The velocity in static spherical metric of *f(R)* quadratic langrangian gravity

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Abstract. **This paper will consider the behavior of the velocity obtained from the static spherically symmetric metric resulting from the square langragian of f(R) gravity. Metric is given by E. Pachlaner and R. Sexl in their paper [1]. Also It will be shown the expressions for radial and total velocity of cosmos in one and two space dimensions and to discuss their behavior. The metric obtained from the langragian which is different from Einstein's by the square term of the Ricci scalar. The exact equations of fields are too complicated to be solved, it solved in weak field approximation.**

Action integral is: $L=\int d^4x\left[\left(R+\frac{a^2}{6}R^2\right)\sqrt{-g}+2k\Lambda(x)\right]$

Where: $g=g_{00}g_{11}g_{22}g_{33}=-ABC^2r^4sin^2\theta$ **and** k **is Einstein gravitation constant.** a **is constant of dimension of length and** R **is Ricci scalar.** $\Lambda(x)$ **is action integral of matter fields. We used the static spherically symmetric metric proposed by E. Pachlaner and R. Sexl in their paper: On quadratic lagrangians in general relativity, Communication mathematics physics 2 (1966), pp. 165-175.**

$ds^2=A(r)dt^2-B(r)dr^2-C(r)r^2d\theta^2-C(r)r^2sin^2\theta d\varphi^2$,

Where A, B **and** C **is function of radius. Matric elements of metric tensor are given:**

$$A(r)=1-\frac{r_s}{r}-\frac{r_s}{3r}e^{-\frac{r}{a}}$$

$$B(r)=1+\frac{r_s}{r}+\frac{4r_sa^2}{3r^3}-e^{-\frac{r}{a}}\left[\frac{r_s}{r}+\frac{4r_sa}{3r^2}+\frac{4r_sa^2}{3r^3}\right]$$

$$C(r)=1+\frac{r_s}{r}-\frac{2r_sa^2}{3r^3}-e^{-\frac{r}{a}}\left[\frac{r_s}{3r}+\frac{2r_sa}{3r^2}-\frac{2r_sa^2}{3r^3}\right]$$

$$g_{00}=A(r),g_{11}=-B(r),g_{22}=-r^2C,g_{33}=-r^2C(r)sin^2\theta$$

Cristoffel symbols : $\Gamma^{\alpha}_{\varepsilon\nu}=\frac{1}{2}g^{\alpha\sigma}(g_{\sigma\varepsilon,\nu}+g_{\sigma\nu,\varepsilon}-g_{\varepsilon\nu,\sigma})$.

Cristoffel symbols $\Gamma^{\alpha}_{\varepsilon\nu}$ **different from zero for static spherically symmetric metric proposed by E. Pachlaner and R. Sexl are:**

$$\Gamma^1_{00}=\frac{1}{2B}\frac{dA}{dr},\ \Gamma^0_{10}=\Gamma^0_{01}=\frac{1}{2A}\frac{dA}{dr},\Gamma^1_{11}=\frac{1}{2B}\frac{dB}{dr},$$

$$\Gamma^2_{33}=-sin\theta cos\theta,\ \Gamma^3_{23}=\Gamma^3_{32}=ctg\theta$$

$$\Gamma^2_{12}=\Gamma^2_{21}=\Gamma^3_{13}=\Gamma^3_{31}=\frac{1}{2Cr^2}\frac{d[Cr^2]}{dr},\Gamma^1_{22}=-\frac{1}{2B}\frac{d[Cr^2]}{dr}$$

$$\Gamma^1_{33}=-\frac{1}{2B}\frac{d[Cr^2]}{dr}sin^2\theta$$

Geodesic equation for metric proposed by E. Pachlaner and R. Sexl in four dimensions are:

$$\frac{d^2t}{dp^2}+\frac{1}{A}\frac{dA}{dr}\frac{dt}{dp}\frac{dr}{dp}=0$$

$$\frac{d^2r}{dp^2}+\frac{1}{2B}\frac{dA}{dr}\left(\frac{dt}{dp}\right)^2+\frac{1}{2B}\frac{dB}{dr}\left(\frac{dr}{dp}\right)^2-\frac{1}{2B}\frac{d[Cr^2]}{dr}\left(\frac{d\theta}{dp}\right)^2$$
$$-\frac{1}{2B}\frac{d[Cr^2]}{dr}sin^2\theta\left(\frac{d\varphi}{dp}\right)^2=0$$

$$\frac{d^2\theta}{dp^2}+\frac{1}{Cr^2}\frac{d[Cr^2]}{dr}\frac{d\theta}{dp}\frac{dr}{dp}-sin\theta cos\theta\left(\frac{d\varphi}{dp}\right)^2=0$$

$$\frac{d^2\varphi}{dp^2}+\frac{1}{Cr^2}\frac{d[Cr^2]}{dr}\frac{d\varphi}{dp}\frac{dr}{dp}+2ctg\theta\frac{d\varphi}{dp}\frac{d\theta}{dp}=0$$

