BPU11 CONGRESS
International Conference of the Balkan Physical Union
28 August – 1 September 2022, Belgrade, Serbia

Domains of scaling in the three-dimensional random field Ising model

D. Spasojević[1]

[1]*Faculty of Physics, University of Belgrade, POB 44, 11001 Belgrade, Serbia*

[1] S. Janićević, D. Knežević, S. Mijatović and D. Spasojević, J. Stat. Mech. 2021, 013202 (2021).

MOTIVATION

Rising interest in the study of nonequilibrium properties of the systems that evolve through avalanche-like response to slowly varying external conditions.

Nonequilibrium zero-temperature random field Ising model – paradigm for disordered ferromagnetic systems

- Scaling in the thermodynamic limit

Two domains
- $R < R_c$
- $R > R_c$

➢ **What with systems of finite size?**

Three domains
- $R < R_c$
- $R_c < R < R_c^{\mathrm{eff}}(L)$
- $R > R_c^{\mathrm{eff}}(L)$

➢ **Motivation:** To identify and make a clear distinction between the different domains of scaling in finite systems.

* Findings could be of importance for mesoscopic experimental systems (magnetic/superconducting/martensitic) and other finite model systems manifesting criticality in their avalanche-like response in the thermodynamic limit (percolation/front propagation model of ferromagnets) having three (or more) domains of the related criticality controlling variable that have to be distinguished in their scaling analysis.

NONEQUILIBRIUM ZERO-TEMPERATURE RANDOM FIELD ISING MODEL (NE-ZT-RFIM)

- Hamiltonian:

$$\mathcal{H} = -J\sum_{\langle i,j\rangle} s_i s_j - H\sum_i s_i - \sum_i h_i s_i$$

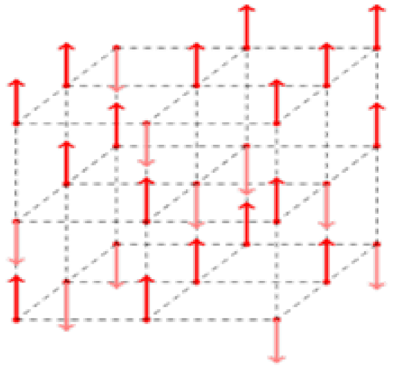
(responsible for equilibrium properties)

- ferromagnetic interaction between nearest neighboring spins s_i, s_j $(J = 1)$

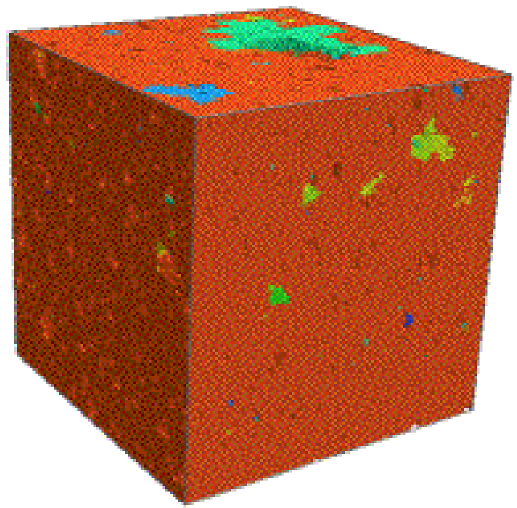
- external magnetic field H increased adiabatically, i.e. to the minimal value causing avalanche and kept constant during its propagation

- random magnetic field h_i chosen so that $\langle h_i \rangle = 0, \langle h_i h_j \rangle = 0$; here from the Gaussian distribution

$$\rho(h) = \frac{1}{\sqrt{2\pi}R}\exp(-h^2/2R^2)\,.$$



Ising spins $s_i = \pm 1$



Nonequilibrium dynamics is set by the

Local dynamical rule: spin flips when $\mathrm{sgn}\left(s_i, h_i^{\mathrm{eff}}\right) < 0$, where

$h_i^{\mathrm{eff}} = J\sum_j s_j + H + h_i$ is the effective magnetic field acting on spin s_i

RESULTS

Numerical simulations of 3D NE-ZT-RFIM

- **Lattice geometry:** cubic, equilateral
- **Boundary conditions:** periodic
- **Magnetic field range:** along the whole rising part of hysteresis loop
- **Driving type:** adiabatic
- **Critical disorder:** R_c= 2.16
- **Critical magnetic field:** H_c=1.435

<u>**Preceding studies**</u> (mostly by Sethna et al) **system size:** up to 10^9 spins (linear dimension L up to 1024)

This study:

✓ **System size:** up to $8.5 \cdot 10^9$ spins (linear dimensions in range $L = 16 - 2048$ spins)

✓ **Disorder:** domains $R < R_c$ (low), $R_c < R < R_c^{\text{eff}}(L)$ (transitional), $R > R_c^{\text{eff}}(L)$ (high)

✓ **Statistics:** data averaged over up to $200\ 000$ realizations of random magnetic field

DOMAINS OF SCALING IN FINITE SYSTEMS

1. Domain below R_c

Correlation length is infinite, only $3D$ spanning avalanches exist

2. Transitional domain $R_c < R < R_c^{\mathrm{eff}}(L)$

(vanishing in thermodynamical limit)

Finite, but large correlation length, all kinds of spanning avalanches are likely to appear

3. Domain above $R_c^{\mathrm{eff}}(L)$

We proved that the system response is INDEPENDENT on system size L; all avalanches are finite

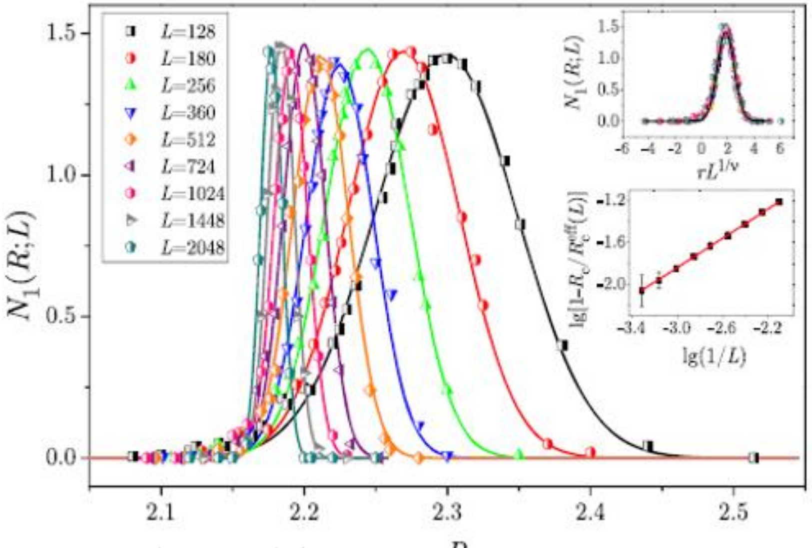
Three domains

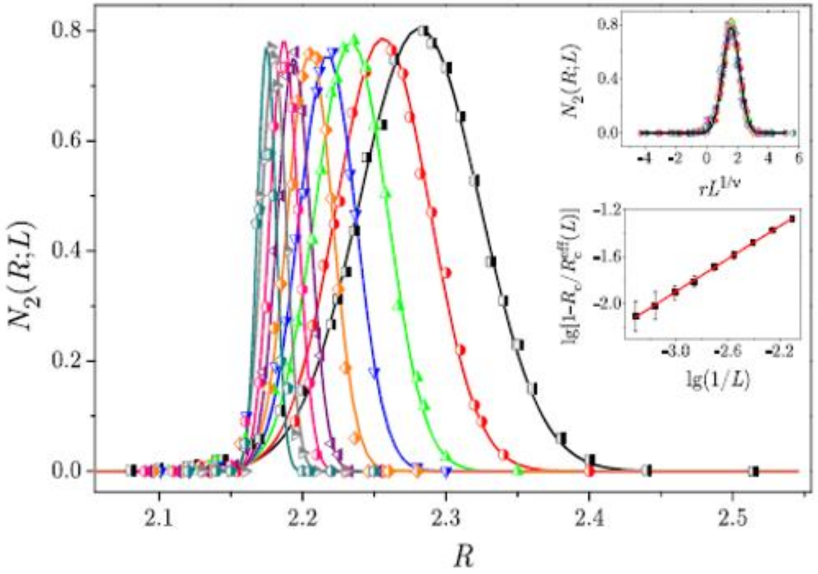
- $R < R_c$
- $R_c < R < R_c^{\mathrm{eff}}(L)$
- $R > R_c^{\mathrm{eff}}(L)$

$R_c^{\mathrm{eff}}(L)$ - the effective critical disorder, separating the domains in which the spanning avalanches appear/disappear

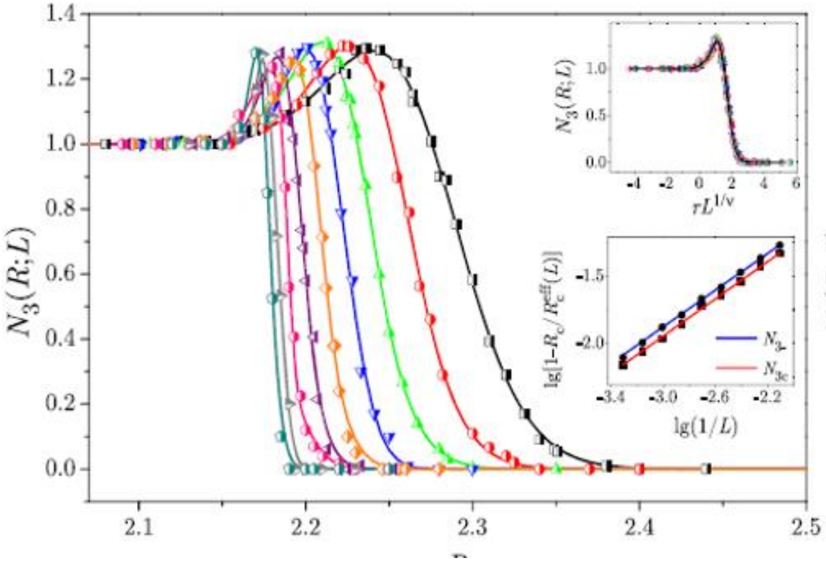
Transition between the domain above and below is not sharp, but has a width $W(L)$, meaning that in the domain $R > R_c^{\mathrm{eff}}(L)$ spanning avalanches may still appear provided that $R - R_c^{\mathrm{eff}}(L) \leq W(L)$.

