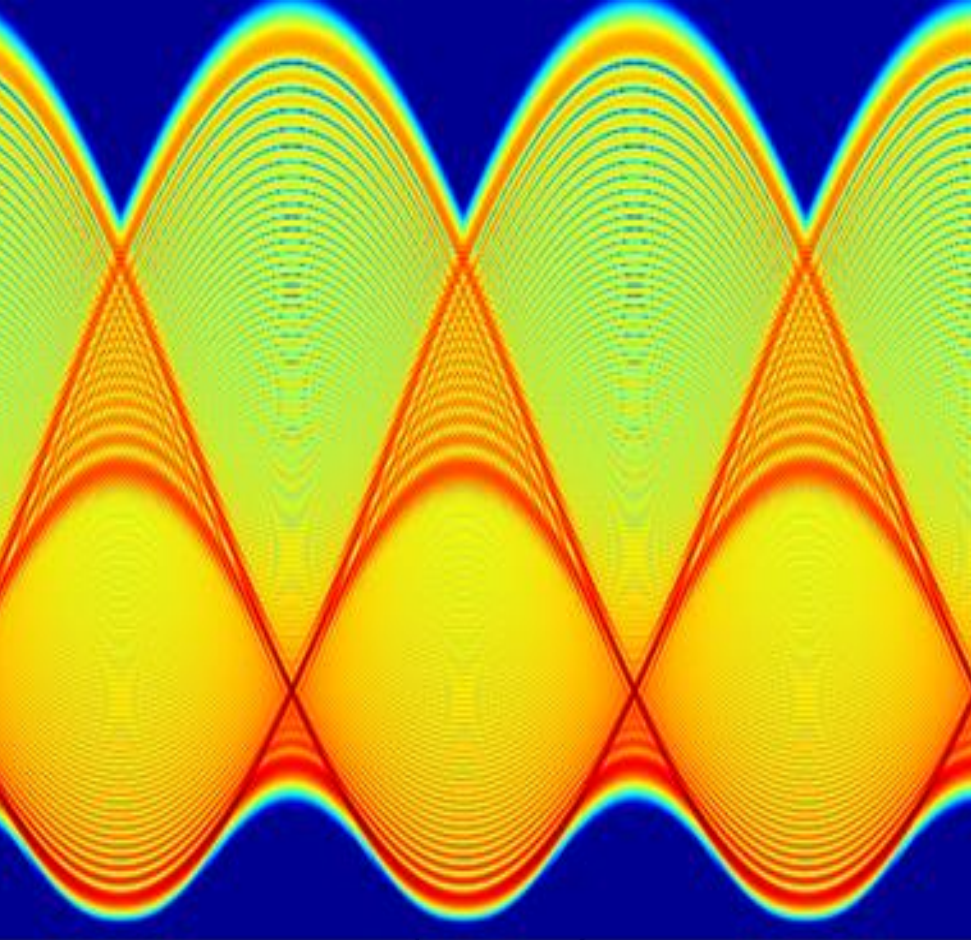
Magnetic-field-induced intensity revivals in HHG

Milosevic, Starace, PRL 82, 2653 (1999)



- HHG, cir + static, Phys. Lett. A 355, 368 (2006)
- Bicircular ω - ω fc, Las. Phys. Lett. 3, 200 (2006)
- OTC + crossing angle, ...





*Thank you for
your attention!*