In one space dimension the geodesic equations are:

$$\frac{d^2t}{dp^2}+\frac{1}{A}\frac{dA}{dr}\frac{dt}{dp}\frac{dr}{dp}=0$$

$$\frac{d^2r}{dp^2}+\frac{1}{2B}\frac{dA}{dr}\left(\frac{dt}{dp}\right)^2+\frac{1}{2B}\frac{dB}{dr}\left(\frac{dr}{dp}\right)^2=0$$

First equation becomes: $\frac{d}{dp}\left[ln\frac{dt}{dp}+lnA\right]=0$

$$\frac{dt}{dp}=\frac{1}{A}$$

Second equation is:

$$2B\frac{d^2r}{dp^2}\frac{dr}{dp}+\frac{dA}{dr}\left(\frac{1}{A}\right)^2\frac{dr}{dp}+\frac{dB}{dr}\left(\frac{dr}{dp}\right)^3=0$$

$$\frac{d}{dp}\left[B\left(\frac{dr}{dp}\right)^2-\frac{1}{A}\right]=0$$

$$\frac{B}{A^2}\left(\frac{dr}{dt}\right)^2-\frac{1}{A}=K=const.\ \mathrm{v}=\frac{dr}{dt}=\sqrt{\frac{A}{B}+K\frac{A^2}{B}}$$

In two space dimensions geodesic equations are:

$$\frac{d^2t}{dp^2}+\frac{1}{A}\frac{dA}{dr}\frac{dt}{dp}\frac{dr}{dp}=0,\rightarrow\frac{dt}{dp}=\frac{1}{A}$$

$$\frac{d^2r}{dp^2}+\frac{1}{2B}\frac{dA}{dr}\left(\frac{dt}{dp}\right)^2+\frac{1}{2B}\frac{dB}{dr}\left(\frac{dr}{dp}\right)^2-\frac{1}{2B}\frac{d[Cr^2]}{dr}\left(\frac{d\varphi}{dp}\right)^2=0$$

$$\frac{d^2\varphi}{dp^2}+\frac{1}{Cr^2}\frac{d[Cr^2]}{dr}\frac{d\varphi}{dp}\frac{dr}{dp}=0\rightarrow\frac{d\varphi}{dp}=\frac{K}{Cr^2}$$

After substitute $\frac{dt}{dp}=\frac{1}{A}$ **and** $\frac{d\varphi}{dp}=\frac{K}{Cr^2}$ **in second equation it gets:**

$$\frac{d^2r}{dt^2}+\left[-\frac{1}{A}\frac{dA}{dr}+\frac{1}{2B}\frac{dB}{dr}\right]\left(\frac{dr}{dt}\right)^2+\frac{1}{2B}\frac{dA}{dr}-\frac{1}{2B}\left(\frac{KA}{Cr^2}\right)^2\frac{d[Cr^2]}{dr}=0$$

where are:

$$f(r)=-\frac{1}{A}\frac{dA}{dr}+\frac{1}{2B}\frac{dB}{dr}\ and\ g(r)=\frac{1}{2B}\frac{dA}{dr}-\frac{1}{2B}\left(\frac{KA}{Cr^2}\right)^2\frac{d[Cr^2]}{dr}$$

Second equation becomes:

$$\frac{d^2r}{dt^2}+f(r)\left(\frac{dr}{dt}\right)^2+g(r)=0\ and\ \frac{dr}{dt}=\frac{1}{\frac{dt}{dr}}\ and\ \frac{d^2r}{dt^2}=-\left(\frac{dr}{dt}\right)^3\frac{d^2t}{dr^2}$$