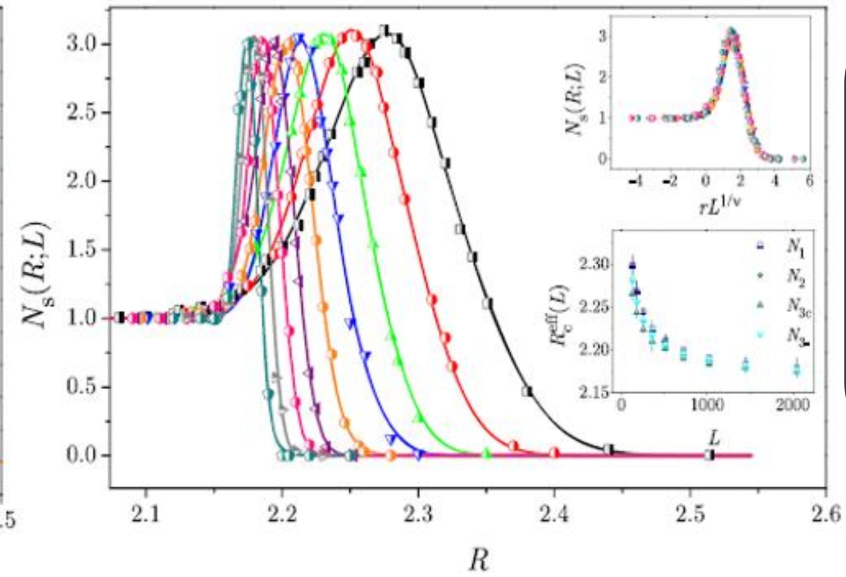
NUMBER OF SPANNING AVALANCHES





$r \equiv (R - R_c)/R$





- **Spanning avalanches** – avalanches that span the system per at least one of its dimensions.

In $3D$ cubic systems can be classified as: $1D, 2D,$ and $3D$ according to the number of spatial directions they span.

$$N_s = N_1 + N_2 + N_3$$

- Fits of N_1 and N_2 to Gaussians

$$y_G(x) = h_G \ \exp[-(x - x_c)^2/2w_x^2]$$

and N_3 to

$$\tilde{N}_3^{\mathrm{mod}}(x) = H_{3c} \ \exp[-(x - c_{3c})^2/2w_{3c}^2] + $$
$$+ 0.5\mathrm{erfc}\ [(x - c_{3-})/w_{3-}]$$

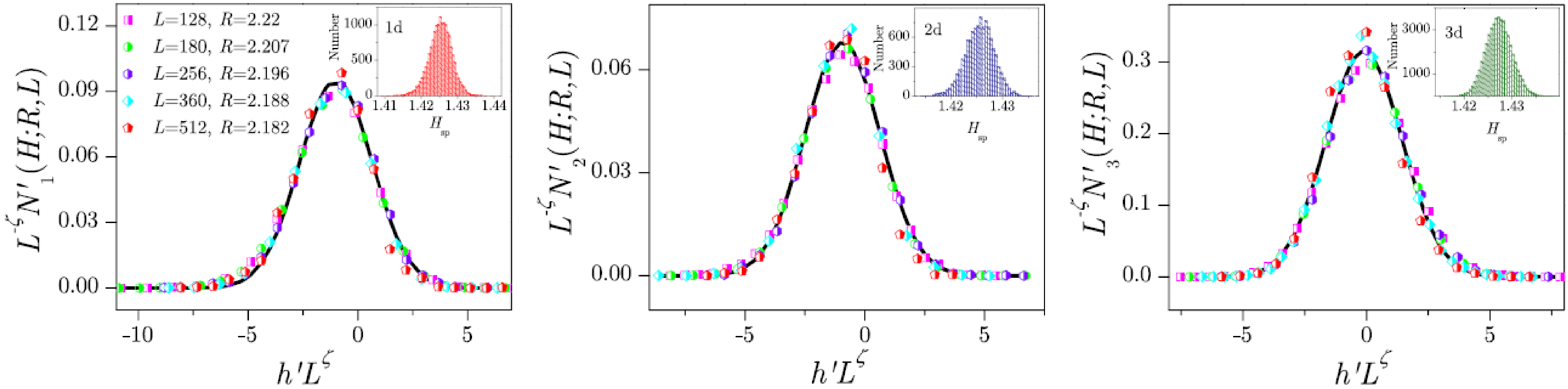
$$\boxed{N_\alpha(R, L) = L^\theta \tilde{N}_\alpha(rL^{1/\nu})}$$

$\theta_\alpha = 0, \ \alpha = 1, 2, 3, s$ $\quad (r = (R - R_c)/R \ ; \ 1/\nu = 0.70)$

$R_c^{\mathrm{eff}}(L)$ estimates

DISTRIBUTIONS OF SPANNING FIELD

- **Spanning field, H_{sp}** – the value of the external magnetic field which triggered the spanning avalanche.



The distribution $N_\alpha(H_{\mathrm{sp}}; R, L)$ of spanning field H_{sp} satisfies:

$$\boxed{N'_\alpha(H_{\mathrm{sp}}\,; R, L) = L^\zeta \widetilde{N}'_\alpha(h'_{\mathrm{sp}} L^\zeta, rL^{1/\nu})}$$

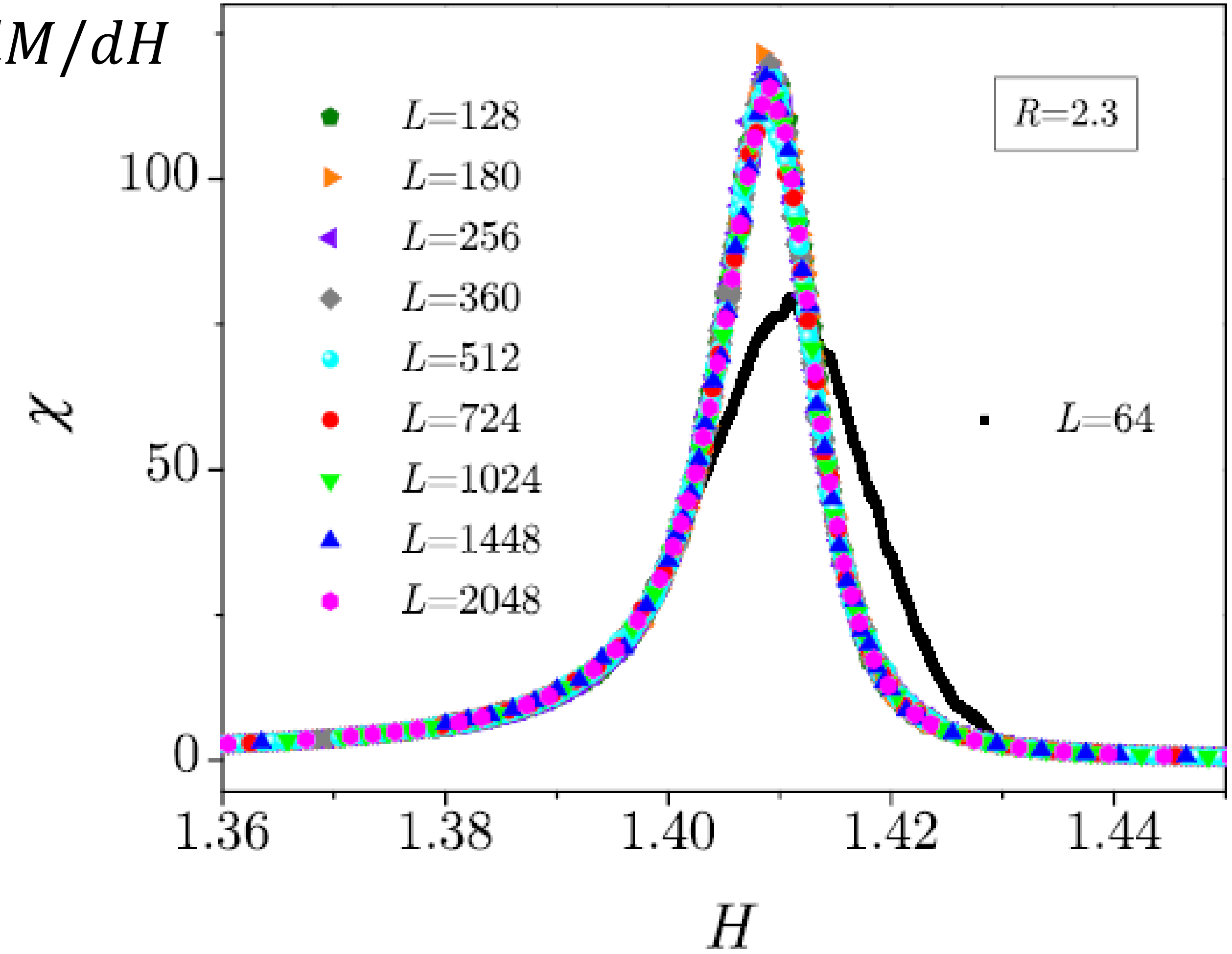
$\zeta = \beta\delta/\nu = 1.35$ is mutual critical exponent,

$$h'_{\mathrm{sp}} = H_{\mathrm{sp}} - H_{\mathrm{c}}^{\mathrm{eff}}(R, L)$$

*is the **reduced spanning field***, while $H_c^{\mathrm{eff}}(R, L)$ is the effective critical field (i.e. the value of H at which the susceptibility curve for disorder R and lattice size L attains its maximum).

SUSCEPTIBILITIES, $R > R_c^{\mathrm{eff}}(L)$

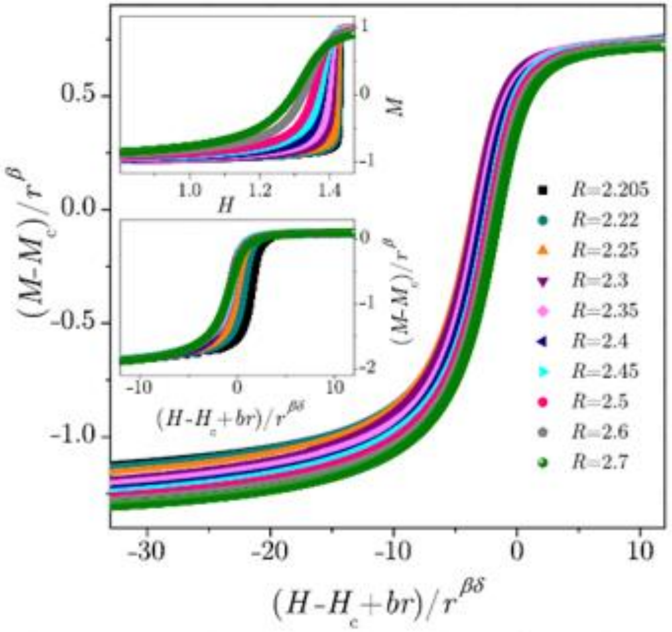
susceptibility $\chi = dM/dH$



Susceptibility curves in a broad range of lattice sizes obtained for the same value of disorder surpassing the effective critical $R_c^{\mathrm{eff}}(L)$.

- We found that in this disorder domain the susceptibility curves and the values of effective critical field $H_c^{\mathrm{eff}}(L)$ are **independent on the lattice size L**!

MAGNETIZATIONS AND SUSCEPTIBILITIES (SAME $L, R > R_c^{\mathrm{eff}}(L)$)



$\boxed{H_c^{\mathrm{eff}}(R) \cong H_c - br}$

(linear approximation)

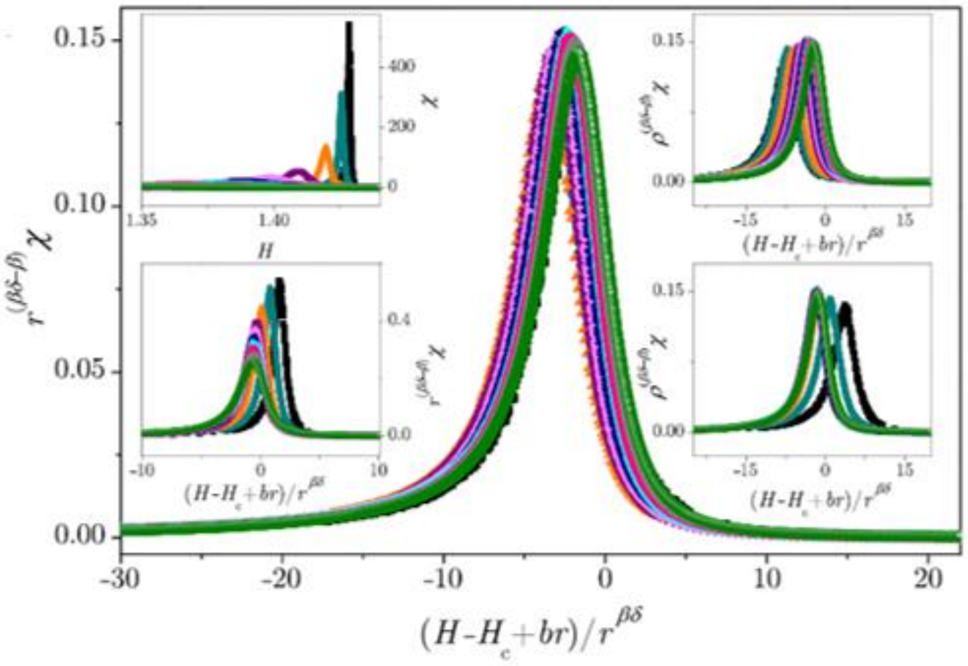
- rotational parameter b, no collapse of satisfactory quality

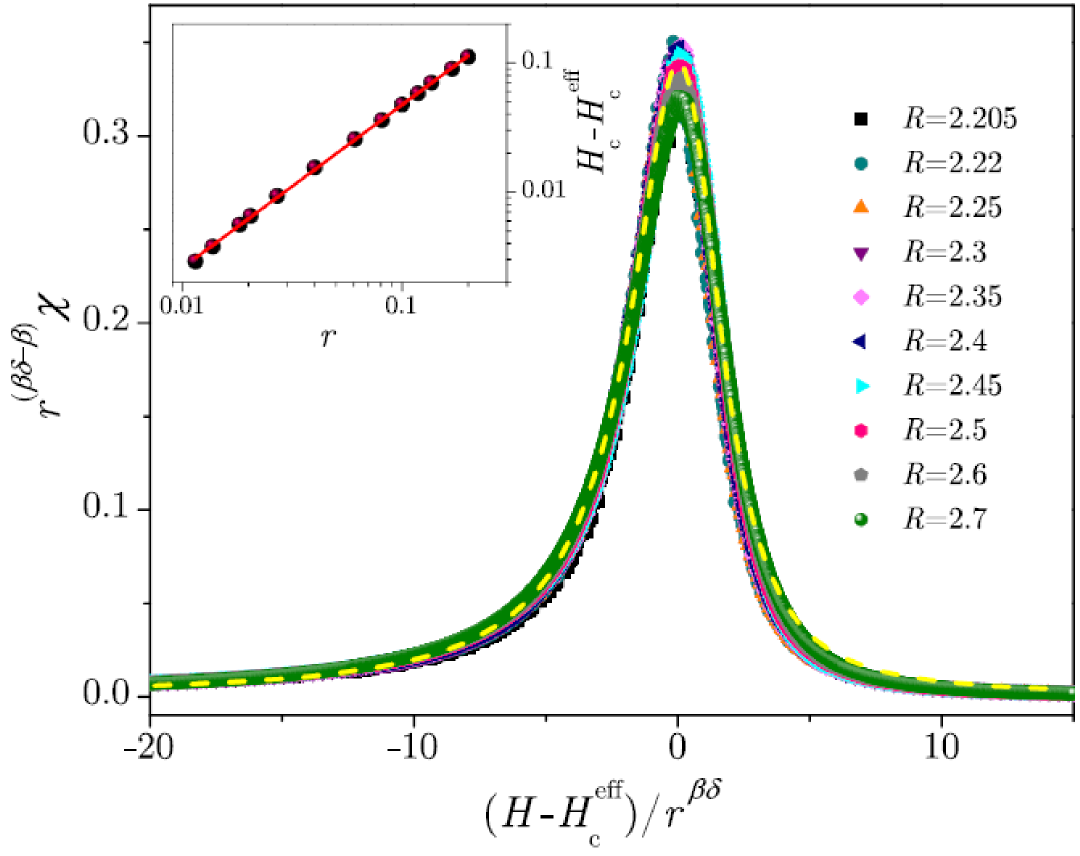
(e.g., B=0.39, Sethna et al, *Phys. Rev.* B **59** 6106 (1999))

$\leftarrow (b = 0.39, M_c = 0.9, \beta = 0.035, \beta\delta = 1.81)$

➢ Instead, the ***effective reduced magnetic field*** $\boxed{\hat{h}_R^{\mathrm{eff}} \equiv H - H_c^{\mathrm{eff}}(R)}$

$\boxed{H_c^{\mathrm{eff}}(R) \approx H_c - b_h r^{c_h}}$

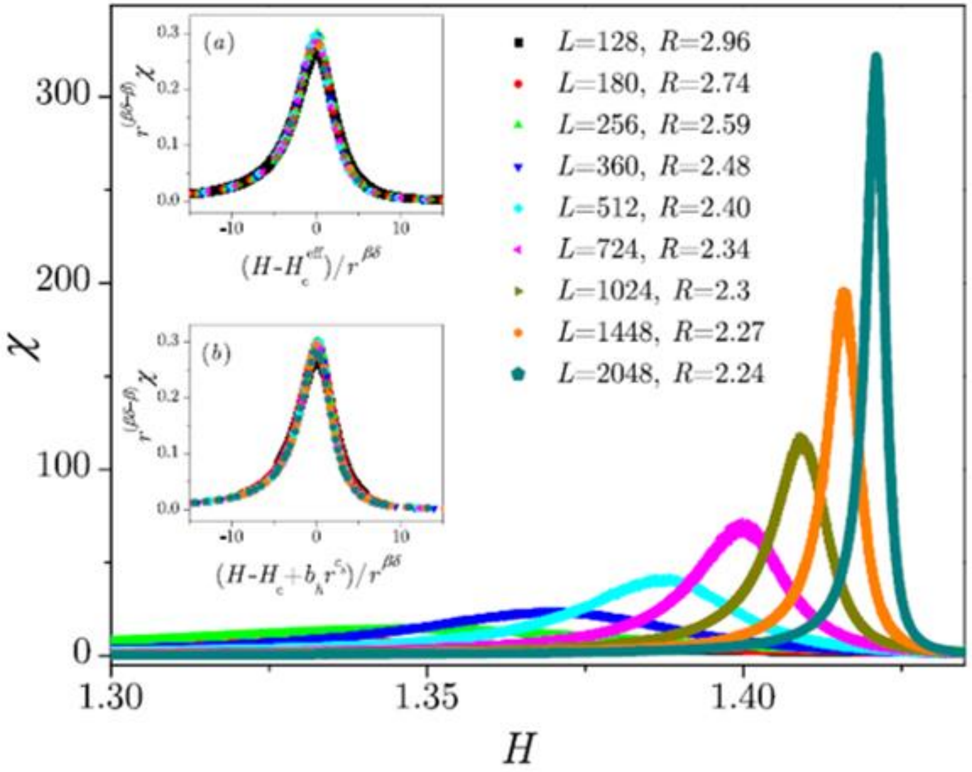




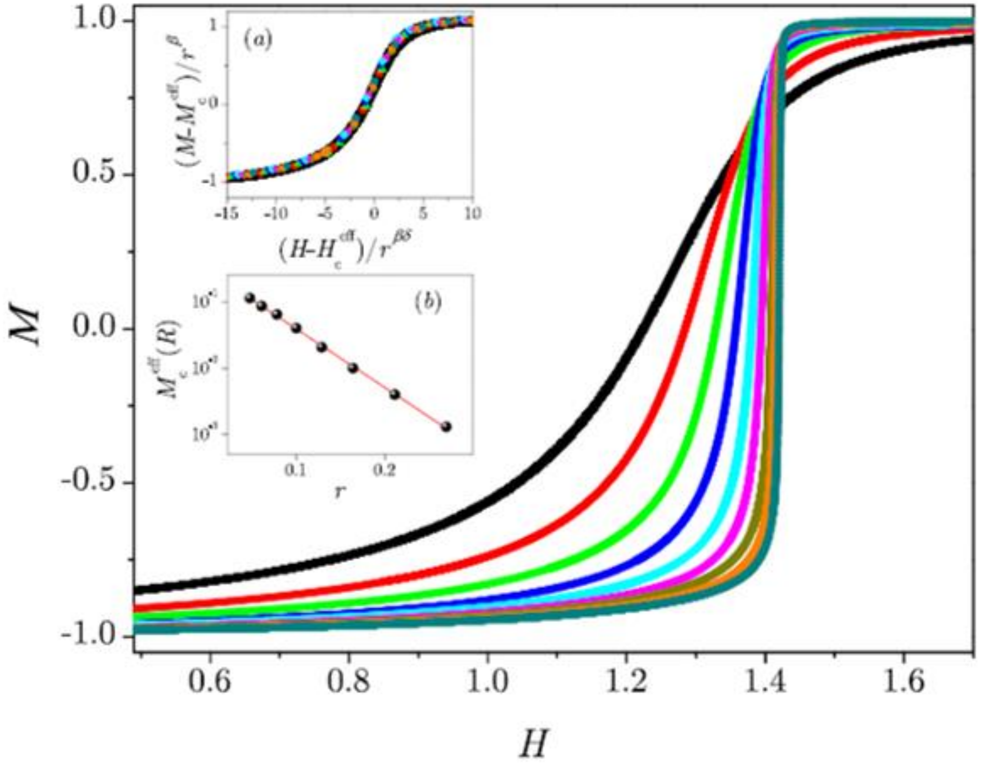
$Lr^{1/\nu} = const$ $R > R_c^{\text{eff}}(L)$

collapse with standard 3D **NE-ZT-RFIM** exponents

$$\chi_{R,L}(H) = |r|^{\beta-\beta\delta}\widetilde{\mathcal{X}}_{\pm}(h'/|r|^{\beta\delta}, 1/L|r|^{\nu})$$



$$m_{R,L}(H) = |r|^{\beta}\widetilde{\mathcal{M}}_{\pm}(h'/|r|^{\beta\delta}, 1/L|r|^{\nu})$$