Substitute first and second derivate in equation, equation becomes:

$$\frac{d^2t}{dr^2}-f(r)\frac{dt}{dr}-g(r)\left(\frac{dt}{dr}\right)^3=0$$

Solution above equation is:

$$\left(\frac{dr}{dt}\right)^2=\frac{C_1-\int 2ge^{\int 2fdr}dr}{e^{\int 2fdr}}$$

Substitute function f(r) and g(r) in the above expression is obtained:

$$v_r=\frac{dr}{dt}=\frac{A}{\sqrt{B}}\sqrt{C_1+\frac{1}{A}-\frac{K^2}{Cr^2}}$$

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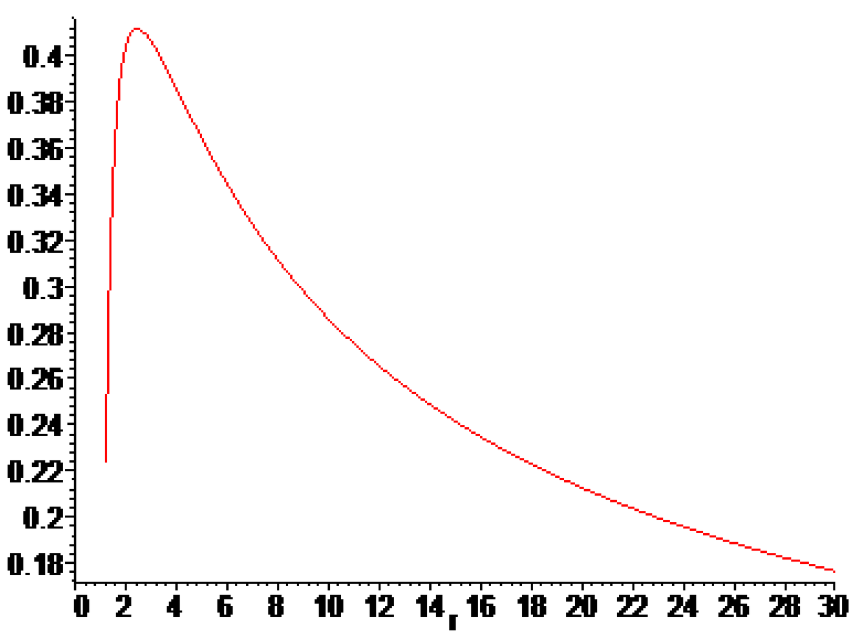
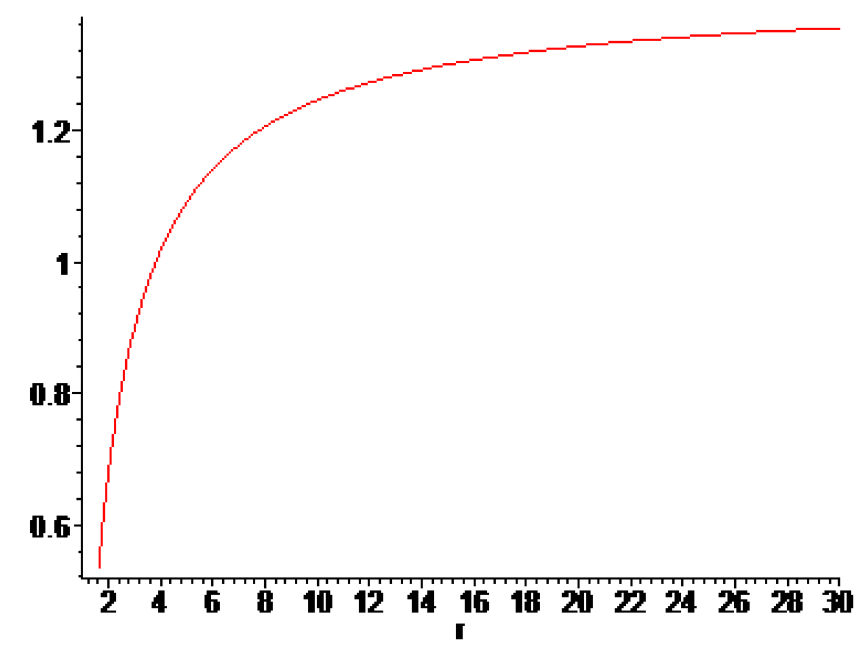
$$v_{\varphi}=r\frac{d\varphi}{dt}=\frac{AK}{Cr}$$