- Problem of determining the value of critical magnetization M_c

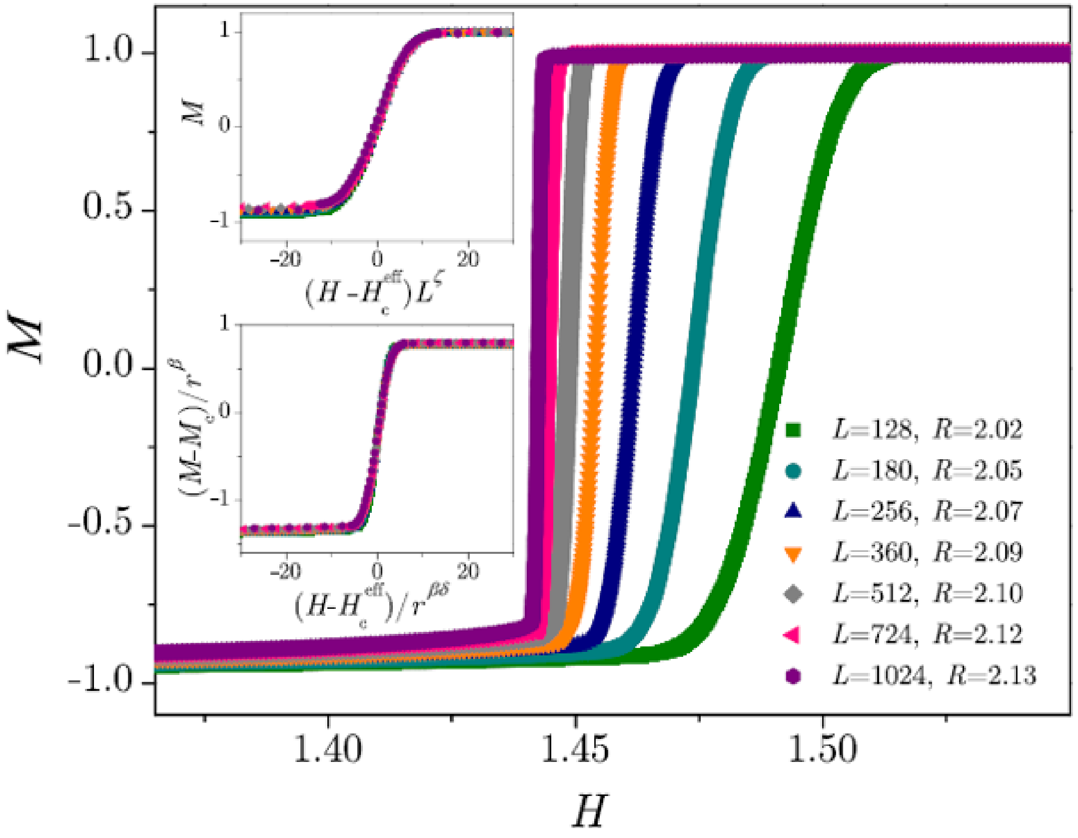
✓ Finding the effective critical magnetization $M_c^{\text{eff}}(R) = M_R(H_c^{\text{eff}}(R))$ and extrapolating data to $r = 0$; $M_c = 0.30 \pm 0.12$

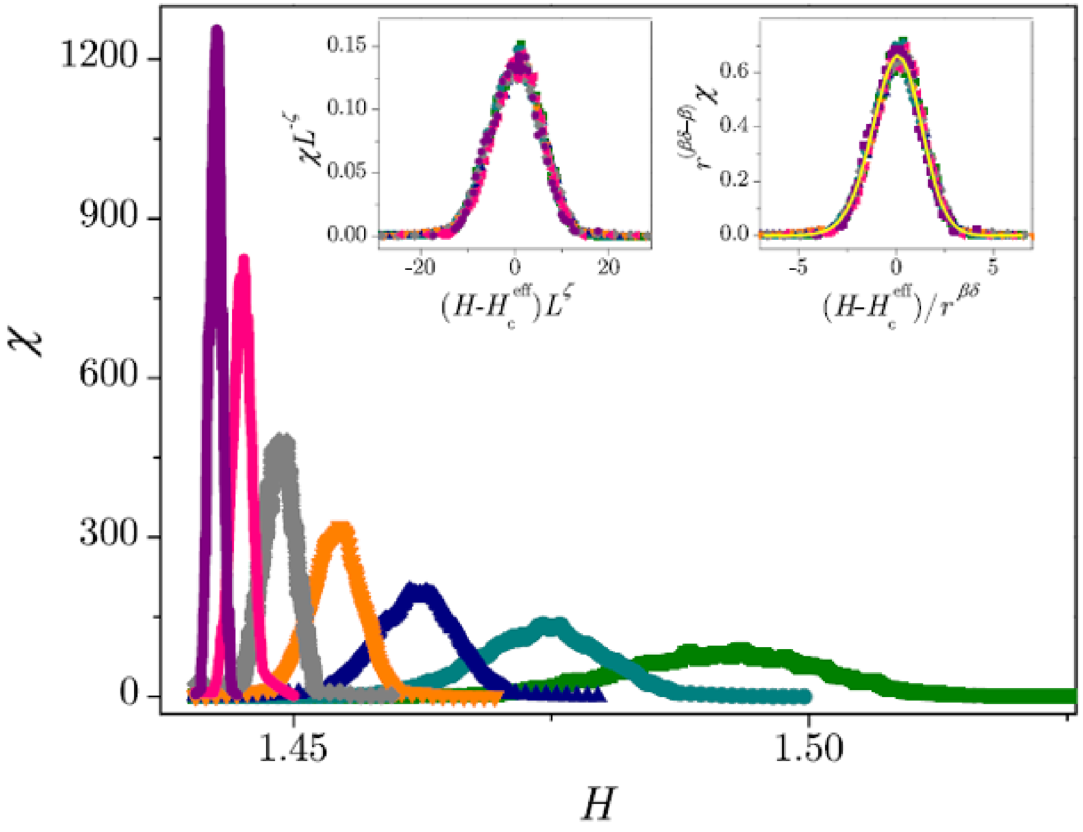
✓ Collapsing by using the ***effective reduced magnetization*** $\hat{m}_R(H) \equiv M - M_c^{\text{eff}}(R)$; $M_c^{\text{eff}}(R) = M_c e^{-c_m r}$

$Lr^{1/\nu} = const$ $R < R_c$

$$M_{R,L}(H) = |r|^{\beta}\widetilde{\mathcal{M}}_{-}(\hat{h}_R^{\text{eff}}/|r|^{\beta\delta}, 1/L|r|^{\nu})$$

$$\chi_{R,L}(H) = |r|^{\beta-\beta\delta}\widetilde{\mathcal{X}}_{-}(\hat{h}_R^{\text{eff}}/|r|^{\beta\delta}, 1/L|r|^{\nu})$$





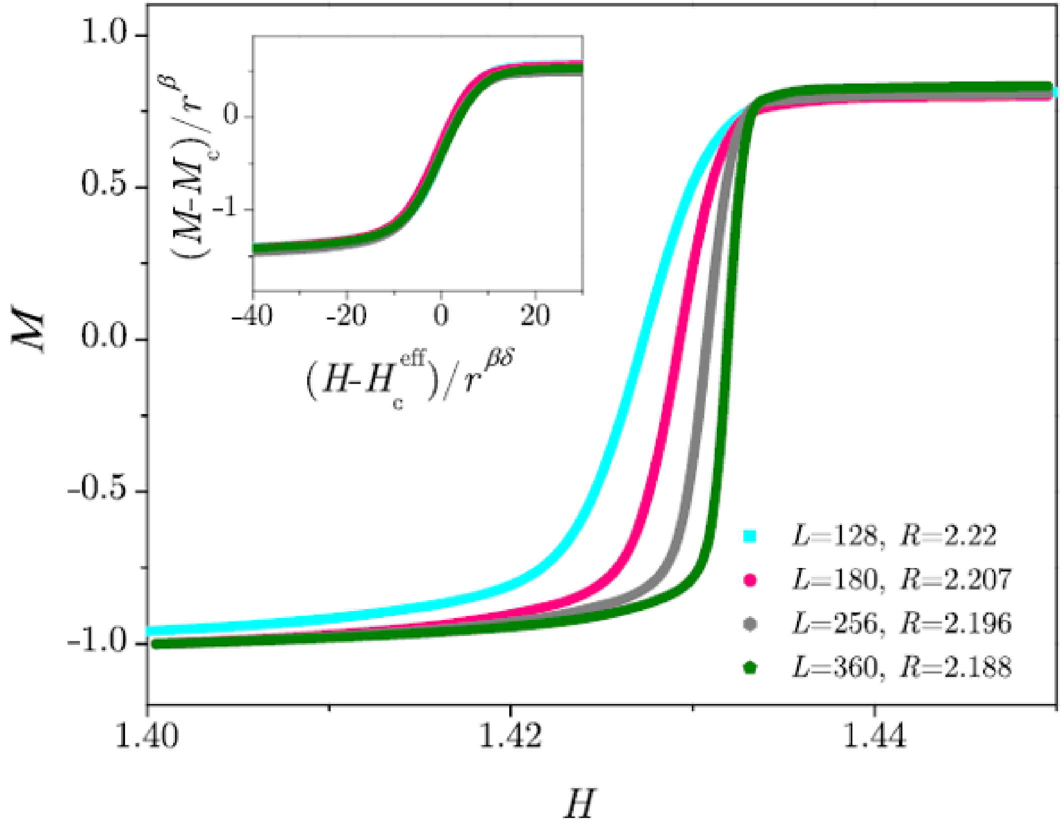
Size-dependent behavior with notable jumps due to appearance of spanning avalanche.

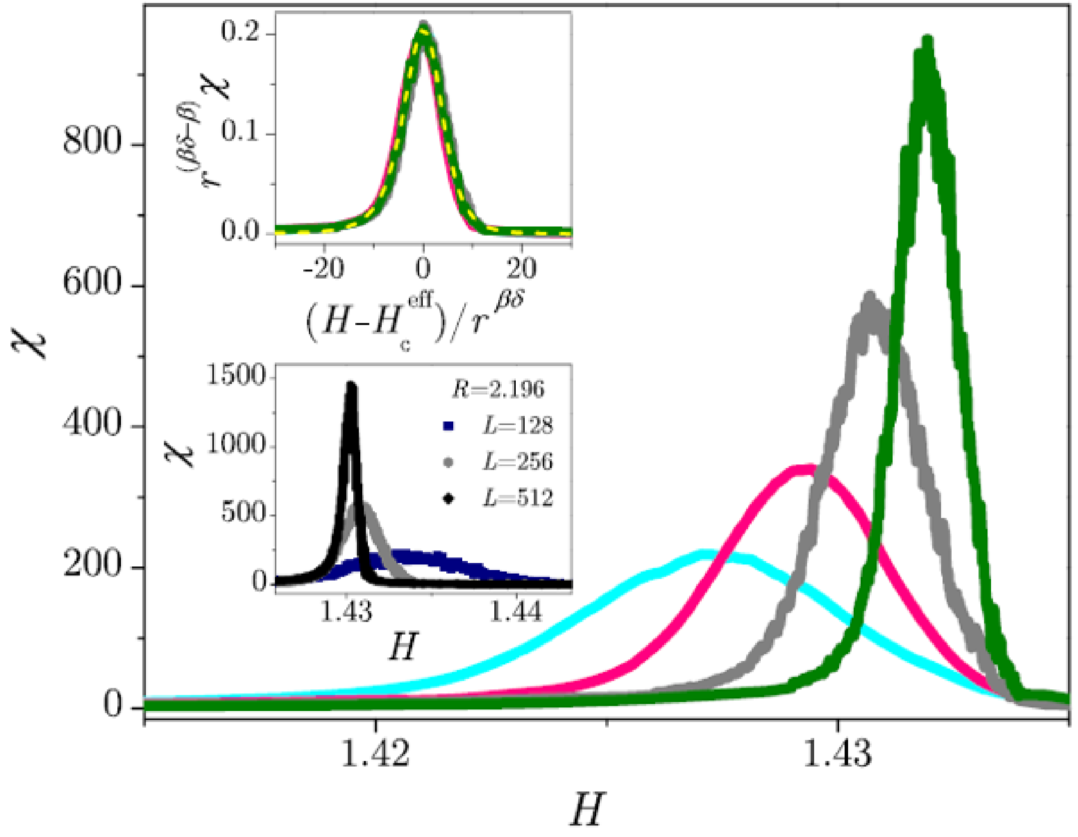
$Lr^{1/\nu} = const$ $R_c < R < R_c^{\mathrm{eff}}(L)$

* transitional domain of disorders

$$M_{R,L}(H) = |r|^{\beta}\widetilde{\mathcal{M}}_{+/-}(\hat{h}_R^{\mathrm{eff}}/|r|^{\beta\delta}, 1/L|r|^{\nu})$$

$$\chi_{R,L}(H) = |r|^{\beta-\beta\delta}\widetilde{\mathcal{X}}_{+/-}(\hat{h}_R^{\mathrm{eff}}/|r|^{\beta\delta}, 1/L|r|^{\nu})$$





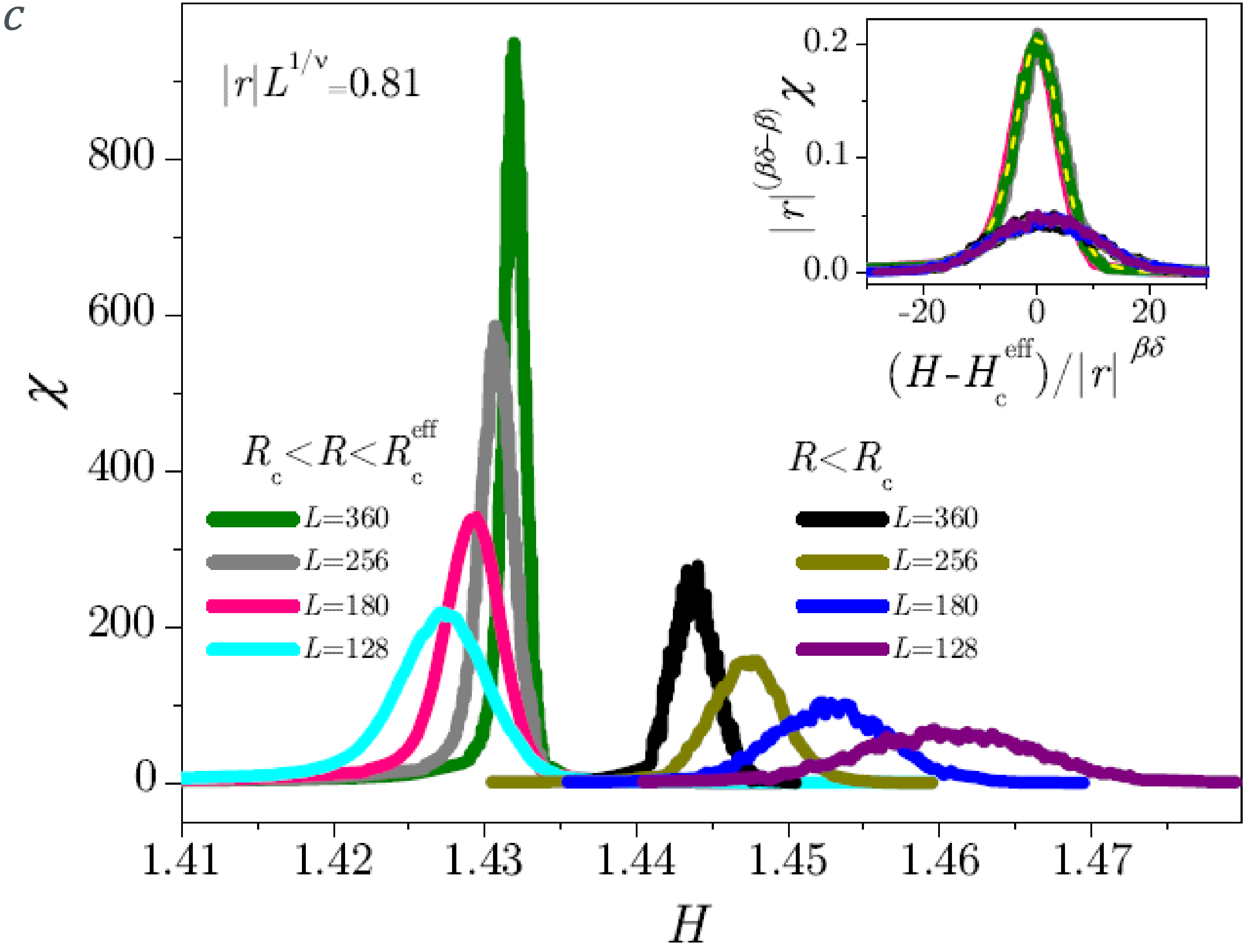
Scaling in the analogous way as the magnetizations and susceptibilities below R_c .

COMPARISON OF THE UNIVERSAL SCALING FUNCTIONS FOR THE SUSCEPTIBILITIES PERTAINING TO THE DOMAINS $R_c < R < R_c^{\text{eff}}(L)$ AND $R < R_c$

✓ Universal scaling functions are different below R_c and in the transitional region!

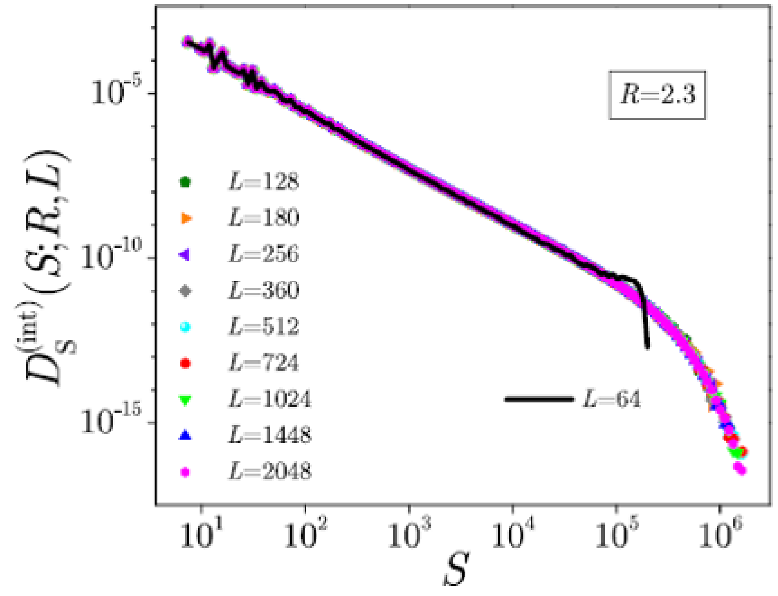
$$\widetilde{\mathcal{X}}_{-}((H - H_c^{\text{eff}})/|r|^{\beta\delta}, |r|L^{1/\nu})$$

$$\widetilde{\mathcal{X}}_{+/-}((H - H_c^{\text{eff}})/|r|^{\beta\delta}, |r|L^{1/\nu})$$

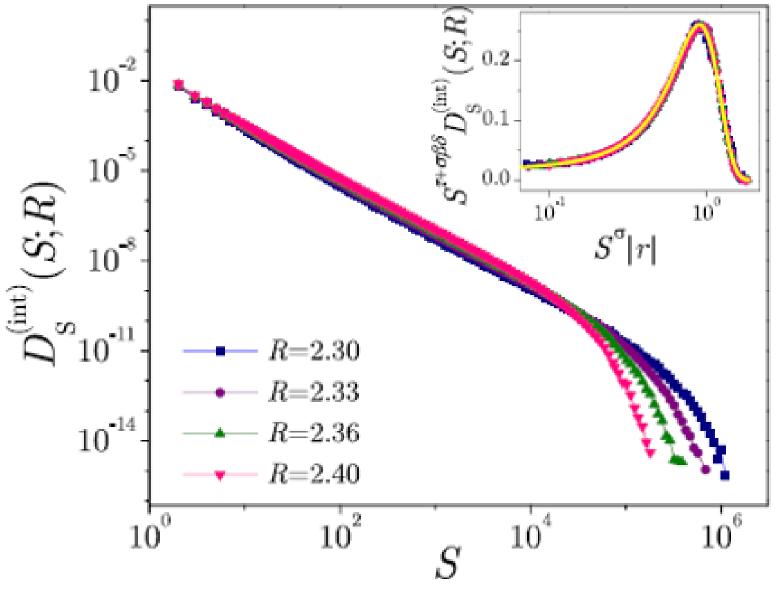


INTEGRATED AVALANCHE SIZE DISTRIBUTIONS

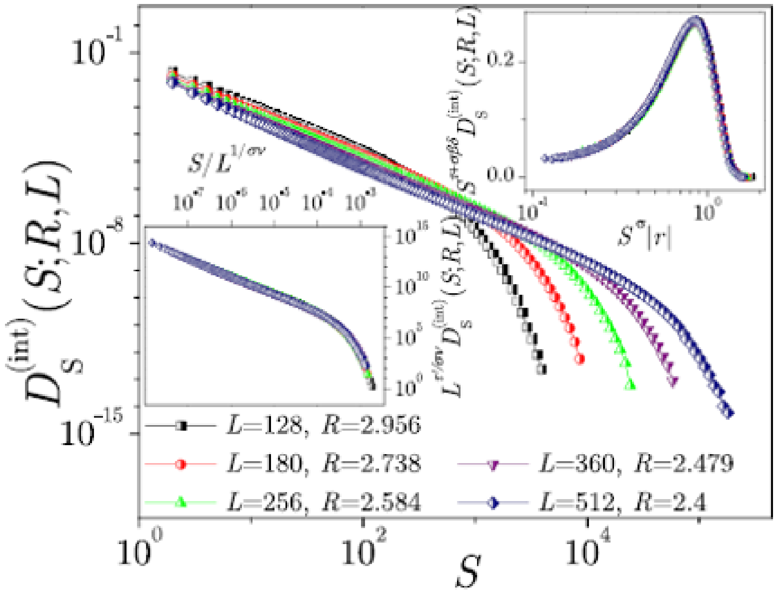
$R > R_c^{\text{eff}}(L)$



$L = 512$



$Lr^\nu = 19.92$



$$\boxed{D_S^{(\text{int})}(S\,;R) = S^{-(\tau+\sigma\beta\delta)}\widetilde{\mathcal{D}}_{S+}^{(\text{int})}(S^\sigma r)}$$