Radial velocity as a function of radius in AU in one dimension

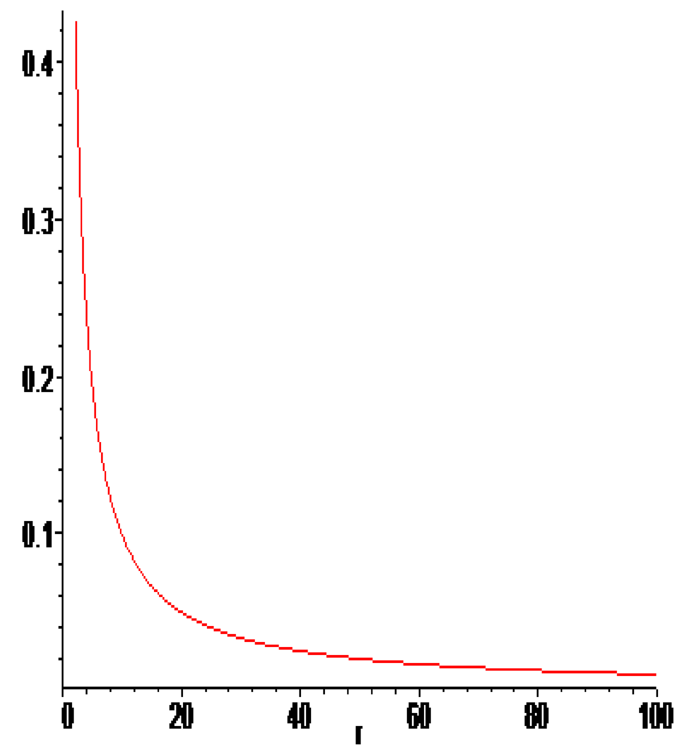


$K=-1,a=1,r_s=1,c=1$

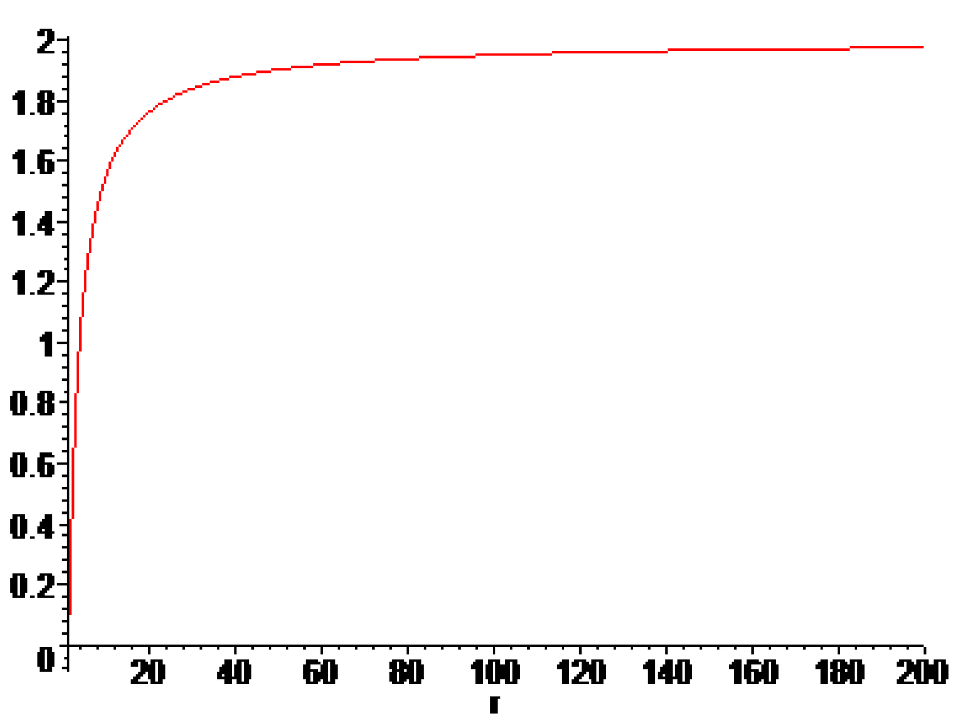


$\mathrm{K}=1,\mathrm{a}=1,\mathrm{r_s}=1,\mathrm{c}=1$

Transverse and radial velocity as a function of radius in AU in two dimensions



$K=1,a=0.1,r_s=0.08,c=1$



$K=1,a=1,r_s=1,c=1$

$C_1=1$

[1] E. Pachlaner and R. Sexl, On quadratic lagrangians in general relativity, Communication mathematics physics 2 (1966), pp. 165-175.