$(\tau + \sigma\beta\delta = 2.02, \sigma = 0.24)$

Phenomenological form: $\bar{\mathcal{D}}_{S+}^{(\text{int})}(X) = \mathrm{e}^{-0.789X^{1/\sigma}} \times (0.021 + 0.002X + 0.531X^2 - 0.266X^3 + 0.261X^4)\,;\ X = S^\sigma r.$

Analytical form: $\bar{\bar{\mathcal{D}}}_{S+}^{(\text{int})}(X) = A\ \exp\left[(X/C_1)^{d_1} - (X/C_2)^{d_2}\right]$

$(A = 0.0147, C_1 = 0.185, d_1 = 0.831, C_2 = 0.941, d_2 = 3.52)$

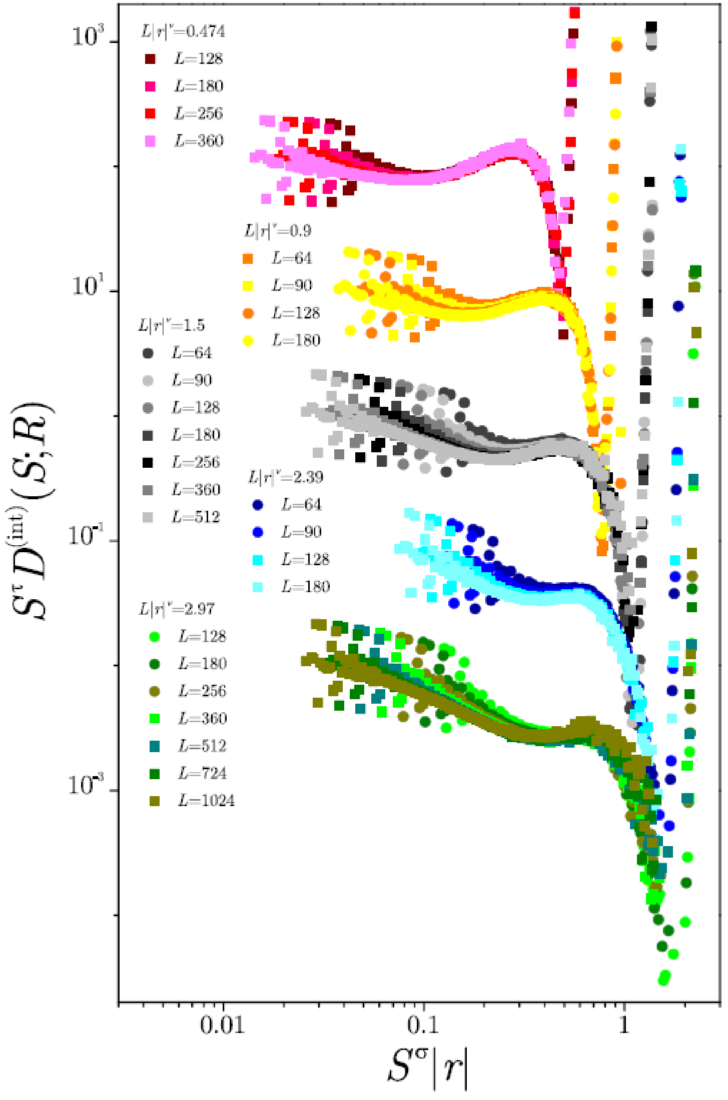
$(\tau' = 2.03, 1/\sigma\nu = 2.98, \nu = 1.41)$

$$\boxed{D_S^{(\text{int})}(S\,;R,L) = L^{-\tau'/\sigma\nu}\,\tilde{D}_{S+}(S/L^{1/\sigma\nu}, 1/Lr^\nu)}$$

INTEGRATED AVALANCHE SIZE DISTRIBUTIONS

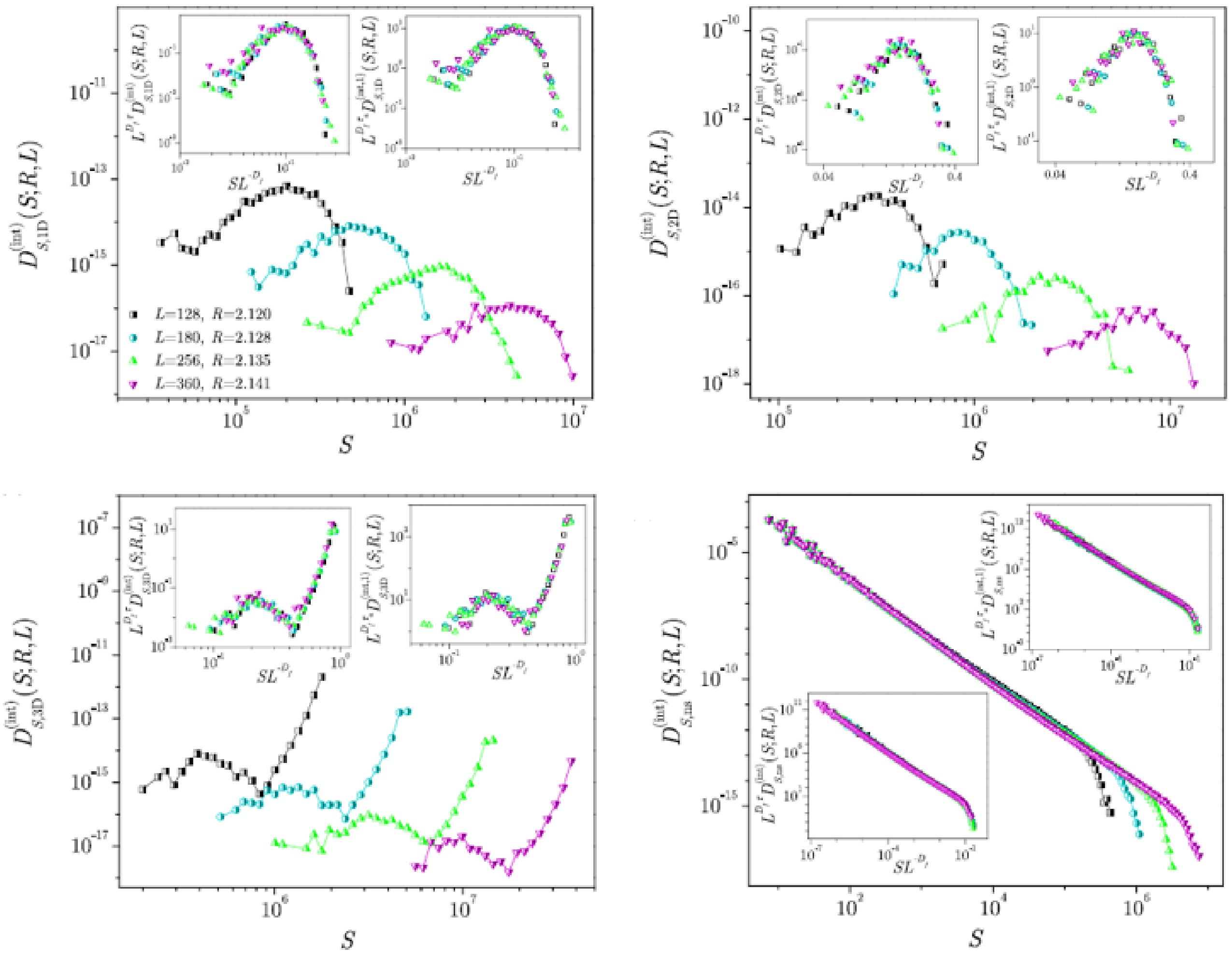
$R < R_c$

decomposition: $D_S^{(\mathrm{int})}(S\,;R,L) = \sum_{\alpha=1,2,3-} D_{S,\alpha}^{(\mathrm{int})}(S\,;R,L) + D_{S,\mathrm{ns}}^{(\mathrm{int})}(S\,;R,L)$



$\tau' = \tau + \sigma\beta\delta$

$D_f = 1/\sigma\nu$



(left insets: $\tau' = 2.03$)

$D_S^{(\mathrm{int})}(S\,;R,L) = S^{-\tau'}\widetilde{\mathcal{D}}_{S-}^{(\mathrm{int})}(S^\sigma|r|\,;L|r|^\nu)$

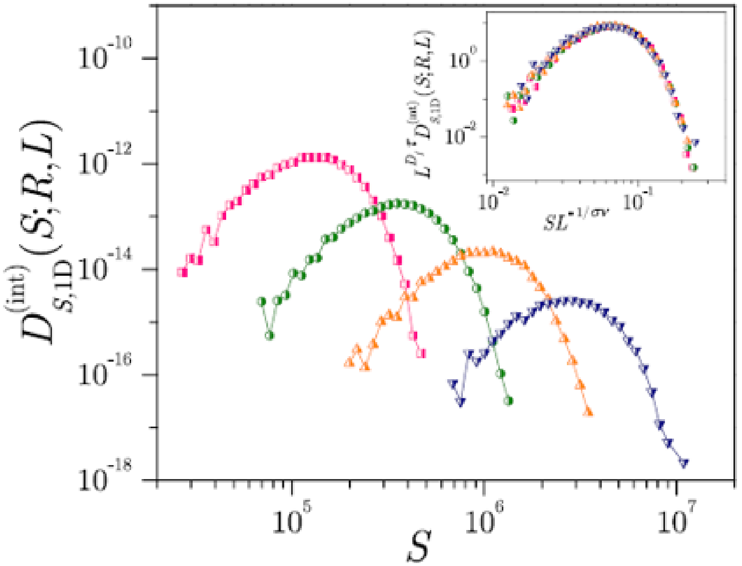
(right insets – distributions normalized to 1: $\tau_\alpha = 1, \tau_{\mathrm{ns}} = 2, D_f = 2.98$)

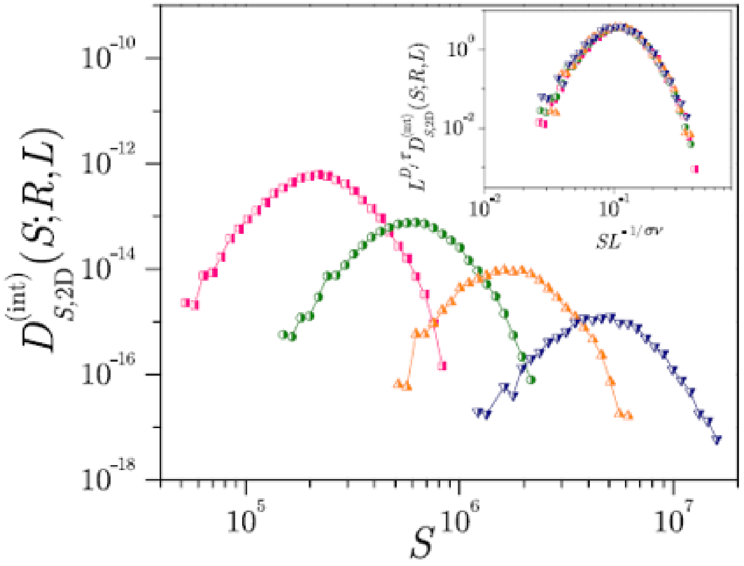
$D_{S,\alpha}^{(\mathrm{int},1)}(S\,;R,L) = L^{-D_f\tau_\alpha}\widetilde{\mathcal{D}}_{S,\alpha}^{(\mathrm{int},1)}(SL^{-D_f}, |r|L^{1/\nu})$

$D_{S,\mathrm{ns}}^{(\mathrm{int},1)}(S\,;R,L) = L^{-D_f\tau_{\mathrm{ns}}}\widetilde{\mathcal{D}}_{S,\mathrm{ns}}^{(\mathrm{int},1)}(SL^{-D_f}, |r|L^{1/\nu})$

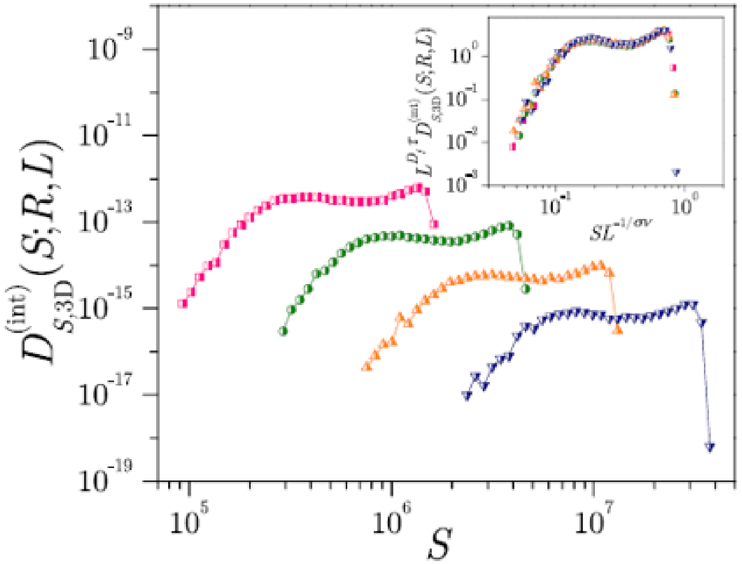
INTEGRATED AVALANCHE SIZE DISTRIBUTIONS

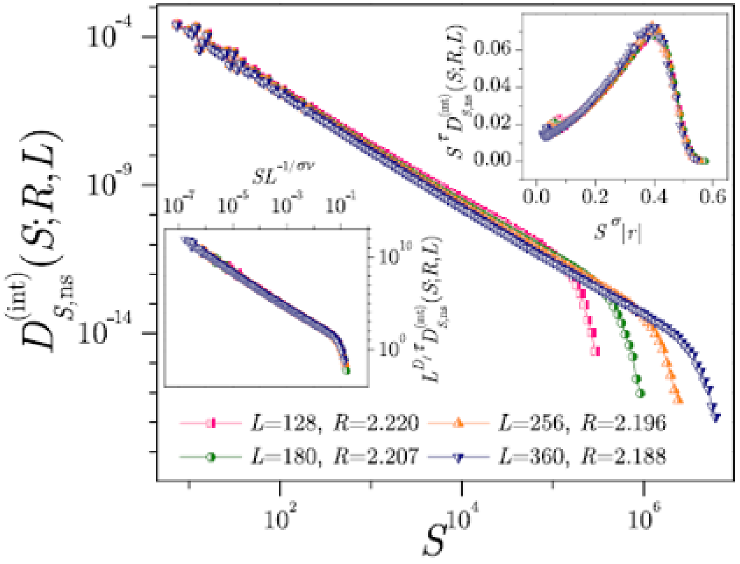
$R_c < R < R_c^{\text{eff}}(L)$





$(\tau' = 2.03, D_f = 2.98, \sigma = 0.24)$

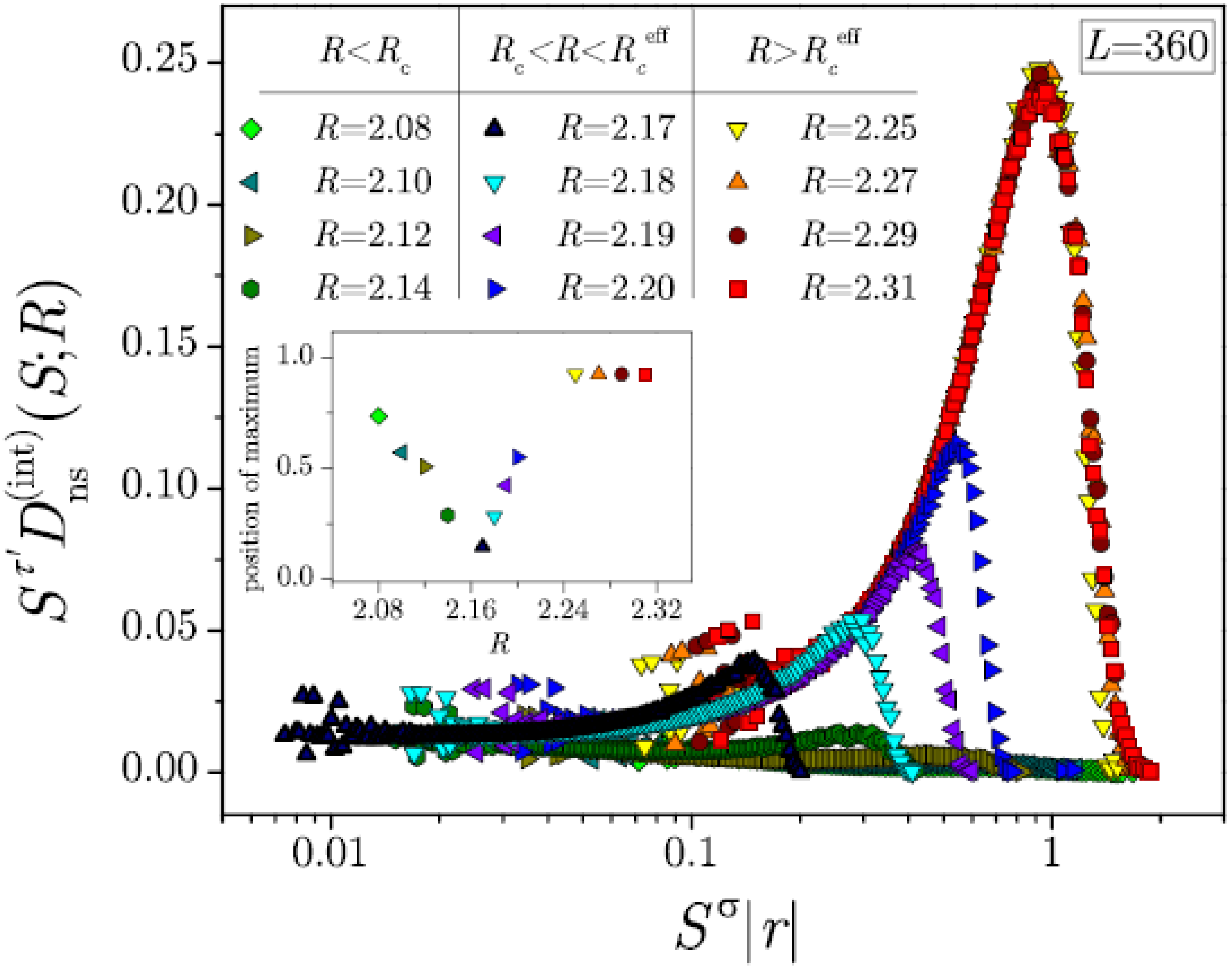




right insets - $D_S^{(\text{int})}(S\,;R,L) = S^{-\tau'}\widetilde{\mathcal{D}}_{S-}^{(\text{int})}(S^{\sigma}|r|\,;L|r|^{\nu})$

$D_{S,\text{ns}}^{(\text{int})}(S\,;R,L) = L^{-D_f\tau'}\widetilde{\mathcal{D}}_{S,\text{ns}}^{(\text{int})}(SL^{-D_f},|r|L^{1/\nu})$ - left inset

INTEGRATED NONSPANNING AVALANCHE SIZE DISTRIBUTIONS

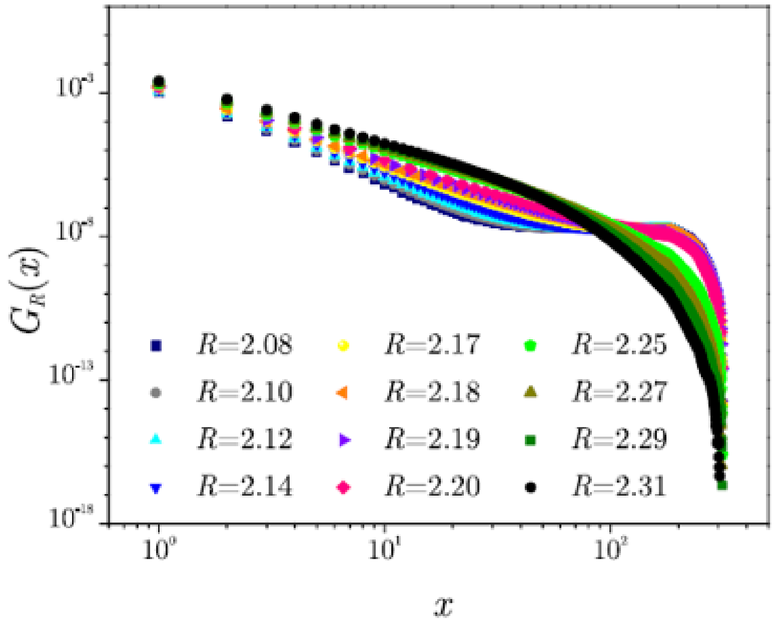


* Scaling collapse of integrated size distributions of nonspanning avalanches for same value of L and a range of disorders covering all domains. Collapse is only possible for $R > R_c^{\mathrm{eff}}(L)$, while in the transitional and domain below R_c, due to $L|r|^{\nu} \neq \mathrm{const}$, collapse cannot be achieved.

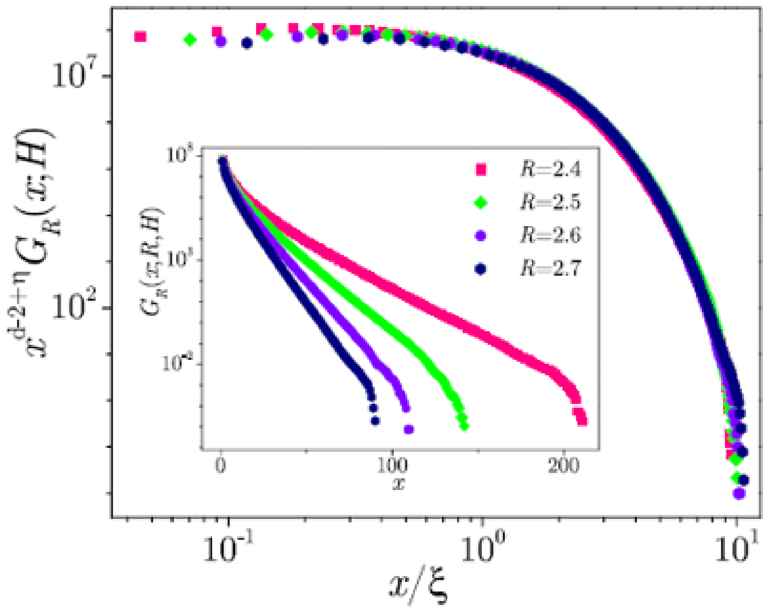
CORRELATION FUNCTIONS

$L = 1024$

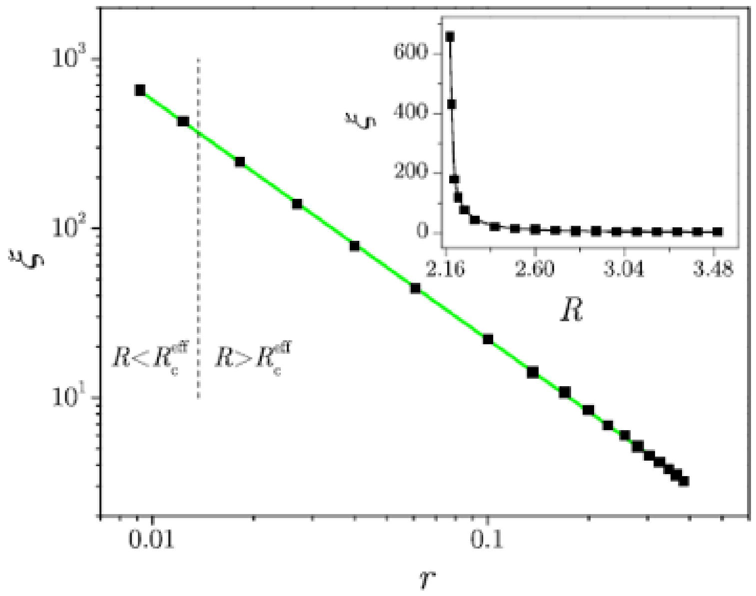
Integrated



Binned $(\Delta H = 0.1)$



Correlation length



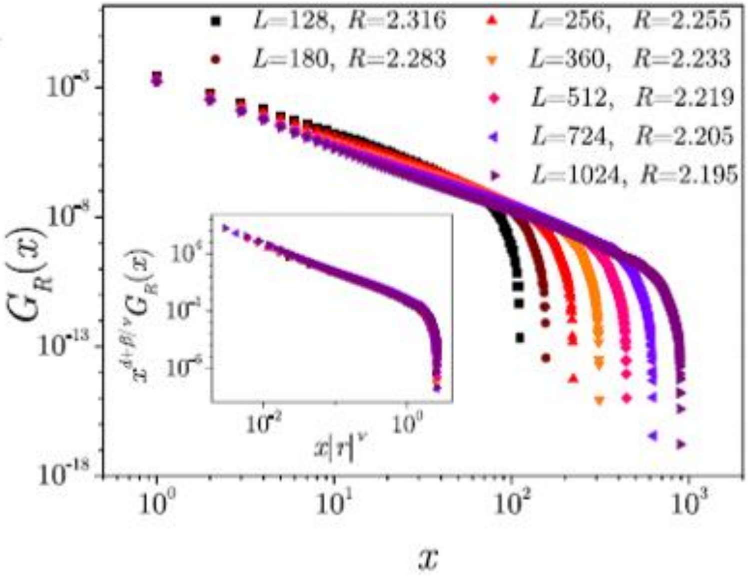
$$G_R(x\,;H) \sim \frac{1}{x^{d-2+\eta}} \mathcal{G}_\pm \left(\frac{x}{\xi(r, h')} \right)$$

$$\xi(r) \sim |r|^{-\nu}$$

$(\eta = 0.53)$

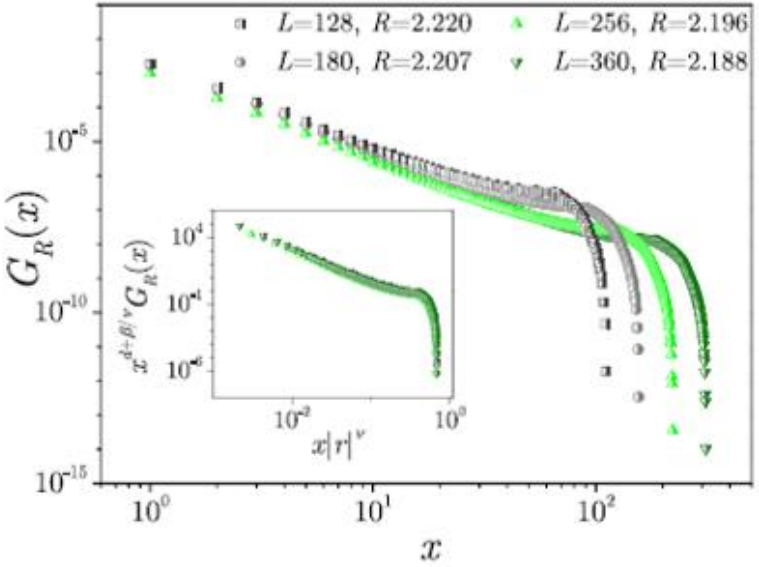
CORRELATION FUNCTIONS

$R > R_c^{\text{eff}}(L)$



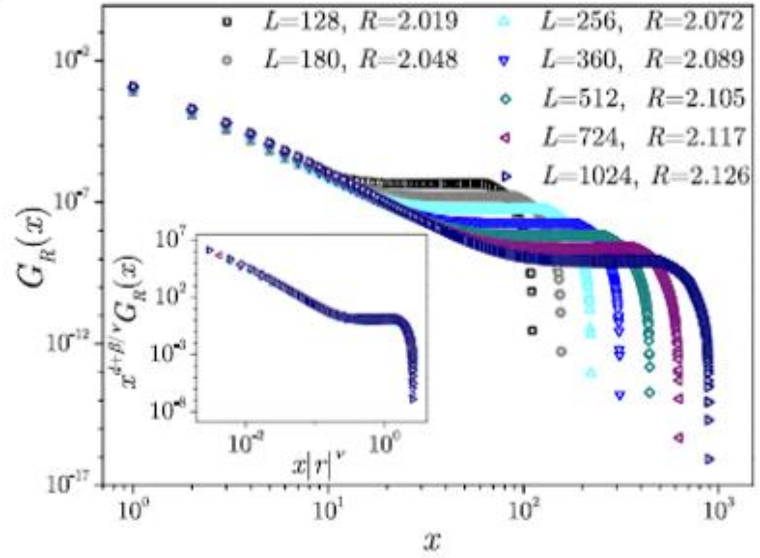
$(Lr^\nu = 2.99)$

$R_c < R < R_c^{\text{eff}}(L)$



$(Lr^\nu = 0.771)$

$R < R_c$



$(L|r|^\nu = 3.005)$

$$G_R(x) \sim \frac{1}{x^{d+\beta/\nu}} \bar{\mathcal{G}}_\pm(x|r|^\nu) \qquad (d + \beta/\nu = 3.05)$$

3D NE-ZT-RFIM EXPONENTS

β	$\beta\delta$	$\tau+\sigma\beta\delta$	σ	θ	η	$1/\sigma\nu z$
0.04 ± 0.03	2.0 ± 0.2	2.02 ± 0.02	0.24 ± 0.02	0.000 ± 0.016	0.53 ± 0.05	1.78 ± 0.04
$1/\nu$	$1/\sigma\nu$	$d+\beta/\nu$	z	$\alpha+\beta\delta/\nu z$	νz	ζ
0.70 ± 0.02	2.99 ± 0.05	3.05 ± 0.01	1.70 ± 0.05	2.75 ± 0.10	2.4 ± 0.1	1.4 ± 0.1
R_c	H_c	M_c	b	b_h	c_h	c_m
2.16 ± 0.02	1.435 ± 0.003	0.30 ± 0.12	0.30 ± 0.06	0.867 ± 0.006	1.255 ± 0.007	20 ± 2

Universal critical exponents (top part) and nonuniversal scaling variables (bottom part) for adiabatically driven 3D NE-ZT-RFIM

CONCLUSION

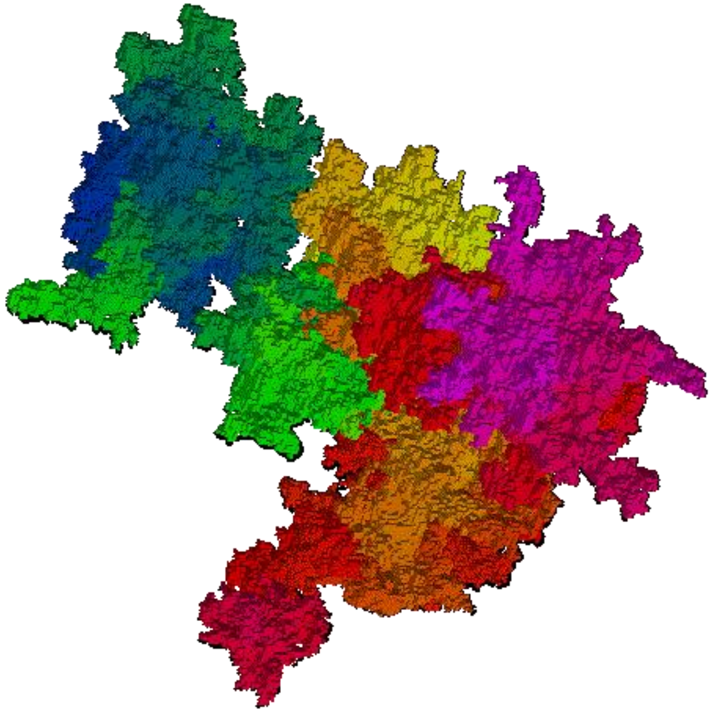
- We show the results of numerical study of scaling domains in the 3D NE-ZT-RFIM on equilateral cubic lattices in adiabatic regime.

- Three different disorder domains identified: above the effective critical disorder $R_c^{\text{eff}}(L)$, below critical disorder R_c and the transitional range of disorder $R_c < R < R_c^{\text{eff}}(L)$ spreading between the first two.

- For disorders above the $R_c^{\text{eff}}(L) + W(L)$ distributions of avalanche size, susceptibility and magnetization, are independent on the lattice size L. System behaves like infinite and follows the scaling predictions valid in the thermodynamic limit. In two remaining ranges of disorder the system behavior is size-dependent permitting collapsing only of distributions with the same value of $Lr^{1/\nu} = const$. The shapes of the scaled avalanche distributions are different in these two regions.

- The value of critical magnetization M_c=0.3 is estimated, an alternative form for the reduced magnetic field h that improves the data collapsing is suggested, and many additional and several alternative analytic forms are proposed along with the values of some of the critical exponents (e.g. $\beta, \beta\delta, \theta, \eta, \zeta$) and of the nonuniversal critical parameter b.

- We believe that the clear recipes introduced here could be beneficial not only for the future studies of the finite RFIM systems, but also in a broader context, e.g. for studies of similar models of finite systems as well as mesoscopic experimental systems.



THANK YOU FOR YOUR ATTENTION

* This work was supported by the Ministry of Education, Science and Technological Development of Republic of Serbia (Agreements No. 451-03-68/2022-14/200162 and 451-03-68/2022-14/200122)